basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

EXAMINATION GUIDELINES

GRADE 12

2021

These guidelines consist of 16 pages.

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1. INTRODUCTION

The *Curriculum and Assessment Policy Statement (CAPS)* for Mathematics outlines the nature and purpose of the subject Mathematics. This guides the philosophy underlying the teaching and assessment of the subject in Grade 12.

The purpose of these Examination Guidelines is to:

- Provide clarity on the depth and scope of the content to be assessed in the Grade 12 National Senior Certificate Examination in Mathematics
- Assist teachers to adequately prepare learners for the examinations

This document deals with the final Grade 12 external examinations. It does not deal in any depth with the school-based assessment (SBA), performance assessment tasks (PATs) or final external practical examinations as these are clarified in a separate PAT document which is updated annually.

These guidelines should be read in conjunction with:

- The *National Curriculum Statement (NCS) Curriculum and Assessment Policy Statement (CAPS)*: Mathematics
- The National Protocol of Assessment: *An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding the National Protocol for Assessment (Grades R–12)*
- National policy pertaining to the programme and promotion requirements of the *National Curriculum Statement, Grades R to 12*

Included in this document is a list of Euclidean Geometry reasons, both in English and Afrikaans, which should be used as a guideline when teaching learners Euclidean Geometry.

The Information Sheet for Paper 1 and 2 is included in this document.

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2. ASSESSMENT IN GRADE 12

All candidates will write two external papers as prescribed.

2.1 Format of Question Papers for Grade 12

Paper	Topics	Duration	Total	Date	Marking
1	Patterns and sequences Finance, growth and decay Functions and graphs Algebra, equations and inequalities Differential Calculus Probability	3 hours	150	October/November	Externally
2	Euclidean Geometry Analytical Geometry Statistics and regression Trigonometry	3 hours	150	October/November	Externally

Questions in both Papers 1 and 2 will assess performance at different cognitive levels with an emphasis on process skills, critical thinking, scientific reasoning and strategies to investigate and solve problems in a variety of contexts.

An Information Sheet is included on p. 15.

2.2 Weighting of Topics per Paper for Grade 12

PAPER 1	MARKS	PAPER 2	MARKS
Algebra, Equations and Inequalities	25	Statistics and Regression	20
Number Patterns	25	Analytical Geometry	40
Functions and Graphs	35	Trigonometry	50
Finance, Growth and Decay	15	Euclidean Geometry	40
Differential Calculus	35		
Counting Principle and Probability	15		
TOTAL	**150**	**TOTAL**	**150**

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2.3 Weighting of Cognitive Levels

Papers 1 and 2 will include questions across four cognitive levels. The distribution of cognitive levels in the papers is given below.

Cognitive Level	Description of Skills to be Demonstrated	Weighting	Approximate Number of Marks in a 150-mark Paper
Knowledge	• Recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers	20%	30 marks
Routine Procedures	• Proofs of prescribed theorems and derivation of formulae • Perform well-known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	35%	52–53 marks
Complex Procedures	• Problems involve complex calculations and/or higher-order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real-world context • Could involve making significant connections between different representations • Require conceptual understanding • Learners are expected to solve problems by integrating different topics.	30%	45 marks
Problem Solving	• Non-routine problems (which are not necessarily difficult) • Problems are mainly unfamiliar • Higher-order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts • Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts.	15%	22–23 marks

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3. ELABORATION OF CONTENT/TOPICS

The purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Integration of topics is encouraged as learners should understand Mathematics as a holistic discipline. Thus questions integrating various topics can be asked.

FUNCTIONS

1. Candidates must be able to use and interpret functional notation. In the teaching process learners must be able to understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$ $f(x)+a$, $af(x)$ and $x=f(y)$ where $a \in R$.
2. Trigonometric functions will ONLY be examined in PAPER 2.

NUMBER PATTERNS, SEQUENCES AND SERIES

1. The sequence of first differences of a quadratic number pattern is linear. Therefore, knowledge of linear patterns can be tested in the context of quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.

FINANCE, GROWTH AND DECAY

1. Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly or semi-annually.
2. With the exception of calculating i in the F_v and P_v formulae, candidates are expected to calculate the value of any of the other variables.
3. Pyramid schemes will NOT be examined in the examination.

ALGEBRA

1. Solving quadratic equations by completing the square will NOT be examined.
2. Solving quadratic equations using the substitution method (k-method) is examinable.
3. Equations involving surds that lead to a quadratic equation are examinable.
4. Solution of non-quadratic inequalities should be seen in the context of functions.
5. Nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

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DIFFERENTIAL CALCULUS

1. The following notations for differentiation can be used: $f'(x)$, D_x, $\frac{dy}{dx}$ or y'.
2. In respect of cubic functions, candidates are expected to be able to:
   - Determine the equation of a cubic function from a given graph.
   - Discuss the nature of stationary points including local maximum, local minimum and points of inflection.
   - Apply knowledge of transformations on a given function to obtain its image.
3. Candidates are expected to be able to draw and interpret the graph of the derivative of a function.
4. Surface area and volume will be examined in the context of optimisation.
5. Candidates must know the formulae for the surface area and volume of the right prisms. These formulae will NOT be provided on the formula sheet
6. If the optimisation question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Candidates will be expected to select the correct formula from this list.

PROBABILITY

1. Dependent events are examinable but conditional probabilities are not part of the syllabus.
2. Dependent events in which an object is not replaced are examinable.
3. Questions that require the learner to count the different number of ways that objects may be arranged in a circle and/or the use of combinations are not in the spirit of the curriculum.
4. In respect of word arrangements, letters that are repeated in the word can be treated as the same (indistinguishable) or different (distinguishable). The question will be specific in this regard.

EUCLIDEAN GEOMETRY AND MEASUREMENT

1. Measurement can be tested in the context of optimisation in calculus and two- and three-dimensional trigonometry.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. The following proofs of theorems are examinable:
   - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
   - The line drawn from the centre of a circle that bisects a chord is perpendicular to the chord;
   - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
   - The opposite angles of a cyclic quadrilateral are supplementary;
   - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
   - A line drawn parallel to one side of a triangle divides the other two sides proportionally;
   - Equiangular triangles are similar.

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4. Corollaries derived from the theorems and axioms are necessary in solving riders:
   - Angles in a semi-circle
   - Equal chords subtend equal angles at the circumference
   - Equal chords subtend equal angles at the centre
   - In equal circles, equal chords subtend equal angles at the circumference
   - In equal circles, equal chords subtend equal angles at the centre.
   - The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
   - If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
   - Tangents drawn from a common point outside the circle are equal in length.
5. The theory of quadrilaterals will be integrated into questions in the examination.
6. Concurrency theory is excluded.

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

ANALYTICAL GEOMETRY

1. Prove the properties of polygons by using analytical methods.
2. The concept of collinearity must be understood.
3. Candidates are expected to be able to integrate Euclidean Geometry axioms and theorems into Analytical Geometry problems.
4. The length of a tangent from a point outside the circle should be calculated.
5. Concepts involved with concurrency will not be examined.

STATISTICS

1. Candidates should be encouraged to use the calculator to calculate standard deviation, variance and the equation of the least squares regression line.
2. The interpretation of standard deviation in terms of normal distribution is not examinable.
3. Candidates are expected to identify outliers intuitively in both the scatter plot as well as the box and whisker diagram.
   In the case of the box and whisker diagram, observations that lie outside the interval (lower quartile – 1,5 IQR; upper quartile + 1,5 IQR) are considered to be outliers. However, candidates will not be penalised if they did not make use of this formula in identifying outliers.

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4. ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

4.1 ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360°.	∠s round a pt **OR** ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB ∥ CD, then the alternate angles are equal.	alt ∠s; AB ∥ CD
If AB ∥ CD, then the corresponding angles are equal.	corresp ∠s; AB ∥ CD
If AB ∥ CD, then the co-interior angles are supplementary.	co-int ∠s; AB ∥ CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the co-interior angles between two lines are supplementary, then the lines are parallel.	coint ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ **OR** sum of ∠s in Δ **OR** Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** ∠∠S
If in two right-angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS

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THEOREM STATEMENT	ACCEPTABLE REASON(S)
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi-circle **OR** diameter subtends right angle **OR** ∠ in $\frac{1}{2}$ ⊙
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi-circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s

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THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ‖m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ‖
The opposite sides of a parallelogram are equal in length.	opp sides of ‖m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ‖m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ‖m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ‖
The diagonals of a parallelogram bisect its area.	diag bisect area of ‖m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

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4.2 ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (AFRIKAANS)

STELLING	AANVAARBARE REDE
LYNE	
Aangrensende hoeke op 'n reguitlyn is supplementêr.	$\angle^e$ op reguit lyn
As aangrensende hoeke supplementêr is, lê die buitenste bene van die hoeke in 'n reguitlyn.	aangr. $\angle^e$ suppl.
Die som van die aangrensende hoeke om 'n punt is 360°.	$\angle^e$ om 'n punt **OF** $\angle^e$ in 'n omw
Regoorstaande hoeke is gelyk.	regoorst. $\angle^e$
As AB ∥ CD, dan is die verwissellende hoeke gelyk	verw. $\angle^e$; AB ∥ CD
As AB ∥ CD, dan is die ooreenkomstige hoeke gelyk.	ooreenk. $\angle^e$; AB ∥ CD
As AB ∥ CD, dan is die ko-binnehoeke supplementêr.	ko-binne $\angle^e$; AB ∥ CD
As die verwissellende hoeke tussen twee lyne gelyk is, dan is die lyne ewewydig.	verw. $\angle^e$ =
As die ooreenkomstige hoeke tussen twee lyne gelyk is, dan is die lyne ewewydig.	ooreenk. $\angle^e$ =
As die ko-binnehoeke tussen twee lyne supplementêr is, dan is die lyne ewewydig.	ko-binne $\angle^e$ suppl.
DRIEHOEKE	
Die binnehoeke van 'n driehoek is supplementêr.	$\angle$ som van Δ **OF** som van $\angle^e$ in Δ **OF** binne $\angle^e$ Δ
Die buitehoek van 'n driehoek is gelyk aan die som van die twee teenoorstaande binnehoeke.	buite $\angle$ van Δ
Die hoeke teenoor die gelyke sye van 'n gelykbenige driehoek, is gelyk.	$\angle^e$ teenoor gelyke sye
Die sye teenoor die gelyke sye van 'n gelykbenige driehoek, is gelyk.	sye teenoor gelyke $\angle^e$
In 'n reghoekige driehoek is die vierkant op die skuinssy gelyk aan die som van die vierkante op die ander twee sye.	Pythagoras **OF** Stelling van Pythagoras
As die vierkant op een sy van 'n driehoek gelyk is aan die som van die vierkante op die ander twee sye, dan is die driehoek reghoekig.	Omgekeerde Pythagoras **OF** Omgekeerde stelling: Pythagoras
As drie sye van een driehoek onderskeidelik gelyk is aan drie sye van 'n ander driehoek, dan is die driehoeke kongruent.	SSS
As twee sye en 'n ingeslote hoek van een driehoek onderskeidelik gelyk is aan twee sye en 'n ingeslote hoek van 'n ander driehoek, dan is die twee driehoeke kongruent.	SHS **OF** S∠S
As twee hoeke en 'n sy van een driehoek onderskeidelik gelyk is aan twee hoeke en 'n ooreenstemmende sy van 'n ander driehoek, dan is die twee driehoeke kongruent.	HHS **OF** ∠∠S
As die skuinssy en 'n reghoeksy van 'n reghoekige driehoek onderskeidelik gelyk is aan die skuinssy en 'n reghoeksy van 'n ander reghoekige driehoek, dan is die twee driehoeke kongruent.	RHS **OF** 90°HS
Die lynstuk wat die middelpunte van twee sye van 'n driehoek verbind, is ewewydig aan en gelyk aan die helfte van die derde sy.	Midpt.-stelling
Die lynstuk wat van die middelpunt van een sy van 'n driehoek ewewydig aan die tweede sy getrek word, halveer die derde sy.	lyn deur midpt ∥ 2de sy
Die lyn ewewydig aan een sy van 'n driehoek verdeel die ander twee sye in eweredige dele.	lyn ∥ een sy van Δ **OF** eweredige stelling; noem ∥ lyne

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STELLING	AANVAARBARE REDE
As 'n lyn twee sye van 'n driehoek in eweredige dele verdeel, is die lyn ewewydig aan die derde sy.	lyn verdeel twee sye van Δ eweredig
As twee driehoeke gelykhoekig is, is hulle ooreenstemmende sye eweredig (en is driehoeke dus gelykvormig).	||| Δ^e **OF** gelykhoekige Δ^e
As die ooreenstemmende sye van twee driehoeke eweredig is, is die driehoeke gelykhoekig (en is driehoeke dus gelykvormig).	Sye van Δ^e eweredig
Driehoeke (of parallelogramme) op dieselfde basis en tussen dieselfde ewewydige lyne is gelyk in oppervlakte.	dieselfde basis ; dieselfde hoogte **OF** gelyke basis ; gelyke hoogte
SIRKELS	
'n Raaklyn aan 'n sirkel is loodreg op die radius/middellyn van die sirkel by die raakpunt.	raaklyn ⊥ radius raaklyn ⊥ middellyn
As 'n lyn loodreg getrek word na die radius/middellyn by die punt waar die radius/middellyn die sirkel ontmoet, dan is die lyn 'n raaklyn aan die sirkel.	Lyn ⊥ Radius **OF** omgekeerde raaklyn ⊥ radius **OF** omgekeerde raaklyn ⊥ middellyn
Die lynstuk wat die middelpunt van 'n sirkel met die middelpunt van 'n koord verbind, is loodreg op die koord.	lyn vanuit midpt na midpt van koord
Die loodlyn uit die middelpunt van 'n sirkel na 'n koord, halveer die koord.	lyn vanuit midpt ⊥ op koord
Die middelloodlyn van 'n koord gaan deur die middelpunt van die sirkel.	middelloodlyn van koord
Die hoek wat 'n koord by die middelpunt van 'n sirkel onderspan, is dubbel die hoek wat dit by enige punt op die omtrek onderspan (aan dieselfde kant van die koord as die midpt).	Midpts∠ = 2 x Omtreks∠
Die omtrekshoek wat deur die middellyn onderspan word, is 90°.	∠ in halwe sirkel **OF** middellyn onderspan regte hoek **OF** ∠ in $\frac{1}{2}\odot$
As 'n koord van 'n sirkel 'n regte hoek by die omtrek onderspan, dan is die koord 'n middellyn.	Koord onderspan 90° **OF** omgekeerde ∠ in halwe sirkel
Hoeke onderspan deur 'n koord van 'n sirkel, aan dieselfde kant van die koord, is gelyk.	$\angle^e$ in dieselfde segment
As 'n lynstuk wat twee punte verbind, gelyke hoeke by twee ander punte aan dieselfde kant van die lynstuk onderspan, dan is die vier punte konsiklies (d.w.s. hulle lê op die omtrek van 'n sirkel).	Lynstuk onderspan gelyke $\angle^e$ **OF** omgekeerde $\angle^e$ in dieselfde segment
Gelyke koorde onderspan gelyke omtrekshoeke.	gelyke koorde; gelyke $\angle^e$
Gelyke koorde onderspan gelyke middelpuntshoeke.	gelyke koorde; gelyke $\angle^e$
Gelyke koorde in gelyke sirkels onderspan gelyke omtrekshoeke.	gelyke sirkels; gelyke koorde ; gelyke $\angle^e$
Gelyke koorde in gelyke sirkels onderspan gelyke middelpuntshoeke.	gelyke sirkels; gelyke koorde ; gelyke $\angle^e$
Die teenoorstaande hoeke van 'n koordvierhoek is supplementêr.	teenoorst. $\angle^e$ van kvh
As die teenoorstaande hoeke van 'n vierhoek supplementêr is, dan is die vierhoek 'n koordevierhoek.	teenoorst. $\angle^e$ van vierhoek is supp **OF** omgekeerde teenoorst $\angle^e$ koordevierhoek
Die buitehoek van 'n koordevierhoek is gelyk aan die teenoorstaande binnehoek.	buite ∠van kvh

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STELLING	AANVAARBARE REDE
As die buitehoek van 'n vierhoek gelyk is aan die teenoorstaande binnehoek, dan is die vierhoek 'n koordevierhoek.	buite ∠van vierhoek = teenoorst. binne ∠ **OF** omgekeerde buite ∠ koordevierhoek
Twee raaklyn wat vanaf dieselfde punt buite 'n sirkel na 'n sirkel getrek word, is ewe lank.	Raaklyne vanuit gemeensk. Punt **OF** raaklyne vanaf dieselfde punt
Die hoek wat gevorm word tussen 'n raaklyn aan 'n sirkel en 'n koord wat vanuit die raakpunt getrek word, is gelyk aan die hoek in die oorstaande segment.	raaklyn koord stelling
As 'n lyn deur die eindpunt van 'n koord 'n hoek met die koord vorm wat gelyk is aan die hoek in die oorstaande segment, dan is die lyn 'n raaklyn aan die sirkel.	∠ tussen lyn en koord **OF** omgekeerde raaklyn koord stelling
VIERHOEKE	
Die som van die binnehoeke van 'n vierhoek is 360°.	som van $\angle^e$ in vierhoek
Die teenoorstaande sye van 'n parallelogram is ewewydig.	teenoorst. sye van ‖m
As die teenoorstaande sye van 'n vierhoek ewewydig is, dan is die vierhoek 'n parallelogram.	teenoorst sye van vierh is ‖
Die teenoorstaande sye van 'n parallelogram is gelyk in lengte.	teenoorst. sye van ‖m
As die teenoorstaande sye van 'n vierhoek gelyk is, dan is die vierhoek 'n parallelogram.	teenoorst sye van vierh = **OF** omgekeerde teenoorst sye van ‖m
Die teenoorstaande hoeke van 'n parallelogram is gelyk.	teenoorst. $\angle^e$ van ‖m
As die teenoorstaande hoeke van 'n vierhoek gelyk is, dan is die vierhoek 'n parallelogram.	teenoorst. $\angle^e$ van vierh = **OF** omgekeerde teenoorst. $\angle^e$ van ‖m
Die hoeklyne van 'n parallelogram halveer mekaar.	hoeklyne van ‖m
As die hoeklyne van 'n vierhoek mekaar halveer, dan is die vierhoek 'n parallelogram.	hoeklyne van vierh halveer mekaar **OF** omgekeerde hoeklyne van ‖m
As een paar teenoorstaande sye van 'n vierhoek gelyk en ewewydig is, dan is die vierhoek 'n parallelogram.	teenoorst. sye = en ‖
Die hoeklyne van 'n parallelogram halveer die oppervlakte van die parallelogram.	hoeklyn van ‖m halveer opp
Die hoeklyne van 'n ruit halveer mekaar reghoekig.	hoeklyne van ruit
Die hoeklyne van 'n ruit halveer die teenoorstaande binnehoeke.	hoeklyne van ruit
Al vier sye van 'n ruit is gelyk.	sye van ruit
Al vier sye van 'n vierkant is gelyk.	sye van vierkant
Die hoeklyne van 'n reghoek is ewe lank.	hoeklyne van reghoek
Die hoeklyne van 'n vlieër sny mekaar reghoekig.	hoeklyne van vlieër
Die een hoeklyn van 'n vlieër halveer die ander hoeklyn.	hoeklyne van vlieër
Een hoeklyn van 'n vlieër halveer die teenoorstaande binnehoeke	hoeklyne van vlieër

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GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2015

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 5 diagram sheets and 1 information sheet.

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QUESTION 1

The table below shows the distances (in kilometres) travelled daily by a sales representative for 21 working days in a certain month.

131	132	140	140	141	144	146
147	149	150	151	159	167	169
169	172	174	175	178	187	189

1.1 Calculate the mean distance travelled by the sales representative. (2)

1.2 Write down the five-number summary for this set of data. (4)

1.3 Use the scaled line on DIAGRAM SHEET 1 to draw a box-and-whisker diagram for this set of data. (2)

1.4 Comment on the skewness of the data. (1)

1.5 Calculate the standard deviation of the distance travelled. (2)

1.6 The sales representative discovered that his odometer was faulty. The actual reading on each of the 21 days was p km more than that which was indicated. Write down, in terms of p (if applicable), the:

1.6.1 Actual mean (1)

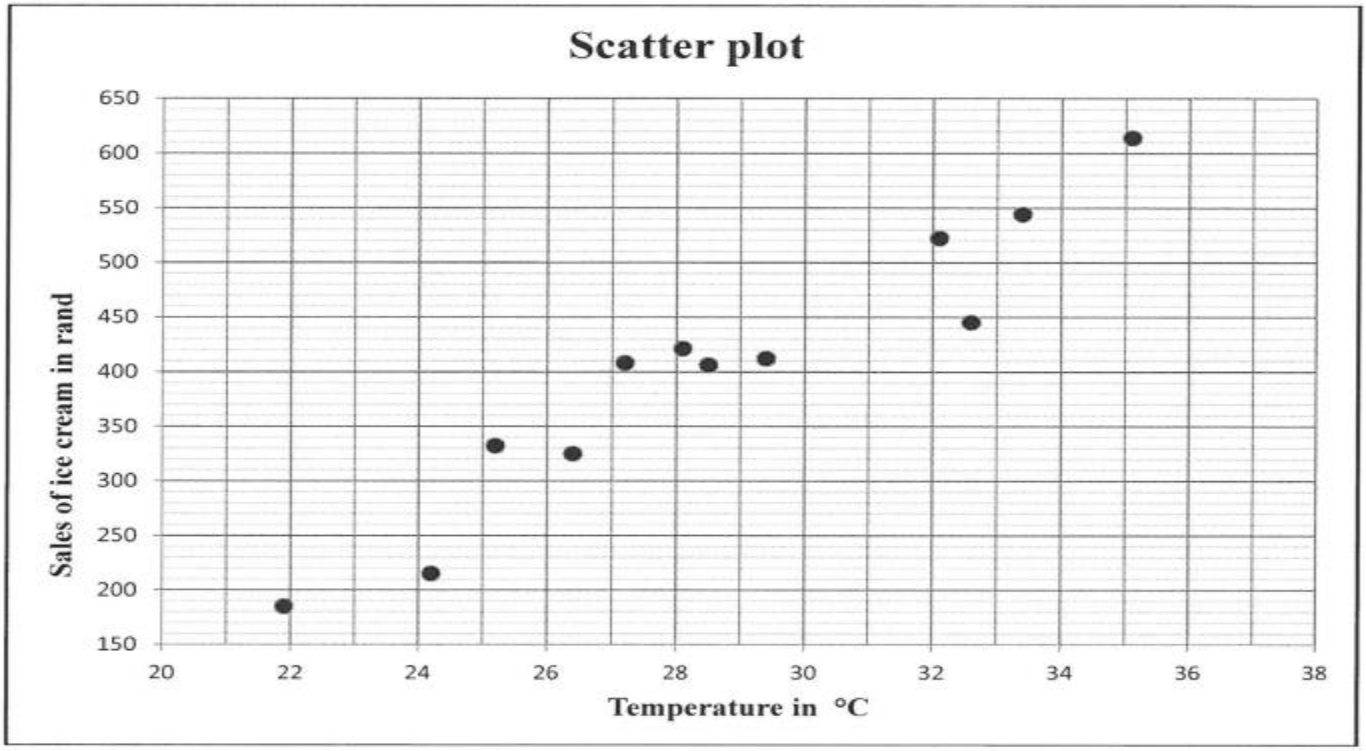
1.6.2 Actual standard deviation (1)
[13]

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QUESTION 2

An ice-cream shop recorded the sales of ice cream, in rand, and the maximum temperature, in °C, for 12 days in a certain month. The data that they collected is represented in the table and scatter plot below.

Temperature in °C	24,2	26,4	21,9	25,2	28,5	32,1	29,4	35,1	33,4	28,1	32,6	27,2
Sales of ice cream in rand	215	325	185	332	406	522	412	614	544	421	445	408



2.1 Describe the influence of temperature on the sales of ice cream in the scatter plot. (1)

2.2 Give a reason why this trend cannot continue indefinitely. (1)

2.3 Calculate an equation for the least squares regression line (line of best fit). (4)

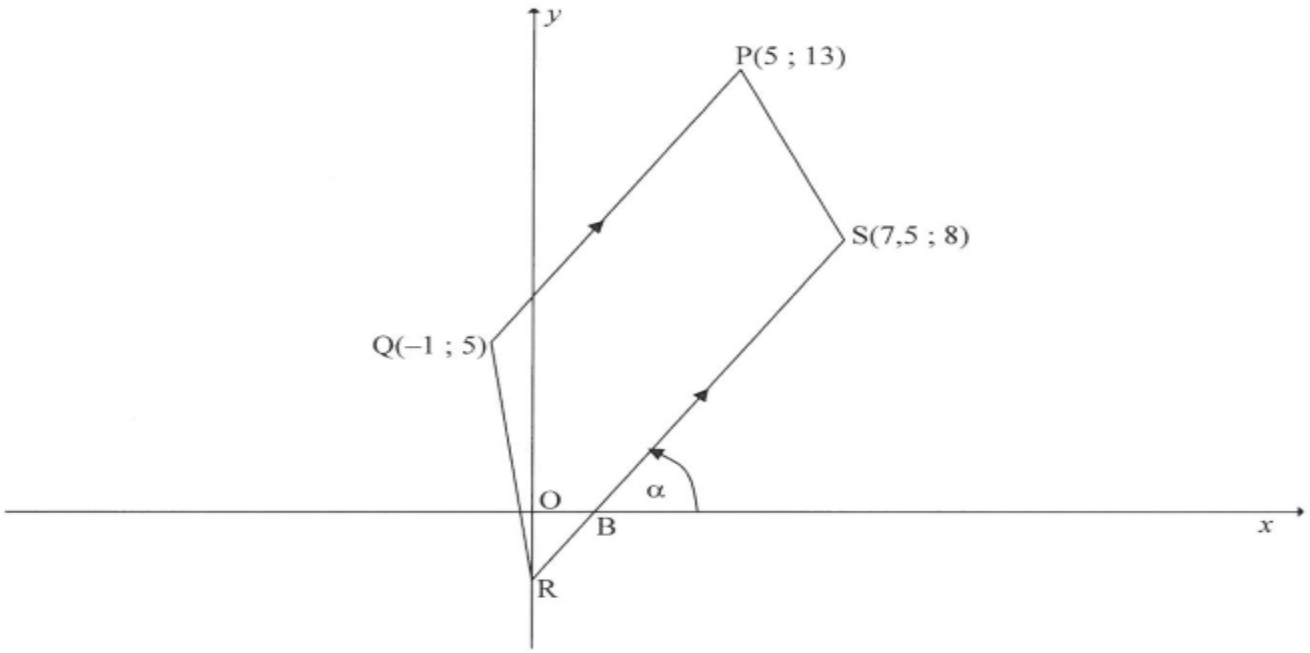
2.4 Calculate the correlation coefficient. (1)

2.5 Comment on the strength of the relationship between the variables. (1)
[8]

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QUESTION 3

In the diagram below points P(5 ; 13), Q(–1 ; 5) and S(7,5 ; 8) are given. SR || PQ where R is the y-intercept of SR. The x-intercept of SR is B. QR is joined.



3.1 Calculate the length of PQ. (3)

3.2 Calculate the gradient of PQ. (2)

3.3 Determine the equation of line RS in the form $ax + by + c = 0$. (4)

3.4 Determine the x-coordinate of B. (2)

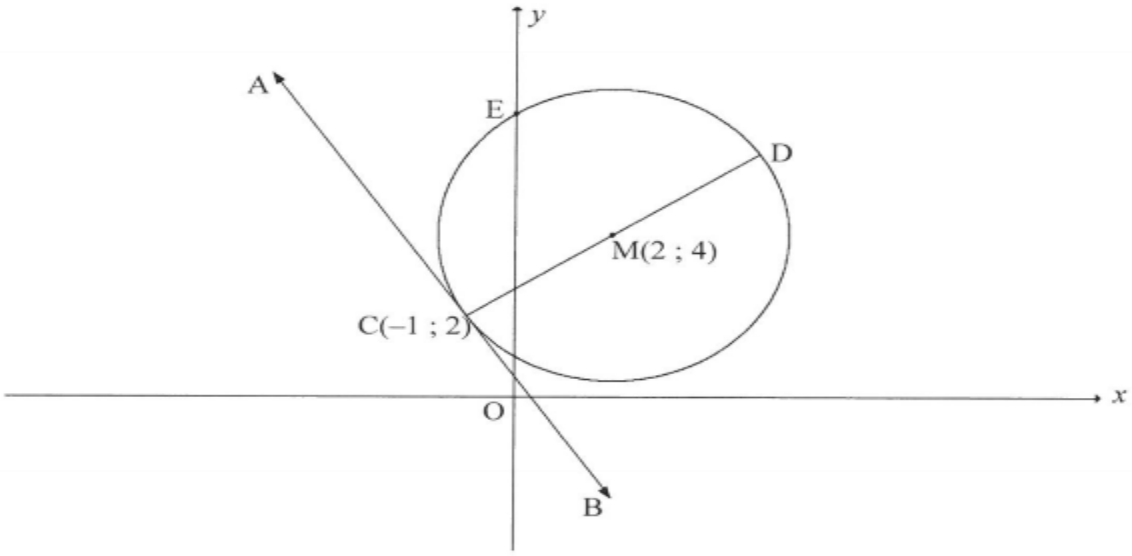
3.5 Calculate the size of $O\hat{R}B$. (3)

3.6 Prove that QBSP is a parallelogram. (4)
[18]

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QUESTION 4

4.1 In the diagram below, the circle centred at M(2 ; 4) passes through C(–1 ; 2) and cuts the y-axis at E. The diameter CMD is drawn and ACB is a tangent to the circle.



4.1.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.1.2 Write down the coordinates of D. (2)

4.1.3 Determine the equation of AB in the form $y=mx+c$. (5)

4.1.4 Calculate the coordinates of E. (4)

4.1.5 Show that EM is parallel to AB. (2)

4.2 Determine whether or not the circles having equations $(x+2)^2+(y-4)^2=25$ and $(x-5)^2+(y+1)^2=9$ will intersect. Show ALL calculations. (6)
[22]

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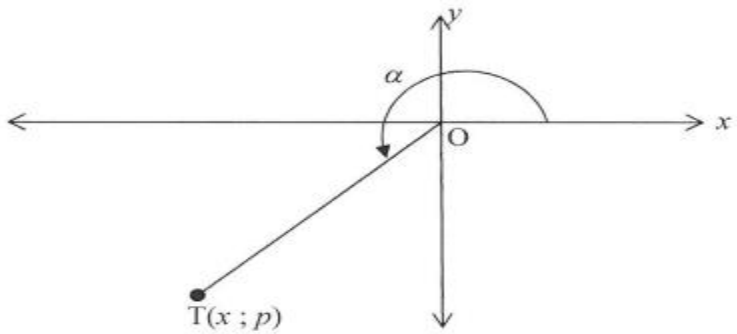
QUESTION 5

5.1 If $x = 3\sin\theta$ and $y = 3\cos\theta$, determine the value of $x^2 + y^2$. (3)

5.2 Simplify to a single term:

$$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$$ (6)

5.3 In the diagram below, $T(x\ ;\ p)$ is a point in the third quadrant and it is given that $\sin\alpha = \frac{p}{\sqrt{1+p^2}}$.



5.3.1 Show that $x = -1$. (3)

5.3.2 Write $\cos(180° + \alpha)$ in terms of p in its simplest form. (2)

5.3.3 Show that $\cos 2\alpha$ can be written as $\frac{1-p^2}{1+p^2}$. (3)

5.4 5.4.1 For which value(s) of x will $\frac{2\tan x - \sin 2x}{2\sin^2 x}$ be undefined in the interval $0° \le x \le 180°$? (3)

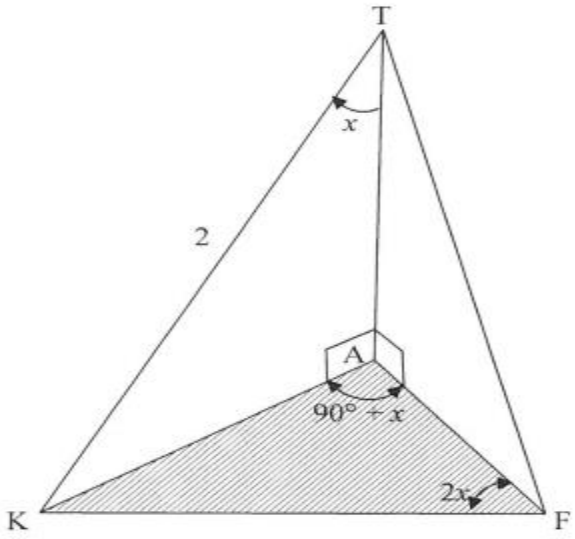
5.4.2 Prove the identity: $\frac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

[26]

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QUESTION 6

6.1 In the figure, points K, A and F lie in the same horizontal plane and TA represents a vertical tower. $A\hat{T}K = x$, $K\hat{A}F = 90° + x$ and $K\hat{F}A = 2x$ where $0° < x < 30°$. TK = 2 units.

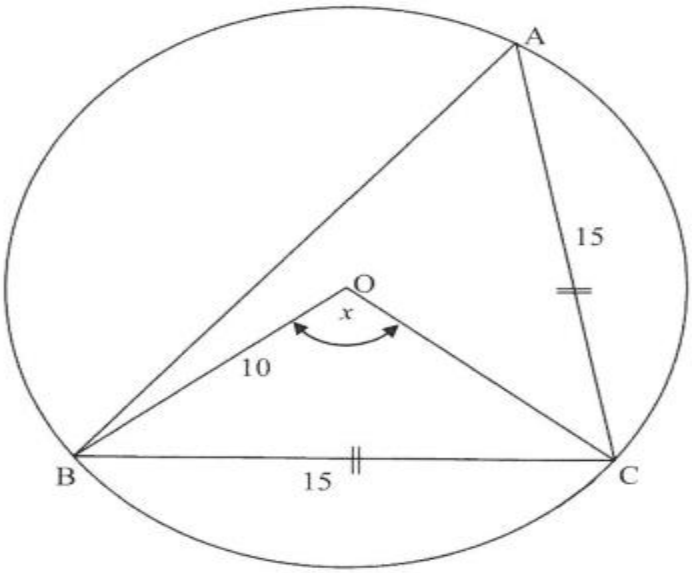


6.1.1 Express AK in terms of $\sin x$. (2)

6.1.2 Calculate the numerical value of KF. (5)

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6.2 In the diagram below, a circle with centre O passes through A, B and C. BC = AC = 15 units. BO and OC are joined. OB = 10 units and $B\hat{O}C = x$.



Calculate:

6.2.1 The size of x (4)

6.2.2 The size of $A\hat{C}B$ (3)

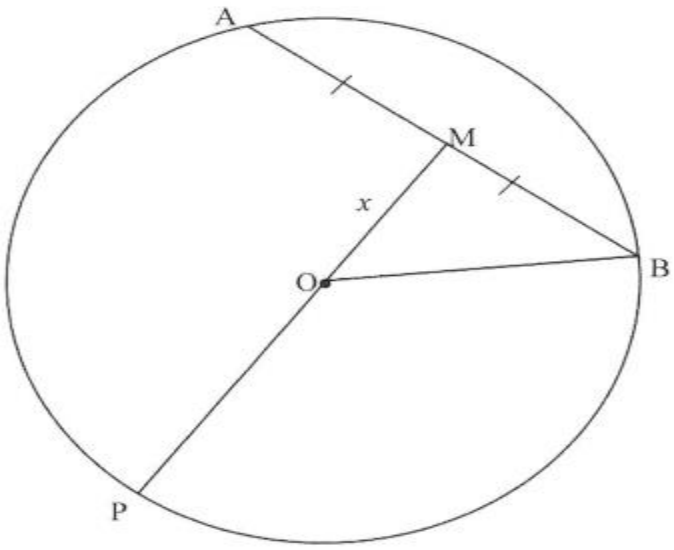
6.2.3 The area of ΔABC (2)
[16]

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GIVE REASONS FOR YOUR ANSWERS IN QUESTIONS 7, 8, 9 AND 10.

QUESTION 7

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. MO is produced to P, where P is a point on the circle. OM = x units, AB = 20 units and $\frac{PM}{OM} = \frac{5}{2}$.



7.1 Write down the length of MB. (1)

7.2 Give a reason why OM ⊥ AB. (1)

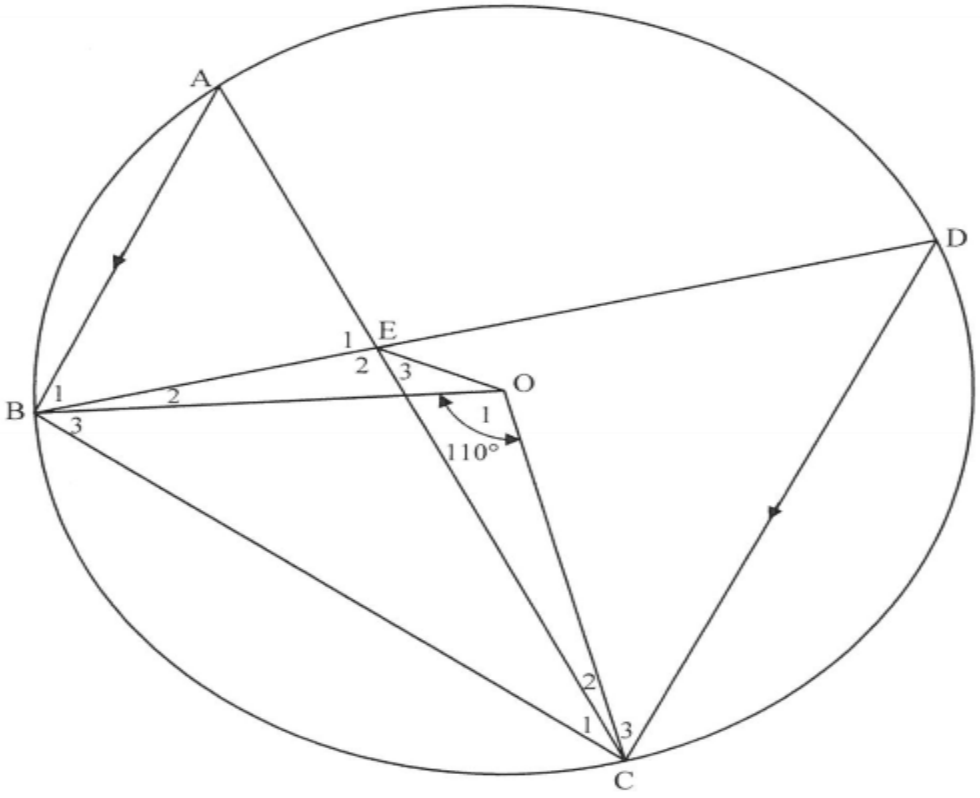
7.3 Show that OP = $\frac{3x}{2}$ units. (2)

7.4 Calculate the value of x. (3)
[7]

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QUESTION 8

In the diagram below, the circle with centre O passes through A, B, C and D.
AB || DC and $B\hat{O}C = 110°$.
The chords AC and BD intersect at E.
EO, BO, CO and BC are joined.



8.1 Calculate the size of the following angles, giving reasons for your answers:

8.1.1 $\hat{D}$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{E}_2$ (4)

8.2 Prove that BEOC is a cyclic quadrilateral. (2)
[10]

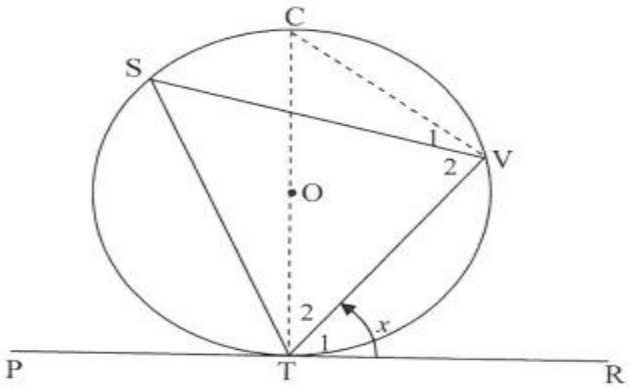
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QUESTION 9

9.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to ... (1)

9.2 In the diagram below the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined.



Given below is the partially completed proof of the theorem that states that $V\hat{T}R = \hat{S}$. **Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 3.**

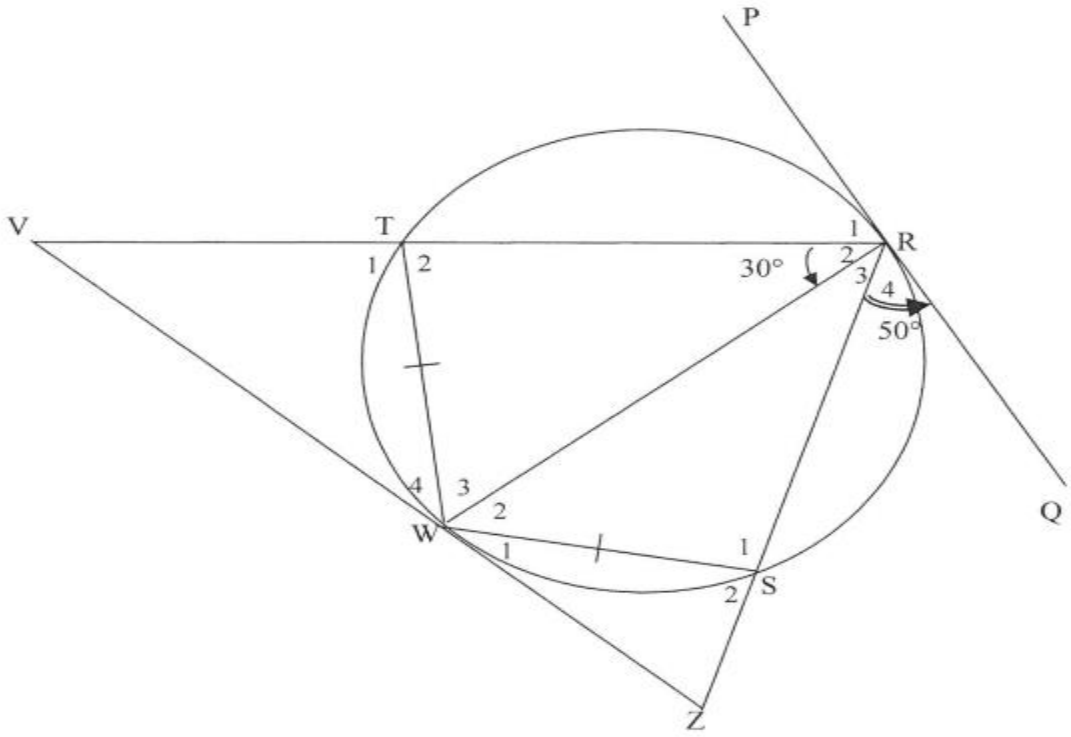
Construction: Draw diameter TC and join CV.

Statement	Reason
Let: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$.....	.....
$\hat{T}_2 = 90° - x$	.....
$\therefore \hat{C} =$	Sum of the angles of a triangle
$\therefore \hat{S} = x$	.....
$\therefore V\hat{T}R = \hat{S}$	

(5)

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9.3 In the figure, TRSW is a cyclic quadrilateral with TW = WS. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent to the circle at R. RW is joined. $\hat{R}_2 = 30°$ and $\hat{R}_4 = 50°$.



9.3.1 Give a reason why $\hat{R}_3 = 30°$. (1)

9.3.2 State, with reasons, TWO other angles equal to 30°. (3)

9.3.3 Determine, with reasons, the size of:

(a) $\hat{S}_2$ (3)

(b) $\hat{V}$ (4)

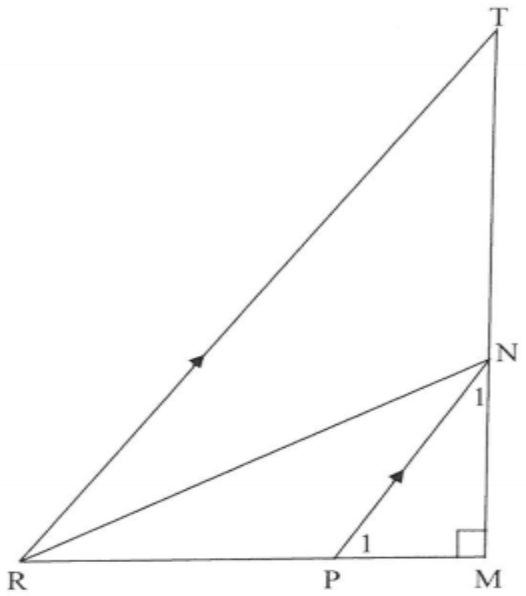
9.3.4 Prove that $WR^2 = RV \times RS$. (5)

[22]

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QUESTION 10

In ΔTRM, $\hat{M} = 90°$. NP is drawn parallel to TR with N on TM and P on RM. It is further given that RT = 3PN.



10.1 Give reasons for the statements below.
Use DIAGRAM SHEET 5.

	Statement	**Reason**
	In ΔPNM and ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	.....
	$\hat{M}$ is common	
10.1.2	∴ ΔPNM ||| ΔRTM	.....

(2)

10.2 Prove that $\frac{PM}{RM} = \frac{1}{3}$. (2)

10.3 Show that $RN^2 - PN^2 = 2RP^2$. (4)
[8]

TOTAL: 150

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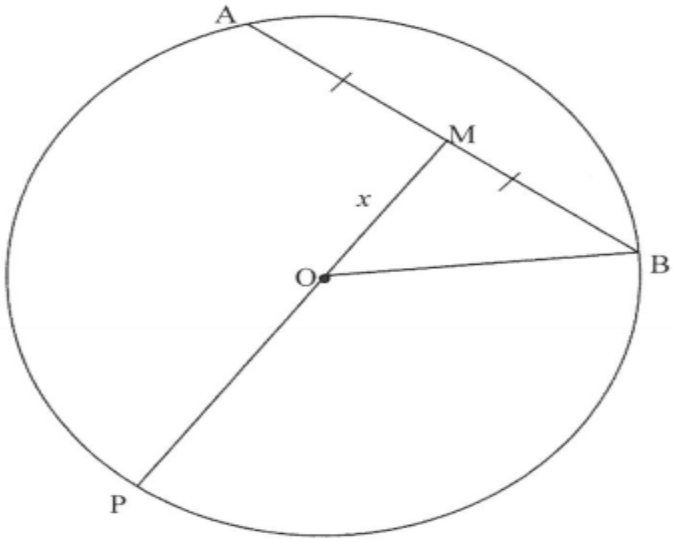
EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 1.3



QUESTION 7



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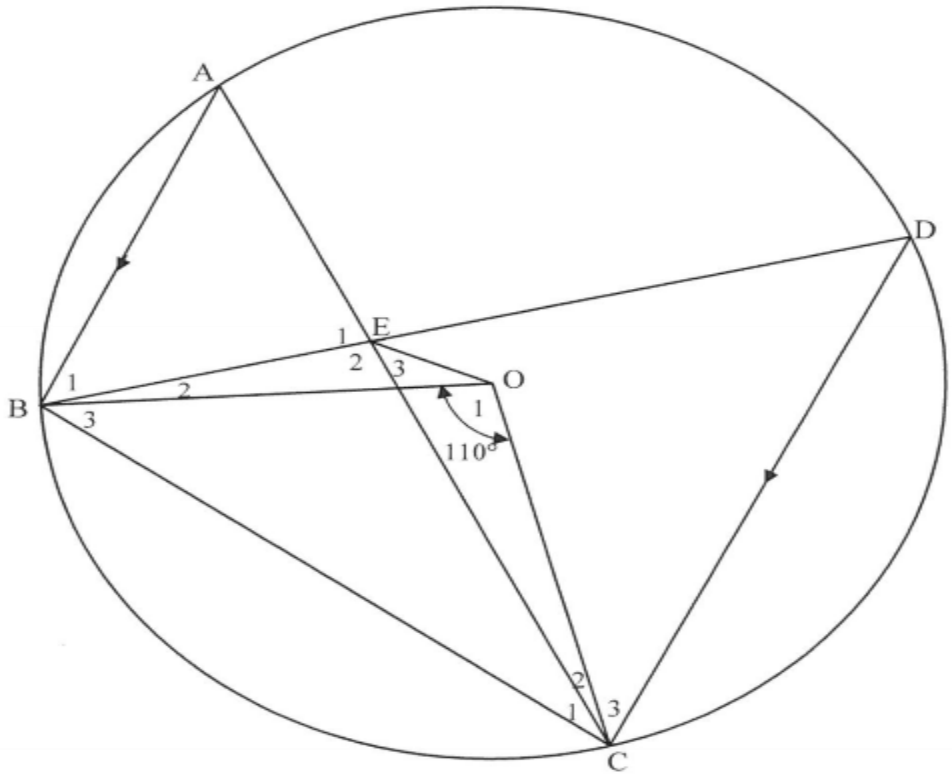
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CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 8



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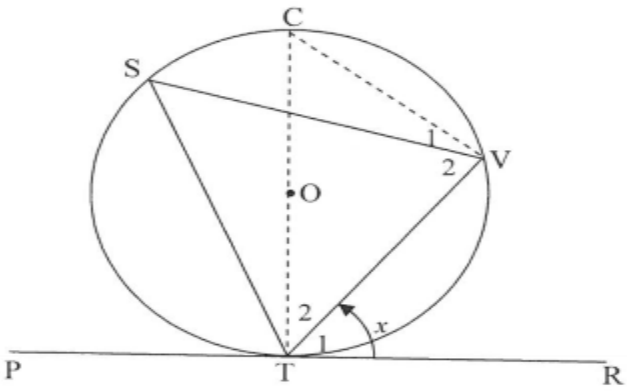
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CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 3

QUESTION 9.2



Construction: Draw diameter CT and join CV.

Statement	Reason
Let: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$.....	.....
$\hat{T}_2 = 90° - x$	.....
$\therefore \hat{C} =$	Sum of the angles of a triangle
$\therefore \hat{S} = x$	.....
$\therefore V\hat{T}R = \hat{S}$	

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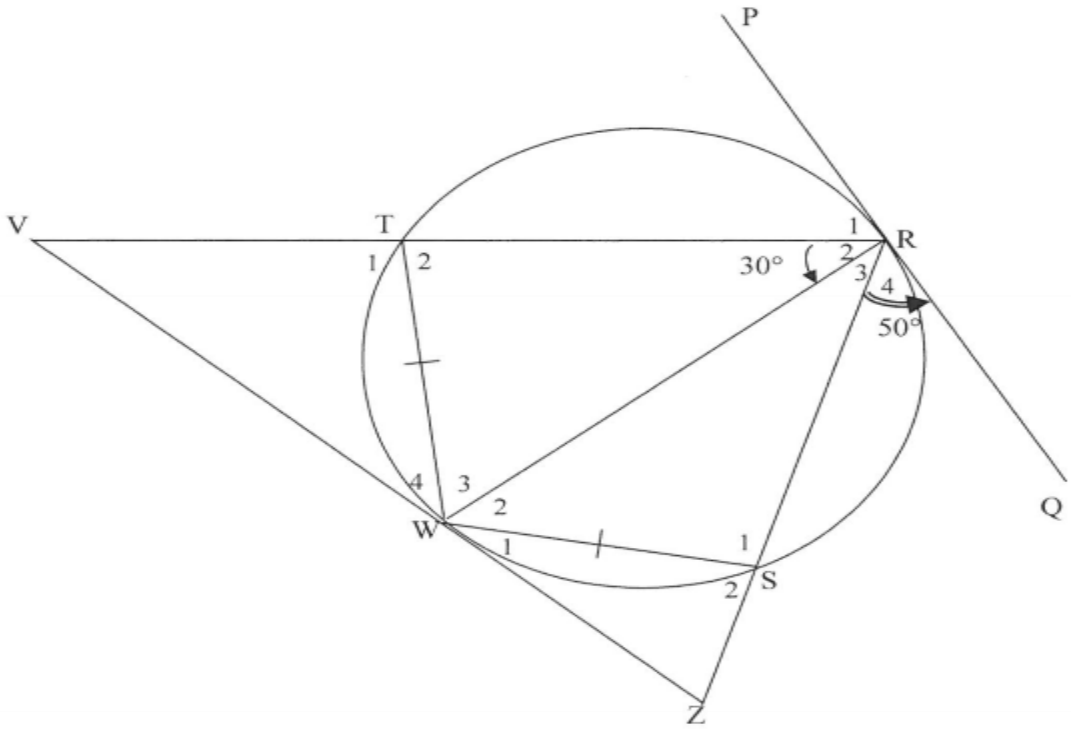
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CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 4

QUESTION 9.3



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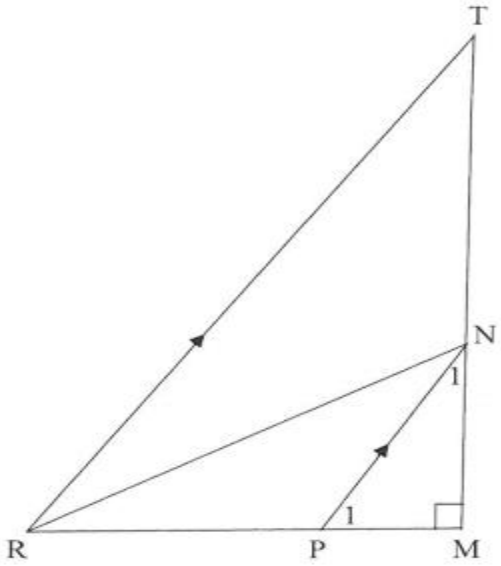
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CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 5

QUESTION 10



10.1

	Statement	Reason
	In ΔPNM and ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	.....
	$\hat{M}$ is common	
10.1.2	∴ ΔPNM ||| ΔRTM	.....

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GRADE/GRAAD 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2015

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 24 pages.
Hierdie memorandum bestaan uit 24 bladsye.

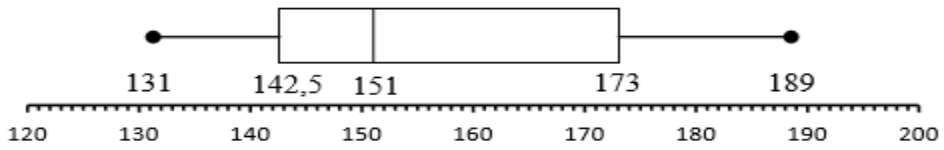
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Please turn over/*Blaai om asseblief*

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:
- *As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.*
- *Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.*

QUESTION/*VRAAG* 1

1.1	$\bar{x} = \frac{3310}{21}$ $= 157{,}62$ Answer only: Full marks *slegs antw: volpunte*	✓ $\frac{3310}{21}$ ✓157,62 (2)
1.2	(131 ; 142,5 ; 151 ; 173 ; 189)	✓ 131 and/ *en* 189 ✓ 142,5 ✓ 173 ✓ 151 (4)
1.3		✓box/*mond* ✓ whiskers/ *snor* (2)
1.4	positively skewed/*positief skeef* **OR/*OF*** skewed to the right/*skeef na regs*	✓ answer/ *antwoord* (1)
1.5	$\sigma = 17{,}27$	✓✓answer/ *antwoord* (2)
1.6.1	$\bar{x} = 157{,}62 + p$	✓ answer (1)
1.6.2	$\sigma = 17{,}27$	✓ answer/ *antwoord* (1) **[13]**

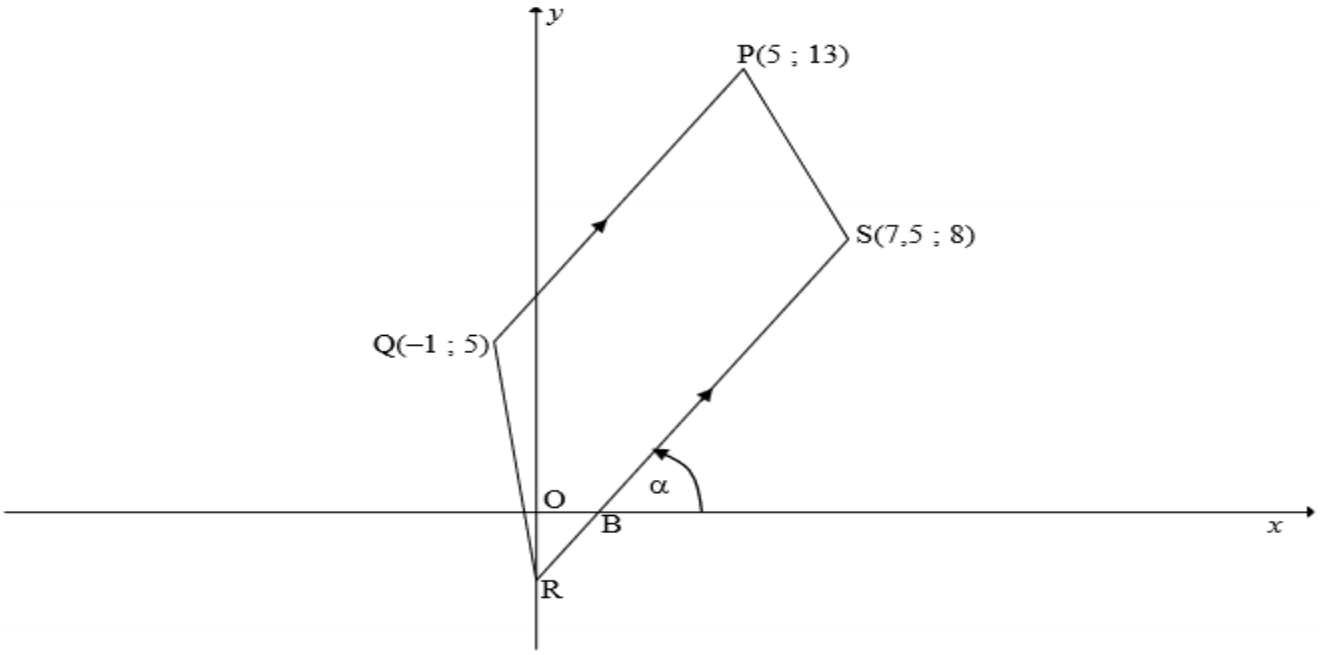
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QUESTION/*VRAAG* 2

2.1	As the temperature increases, the sales of ice-creams increase/*Soos die temperatuur styg, neem die verkope toe.* **OR/*OF*** As the temperature decreases, the sales of ice-creams decrease/*Soos die temperatuur daal, neem die verkope af.*	✓ reason/*rede* (1) ✓ reason/*rede* (1)
2.2	The liveable temperature cannot keep on increasing/*Die leefbare temperatuur kan nie aanhou styg nie.*	✓ reason/*rede* (1)
2.3	$a = -460{,}35$ $b = 30{,}09$ $\hat{y} = 30{,}09x - 460{,}35$ **OR/*OF*** $\hat{y} = -460{,}35 + 30{,}09x$ Answer only: Full marks *slegs antw: volpunte*	✓✓ –460,35 ✓ 30,09 ✓ equation/*vgl* (4)
2.4	$r = 0{,}96$	✓ 0,96 (1)
2.5	There is a very strong positive relationship (correlation)/*Daar is 'n baie sterk positiewe verband (korrelasie).*	✓ very strong/*baie sterk* (1) **[8]**

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QUESTION/VRAAG 3



3.1	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5+1)^2 + (13-5)^2}$ $= 10$	✓ use of distance formula/*gebruik afstandformule* ✓ correct subst into form/*korrekte subst in formule* ✓ 10 (3)
3.2	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{13-5}{5-(-1)}$ $= \frac{8}{6} = \frac{4}{3}$ Answer only: Full marks *slegs antw: volpunte*	✓ correct subst into gradient formula/ *korrekte subst in gradiëntformule* ✓ gradient/*gradiënt* (2)

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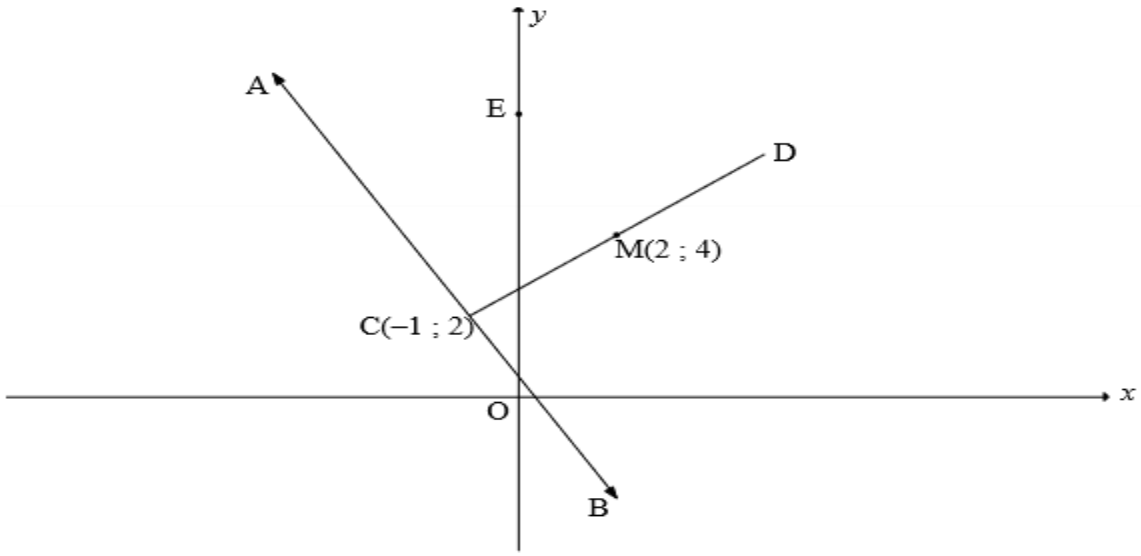
Please turn over/*Blaai om asseblief*

3.3	Equation of line RS/*Vgl van lyn RS*: $m_{RS} = m_{PQ} = \frac{4}{3}$ (= gradients, || lines/=*gradiënte, || lyne*) $y = mx + c$ $8 = \frac{4}{3}\left(\frac{15}{2}\right) + c$ $c = -2$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y - 8 = \frac{4}{3}\left(x - \frac{15}{2}\right)$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$	✓ $m_{RS} = \frac{4}{3}$ ✓ subst of S(7,5 ; 8) and *m* into eq /*subst van S(7,5 ; 8) en m in vgl* ✓ value of *c* /*waarde van c* or/of st form/*st vorm* ✓ equation/*vgl* (4)
3.4	B is the *x*-intercept of/*is die x-afsnit van* $y = \frac{4}{3}x - 2$ $0 = \frac{4}{3}x - 2$ $4x - 6 = 0$ $x = \frac{3}{2}$ **OR/*OF*** $4x - 3(0) - 6 = 0$ $4x - 6 = 0$ $x = \frac{3}{2}$	✓ $y = 0$ ✓ $x = \frac{3}{2}$ (2)
3.5	$\tan \alpha = \frac{4}{3}$ $\alpha = 53{,}13° = \text{O}\hat{\text{B}}\text{R}$ (vert opp ∠s/*regoorst ∠e*) $\text{OR}\hat{\text{B}} = 180° - (90° + 53{,}13°)$ (∠s of Δ/*∠e van Δ*) $= 36{,}87°$	✓ $\tan \alpha = \frac{4}{3}$ ✓ 53,13° ✓ 36,87° (3)
3.6	$\text{BS} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8 - 0)^2}$ $= 10$ PQ || BS and/*en* PQ = BS PQBS = parallelogram (1 pair opp sides = and ||/*1 pr tos sye =en* ||) **OR/*OF*** midpoint of/*midpt van* QS: $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ midpoint of/*midpt van* PB: $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ PQBS = parallelogram (diags bisect each other/*hoekl halv mekaar*) **OR/*OF***	✓ correct subst into form/*korrekte subst in formule* ✓ BS = 10 ✓ BS = PQ ✓ reason/*rede* (4) ✓ $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right)$ ✓ $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right)$ ✓ $\left(\frac{13}{4}; \frac{13}{2}\right)$ ✓ reason/*rede* (4)

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	$m_{QB} = \frac{5-0}{-1-1,5} = \frac{5}{-2,5} = -2$ $m_{PS} = \frac{13-8}{5-7,5} = \frac{5}{-2,5} = -2$ $m_{QB} = m_{PS}$ $\therefore$ QB || PS PQ ∥ BS PQBS = parallelogram (both pairs opp sides ||/*beide pr tos sye* ||) **OR/*OF*** $BS = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{\left(\frac{15}{2}-\frac{3}{2}\right)^2+(8-0)^2}$ $\therefore$ PQ = BS $= 10$ $QB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(-1-1,5)^2+(5-0)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{5\sqrt{5}}{2}$ or 5,59 $PS = \sqrt{(5-7,5)^2+(13-8)^2} = \sqrt{(2,5)^2+(5)^2} = \frac{\sqrt{125}}{2}$ or 5,59 QB = PS PQBS = parallelogram (both pairs opp sides =/ *beide pr tos sye* =)	✓ m_{QB} ✓ m_{PS} ✓ QB || PS ✓ reason/*rede* (4) ✓ correct subst into form/*korrekte subst in formule* ✓ PQ = 10 ✓ QB = PS ✓ reason/*rede* (4) **[18]**

QUESTION/VRAAG 4



4.1.1	Radius $= \sqrt{(2+1)^2+(4-2)^2}$ $r = \sqrt{13}$ Equation of circle/*vgl van sirkel*: $(x-2)^2+(y-4)^2=13$ **OR/*OF*** $(x-2)^2+(y-4)^2=r^2$ $(-1-2)^2+(2-4)^2=r^2$ $r^2=13$ $\therefore (x-2)^2+(y-4)^2=13$	✓ $\sqrt{(2+1)^2+(4-2)^2}$ or/*of* $\sqrt{13}$ ✓ $(x-2)^2+(y-4)^2$ ✓ 13 (3) ✓ $(x-2)^2+(y-4)^2$ ✓ $(-1-2)^2+(2-4)^2$ ✓ 13 (3)
4.1.2	At/*by* D: $\frac{-1+x_D}{2}=2$ and/*en* $\frac{2+y_D}{2}=4$ $-1+x_D=4$ $2+y_D=8$ $x_D=5$ $y_D=6$ D(5 ; 6) **OR/*OF*** By inspection/*deur inspeksie*: D(5 ; 6)	✓ x – value/*waarde* ✓ y - value/*waarde* (2) ✓ x - value/*waarde* ✓ y - value/*waarde* (2)

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4.1.3	$m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $m_{AB} \times m_{MC} = -1$ (Tangent ⊥ radius/*raaklyn ⊥ radius*) $m_{AB} = -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ **OR/OF** $y = mx + c$ $y - 2 = -\frac{3}{2}(x+1)$ $2 = -\frac{3}{2}(-1) + c$ $y = -\frac{3}{2}x + \frac{1}{2}$ $y = -\frac{3}{2}x + \frac{1}{2}$	✓ $m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ ✓ $m_{AB} \times m_{MC} = -1$ ✓ $m_{AB} = -\frac{3}{2}$ ✓ subst m and $(-1 ; 2)$ into eq /*subst m en (–1 ; 2) in vgl* ✓ eq in standard form/ *vgl in st vorm* (5)
4.1.4	At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $(y-4)^2 = 9$ $y - 4 = \pm 3$ $y = 7$ or $y = 1$ E(0 ; 7) **OR/*OF*** At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $4 + y^2 - 8y + 16 = 13$ $y^2 - 8y + 7 = 0$ $(y-7)(y-1) = 0$ $y = 7$ or $y = 1$ E(0 ; 7)	✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ y - values/*waardes* ✓ E(0 ; 7) (4) ✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ y - values/*waardes* ✓ E(0 ; 7) (4)
4.1.5	$m_{EM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-7}{2-0}$ $= -\frac{3}{2}$ $m_{AB} = -\frac{3}{2}$ $\therefore$ EM $\|\|$ AB $(m_{EM} = m_{AB})$	✓ $m_{EM} = -\frac{3}{2}$ ✓ reason/*rede* (2)

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4.2	The centres of the circles are / *Die middelpunte van die sirkels is* P(–2 ; 4) and / *en* Q(5 ; –1) $QP^2 = (-2-5)^2 + (4-(-1))^2$ $QP = \sqrt{74} \approx 8{,}60$ units $r_M + r_p = 5 + 3$ $= 8$ $\therefore r_M + r_p < QP$ ∴ The two circles do not intersect/*Die twee sirkels sny nie*	✓ both centres/*albei Midpte* ✓ QP ✓ correct subst into form/*korrekte subst in formule* ✓ distance between 2 centres/*afstand tussen 2 midpte* ✓✓ $r_M + r_p < QP$ (6) **[22]**

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QUESTION/VRAAG 5

5.1	$x^2 + y^2$ $=(3\sin\theta)^2 + (3\cos\theta)^2$ $=9\sin^2\theta + 9\cos^2\theta$ $=9(\sin^2\theta + \cos^2\theta)$ $=9(1)$ $=9$	✓ simpl/*vereenv* ✓ CF/*GF* = 9 ✓ answer/*antw* (3)
5.2	$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $\sin(180° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $= (\sin x)(-\sin x) - (-\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$	✓ $\sin(540° - x) = \sin x$ ✓ $\sin(-x) = -\sin x$ ✓ $\cos(180° - x) = -\cos x$ ✓ $\sin(90° + x) = \cos x$ ✓ $-\sin^2 x + \cos^2 x$ ✓ $\cos 2x$ (6)
5.3.1	$OT = \sqrt{x^2 + p^2}$ $\sin\alpha = \frac{y_T}{OT}$ $= \frac{p}{\sqrt{x^2 + p^2}}$ $\frac{p}{\sqrt{x^2 + p^2}} = \frac{p}{\sqrt{1 + p^2}}$ $x^2 = 1$ $x = -1$ **OR/*OF*** (P lies in 3rd quadrant) $x^2 + y^2 = r^2$ $x^2 + p^2 = \left(\sqrt{1 + p^2}\right)^2$ $x^2 + p^2 = 1 + p^2$ $x^2 = 1$ $x = -1$ (P lies in 3rd quadrant)	✓ $OT = \sqrt{x^2 + p^2}$ ✓ $\sin\alpha = \frac{y_T}{OT}$ ✓ $x^2 = 1$ (3) ✓ $x^2 + y^2 = r^2$ ✓ subst ✓ $x^2 = 1$ (3)
5.3.2	$\cos(180° + \alpha)$ $= -\cos\alpha$ $= -\left(\frac{-1}{\sqrt{1 + p^2}}\right)$ $= \frac{1}{\sqrt{1 + p^2}}$	✓ $-\cos\alpha$ ✓ answer/*antw* (2)

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5.3.3	$\cos 2\alpha$ $= \cos^2\alpha - \sin^2\alpha$ $= \left(\frac{-1}{\sqrt{1+p^2}}\right)^2 - \left(\frac{p}{\sqrt{1+p^2}}\right)^2$ $= \frac{1}{1+p^2} - \frac{p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓✓ squaring each term/*kwadreer elke term* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 1 - 2\sin^2\alpha$ $= 1 - 2\left(\frac{p}{\sqrt{1+p^2}}\right)^2$ $= 1 - 2\left(\frac{p^2}{1+p^2}\right)$ $= 1 - \frac{2p^2}{1+p^2}$ $= \frac{1+p^2-2p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓ squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 2\cos^2\alpha - 1$ $= 2\left(\frac{-1}{\sqrt{1+p^2}}\right)^2 - 1$ $= 2\left(\frac{1}{1+p^2}\right) - 1$ $= \frac{2}{1+p^2} - 1$ $= \frac{2-1-p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓ squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)

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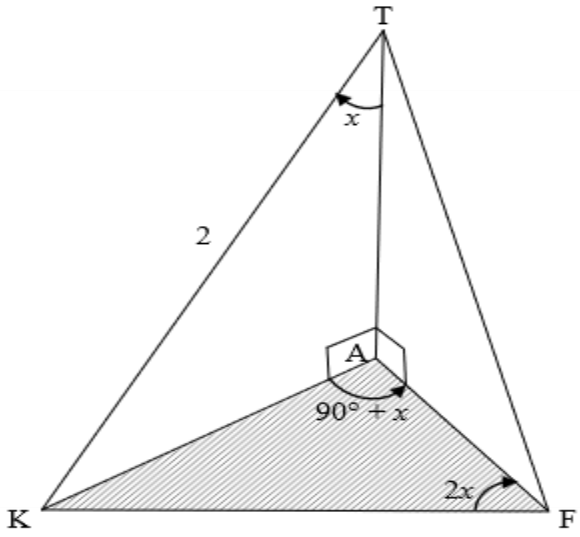
5.4.1	The identity is undefined for/*die identiteit is ongedefinieerd as*: $2\sin^2 x = 0$ $\therefore$ $\underline{\sin x = 0:}$ $x = 0° ; 180°$ or/*of* $\underline{\tan x = \infty:}$ $x = 90°$ $\therefore$ $x = 0° ; 90° ; 180°$	✓ $x = 0°$ ✓ $x = 90°$ ✓ $x = 180°$ (3)
5.4.2	LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ $= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x}$ $= \left(\dfrac{2\sin x - 2\sin x\cos^2 x}{\cos x}\right) \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{2\sin x(1-\cos^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{2\sin x(\sin^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK **OR/*OF*** LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ $= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x} \times \dfrac{\cos x}{\cos x}$ $= \dfrac{2\sin x - 2\sin x\cos^2 x}{2\sin^2 x\cos x}$ $= \dfrac{2\sin x(1-\cos^2 x)}{2\sin^2 x\cos x}$ $= \dfrac{2\sin x.\sin^2 x}{2\sin^2 x\cos x}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK	✓ $\dfrac{\sin x}{\cos x}$ ✓ $2\sin x.\cos x$ ✓ simplify numerator/*vereenv teller* ✓ factorising/*fakt* ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to/*vereenv na* $\dfrac{\sin x}{\cos x}$ (6) ✓ $\dfrac{\sin x}{\cos x}$ ✓ $2\sin x.\cos x$ ✓ simpl/*vereenv* ✓ factorising/*fakt* ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to /*vereenv na* $\dfrac{\sin x}{\cos x}$ (6) **[26]**

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QUESTION/VRAAG 6

6.1



6.1.1	In ΔTAK: $\frac{AK}{KT} = \sin K\hat{T}A$ $AK = KT.\sin x$ $= 2\sin x$ **OR/*OF*** $\frac{\sin K\hat{T}A}{AK} = \frac{\sin K\hat{A}T}{KT}$ $\frac{\sin 90^\circ}{2} = \frac{\sin x}{AK}$ $AK = 2\sin x$	✓ correct trig ratio/ *korrekte trigverh.* ✓answer/*antw* (2) ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓answer/*antw* (2)

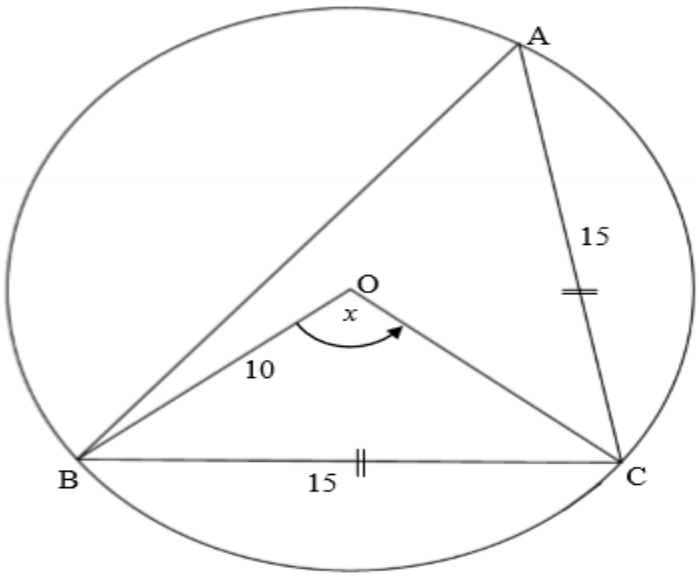
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6.1.2	In ΔAKF: $\frac{KF}{\sin K\hat{A}F} = \frac{AK}{\sin A\hat{F}K}$ $\frac{KF}{\sin(90° + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK.\sin(90° + x)}{\sin 2x}$ $= \frac{2\sin x.\cos x}{2\sin x.\cos x}$ $= 1$ **OR/*OF*** In ΔAKF: $\frac{KF}{\sin K\hat{A}F} = \frac{AK}{\sin A\hat{F}K}$ $\frac{KF}{\sin(90° + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK.\sin(90° + x)}{\sin 2x}$ $= \frac{AT.\tan x.\cos x}{2\sin x.\cos x}$ $= \frac{2\cos x.\frac{\sin x}{\cos x}.\cos x}{2\sin x.\cos x}$ $= 1$ $\cos x = \frac{AT}{2}$ $\therefore AT = 2\cos x$	✓ using sine rule/ *gebruik sin-reël* ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ $\sin(90° + x) = \cos x$ ✓ $2\sin x.\cos x$ ✓ 1 (5) ✓ using sine rule/ *gebruik sin-reël* ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ $\sin(90° + x) = \cos x$ ✓ $2\sin x.\cos x$ ✓ 1 (5)

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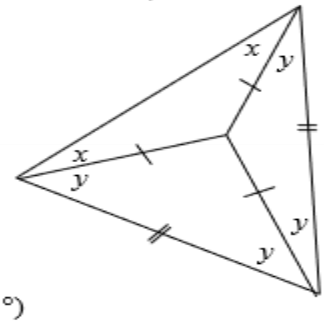
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6.2



6.2.1	In ΔBOC: $BC^2 = BO^2 + CO^2 - 2.BO.\ CO\ .\cos x$ $15^2 = 10^2 + 10^2 - 2(10)(10).\ \cos x$ $200\cos x = -25$ $\cos x = -0{,}125$ $x = 180° - 82{,}82°$ $= 97{,}18°$ **OR/*OF*** Draw a line OD⊥ BC: BD = DC (line from centre ⊥ on chord) ΔOBD ≡ ΔOCD (90°; h; s) $\sin\frac{x}{2} = \frac{7{,}5}{10}$ $\frac{x}{2} = 48{,}59°$ $\therefore\ x = 97{,}18°$	✓ using cosine rule/ *gebruik cos-reël* ✓correct subst/ *korrekte subst* ✓ $\cos x = -0{,}125$ ✓97,18° (4) ✓ S/R ✓correct ratio/ *korrekte verh* ✓ value of/*waarde van* $\frac{x}{2}$ ✓97,18° (4)

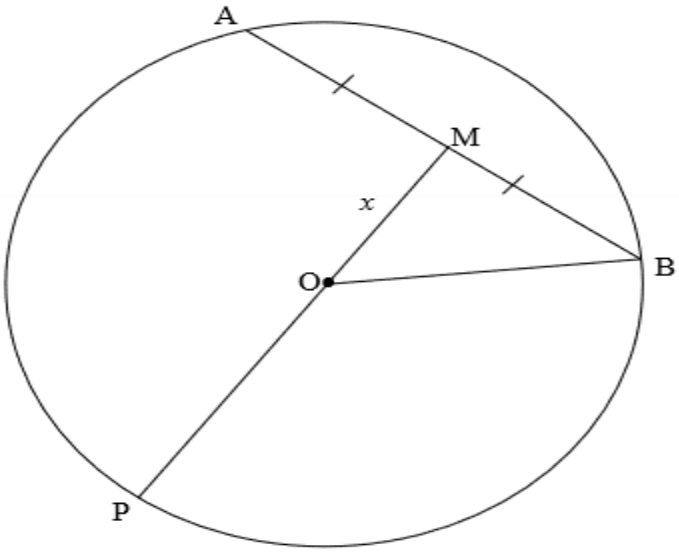
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6.2.2	$B\hat{A}C = 48{,}59°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠omt*) $A\hat{B}C = B\hat{A}C = 48{,}59°$ (∠'s opp equal sides/∠*e teenoor = sye*) $\therefore A\hat{C}B = 82{,}82°$ (sum of ∠s of Δ/*som van ∠e van Δ*) **OR/*OF*** $A\hat{C}B = \frac{1}{2}A\hat{O}B$ (∠at centre = 2× ∠at circle) (∠ *by midpt=2×∠omt*) $= \frac{1}{2}[360° - 2(97{,}18°)]$ $= 82{,}82°$ **OR/*OF*** $O\hat{C}B = \frac{1}{2}(180° - 97{,}18°)$ (∠'s opp equal sides; sum of ∠s of Δ) $= 41{,}41°$ (∠*e teenoor = sye; som van ∠e van Δ*)  $A\hat{C}B = 2(41{,}41°)$ $= 82{,}82°$	✓ S ✓ S ✓82,82° (3) ✓ S ✓ S ✓82,82° (3) ✓ S ✓ S ✓82,82° (3)
6.2.3	Area/*Oppervlakte* ΔABC $= \frac{1}{2}(BC)(AC)\sin A\hat{C}B$ $= \frac{1}{2}(15)(15)(\sin 82{,}82°)$ $= 111{,}62 \text{ cm}^2$	✓correct subst into area rule/*korrekte subst in opp-reël* ✓ $111{,}62 \text{ cm}^2$ (2) **[16]**

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QUESTION/VRAAG 7



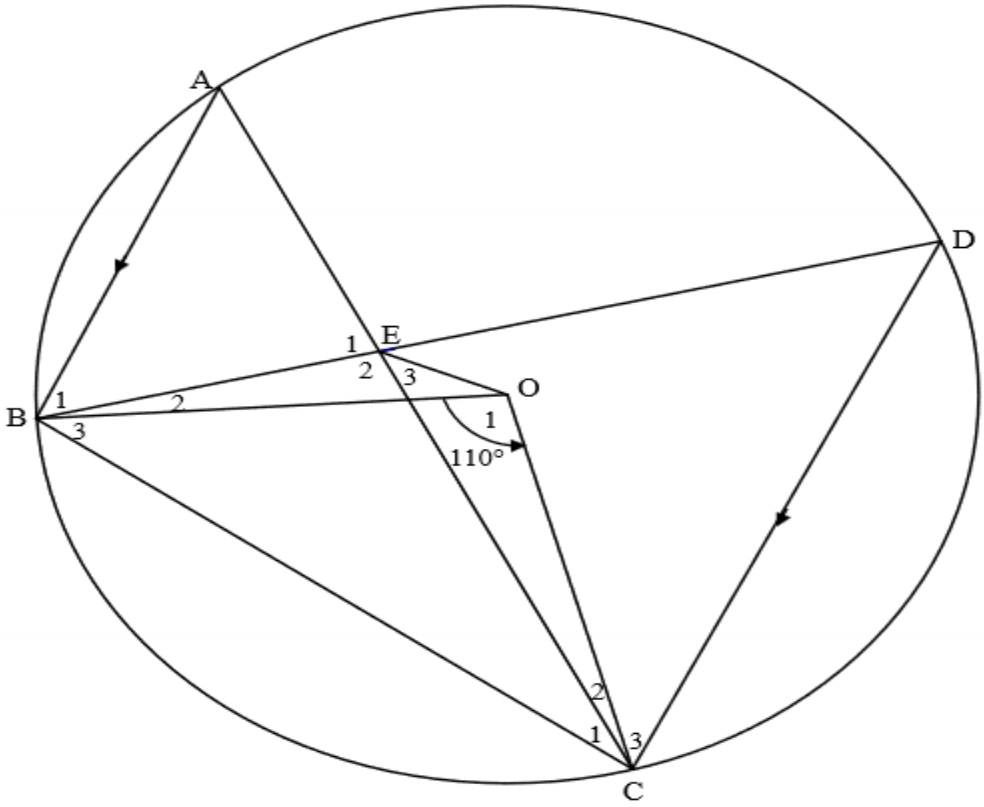
7.1	MB = 10 cm	✓ answer/*antw* (1)
7.2	line from centre to midpoint of chord is perpendicular to chord/*lyn vanaf midpt na midpt van koord is loodreg op koord* **OR/*OF*** line from centre bisects chord/*lyn vanaf midpt halveer koord*	✓ answer/*antw* (1) ✓ answer/*antw* (1)
7.3	$\frac{MP}{OM}=\frac{5}{2}$ $\frac{x+OP}{x}=\frac{5}{2}$ $2x+2OP=5x$ $OP=\frac{3x}{2}$ **OR/*OF*** $\frac{OP}{OM}=\frac{3}{2}$ $OP=\frac{3x}{2}$	✓ $\frac{x+OP}{x}=\frac{5}{2}$ ✓ $OP=\frac{3x}{2}$ (2) ✓ $\frac{OP}{OM}=\frac{3}{2}$ ✓ $OP=\frac{3x}{2}$ (2)

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7.4	$OM^2 + MB^2 = OB^2$ $x^2 + 10^2 = \left(\frac{3x}{2}\right)^2$ $4x^2 + 400 = 9x^2$ $5x^2 = 400$ $x^2 = 80$ $x = 8{,}94$ or $4\sqrt{5}$ or $\sqrt{80}$	✓ subst into/*subst* Pythagoras ✓ $4x^2 + 400 = 9x^2$ ✓ answer/*antw* (3) [7]

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QUESTION/VRAAG 8



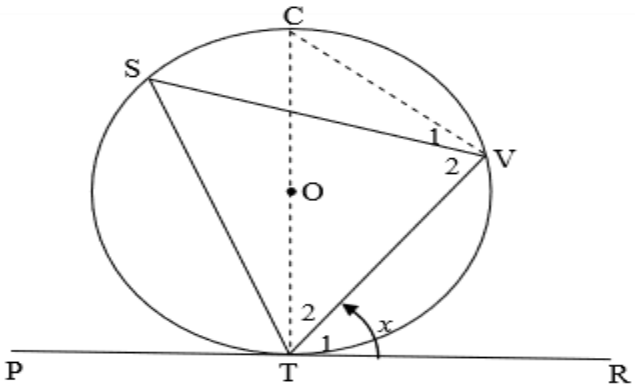
8.1.1	$\hat{D} = \frac{1}{2}\hat{O}_1 = 55°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠by omt*)	✓S ✓R (2)
8.1.2	$\hat{A} = \frac{1}{2}\hat{O}_1 = 55°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠by omt*) **OR/*OF*** $\hat{A} = \hat{D} = 55°$ (∠s in same segment/∠*e in dieselfde segment*)	✓S ✓R (2) ✓S ✓R (2)
8.1.3	$\hat{B}_1 = \hat{D} = 55°$ (alternate ∠s/*verwisselende*; AB || DC) $\hat{E}_2 = \hat{B}_1 + \hat{A}$ (ext∠ of Δ = sum of opp∠^{s}/*buite∠ v Δ=som v tos∠e*) $= 55° + 55°$ $\hat{E}_2 = 110°$	✓S ✓R ✓R ✓ answer/*antw* (4)
8.2	$\hat{E}_2 = \hat{O}_1 = 110°$ (proven in/*bewys in* 8.1.3) BEOC is a cyclic quadrilateral (equal ∠s subtended by line/ *gelyke ∠e onderspan deur lyn*)	✓S ✓R (2) **[10]**

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QUESTION/VRAAG 9

9.1	the interior opposite angle/*die teenoorstaande binnehoek.*	✓ answer/*antw* (1)

9.2

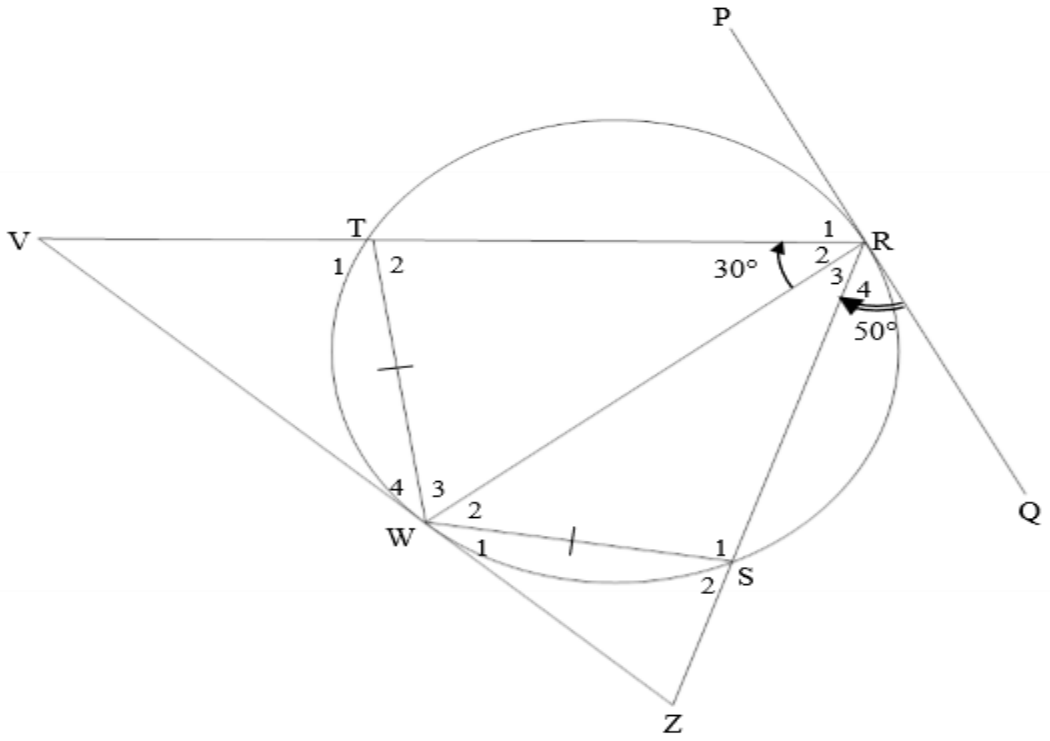


Construction: Draw diameter CT and join CV.
Konstruksie: Trek middellyn CT en verbind CV.

$\hat{V}_1 + \hat{V}_2 = 90°$	**∠ in semi-circle/*∠ in halfsirkel***	✓ S ✓ R
$\hat{T}_2 = 90° - x$	**Tangent ⊥ diameter/radius/*raaklyn ⊥ middellyn/radius***	✓ R
$\therefore \hat{C} =$ ***x***	Sum of the angles of triangle/*Som van die hoeke van 'n driehoek*	✓ S
$\therefore \hat{S} = x$	**∠'s same segment/*∠e in dieselfde segment***	✓ R
$\therefore V\hat{T}R = \hat{S}$		(5)

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9.3



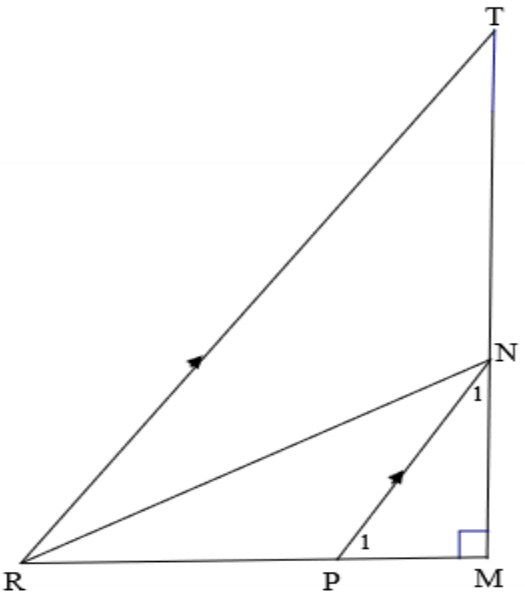
9.3.1	Equal chords subtend equal ∠s/*Gelyke koorde onderspan gelyke ∠e*	✓ R (1)
9.3.2	$\hat{W}_4 = 30°$ (tan chord theorem/*rkl-koordst*) $\hat{W}_1 = 30°$	✓ answer/*antw* ✓ reason/*rede* ✓ answer/*antw* (3)
9.3.3(a)	$\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext∠ of Δ/*buite ∠ v Δ*) $\therefore \hat{S}_2 = 80°$ **OR/*OF*** $\hat{R}_2 = \hat{R}_3 = 30°$ (= chords subtend =∠s /= *kde onderspan=∠e*) $\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\therefore \hat{S}_2 = 80°$	✓ S ✓R ✓ S (3) ✓ S ✓R ✓ S (3)

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9.3.3(b)	$\hat{T}_2 = \hat{S}_2 = 80°$ (ext ∠ of cyclic quad/*buite ∠ van koordevh*) $V + \hat{W}_4 = \hat{T}_2$ (ext∠ of Δ/*buite ∠ van Δ*) $\therefore \hat{V} = 50°$	✓ S ✓ R ✓ S ✓ S (4)
9.3.4	In ΔRVW and/*en* ΔRWS:	✓ using the correct Δs/ *gebruik korrekte Δe*
	$\hat{R}_2 = \hat{R}_3 = 30°$ (proven/*bewys* in 9.3.1) $\hat{V} = \hat{W}_2 = 50°$ (proven/*bewys* in 9.3.3) $V\hat{W}R = \hat{S}_1$ (3rd ∠ in Δ) $\therefore \Delta RVW \mid\mid\mid \Delta RWS$ (∠∠∠)	✓ S ✓ S ✓ R (3rd ∠ in Δ) or (∠∠∠)
	$\therefore \frac{WR}{RV} = \frac{RS}{WR}$ (ΔRVW ||| ΔRWS) $\therefore WR^2 = RV.RS$	✓ S (5) **[22]**

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

QUESTION/VRAAG 10



10.1.1	corresponding ∠s/*ooreenkomstige ∠e*; PN || RT	✓ answer/*antw* (1)
10.1.2	∠; ∠; ∠ **OR/*OF*** ∠; ∠	✓ answer/*antw* (1)
10.2	$\frac{PM}{RM}=\frac{PN}{RT}$ $(\Delta PNM \|\|\| \Delta RTM)$ $=\frac{PN}{3PN}$ $=\frac{1}{3}$	✓ S ✓ S (2)
10.3	$\frac{PM}{RM}=\frac{1}{3}$ $\therefore \frac{RP}{RM}=\frac{2}{3}$ $RN^2-PN^2=(RM^2+NM^2)-(PM^2+NM^2)$ (Pyth) $=RM^2-PM^2$ $=\left(\frac{3}{2}RP\right)^2-\left(\frac{1}{2}RP\right)^2$ $=\frac{9}{4}RP^2-\frac{1}{4}RP^2$ $=2RP^2$ **OR/*OF***	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM=\frac{3}{2}RP$ ✓ $PM=\frac{1}{2}RP$ ✓ $\frac{9}{4}RP^2$ & $\frac{1}{4}RP^2$ (4)

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	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (3PM)^2 - PM^2$ $= 8PM^2$ $= 2(2PM)^2$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ $(3PM)^2 - PM^2$ ✓ RP = 2PM (4)
	OR/*OF*	
	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (RP + PM)^2 - PM^2$ $= RP^2 + 2RP.PM + PM^2 - PM^2$ $= RP^2 + 2RP.\ \frac{1}{2}RP$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ expansion/ *uitbreiding* ✓ $PM = \frac{1}{2}RP$ (4) **[8]**

TOTAL/*TOTAAL*: 150

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GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2016

MARKS: 150

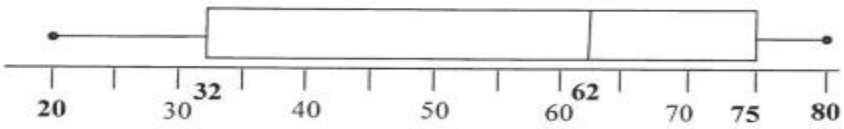
TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and a 25-page answer book.

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QUESTION 1

The box and whisker diagram below shows the marks (out of 80) obtained in a History test by a class of nine learners.



1.1 Comment on the skewness of the data. (1)

1.2 Write down the range of the marks obtained. (2)

1.3 If the learners had to obtain 32 marks to pass the test, estimate the percentage of the class that failed the test. (2)

1.4 In ascending order, the second mark is 28, the third mark 36 and the sixth mark 69. The seventh and eighth marks are the same. The average mark for this test is 54.

	28	36			69			

Fill in the marks of the remaining learners in ascending order. (6)
[11]

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QUESTION 2

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below.

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

2.1 Estimate the mean number of messages sent per day, rounded off to TWO decimal places. (3)

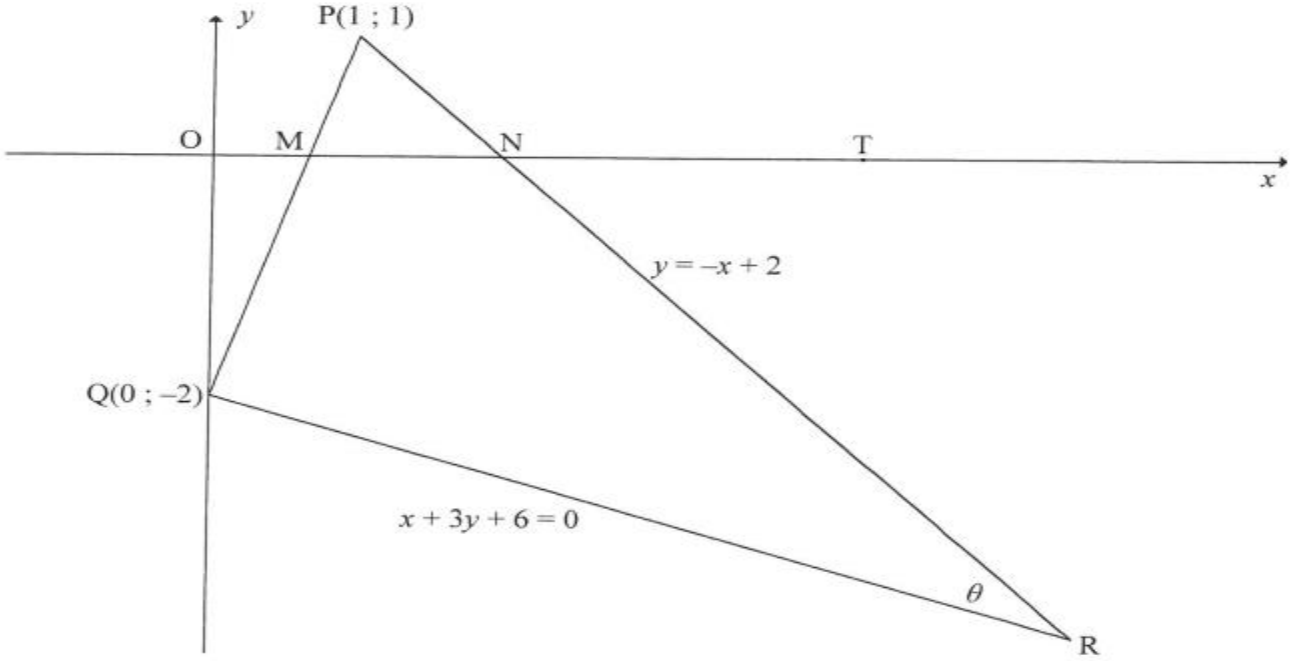
2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided in the ANSWER BOOK. (4)

2.3 Hence, estimate the number of days on which 65 or more messages were sent. (2)
[9]

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QUESTION 3

In the diagram below, P(1 ; 1), Q(0 ; –2) and R are the vertices of a triangle and $P\hat{R}Q = \theta$. The x-intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x-axis, as shown.



3.1 Determine the gradient of QP. (2)

3.2 Prove that $P\hat{Q}R = 90°$. (2)

3.3 Determine the coordinates of R. (3)

3.4 Calculate the length of PR. Leave your answer in surd form. (2)

3.5 Determine the equation of a circle passing through P, Q and R in the form $(x-a)^2 + (y-b)^2 = r^2$. (6)

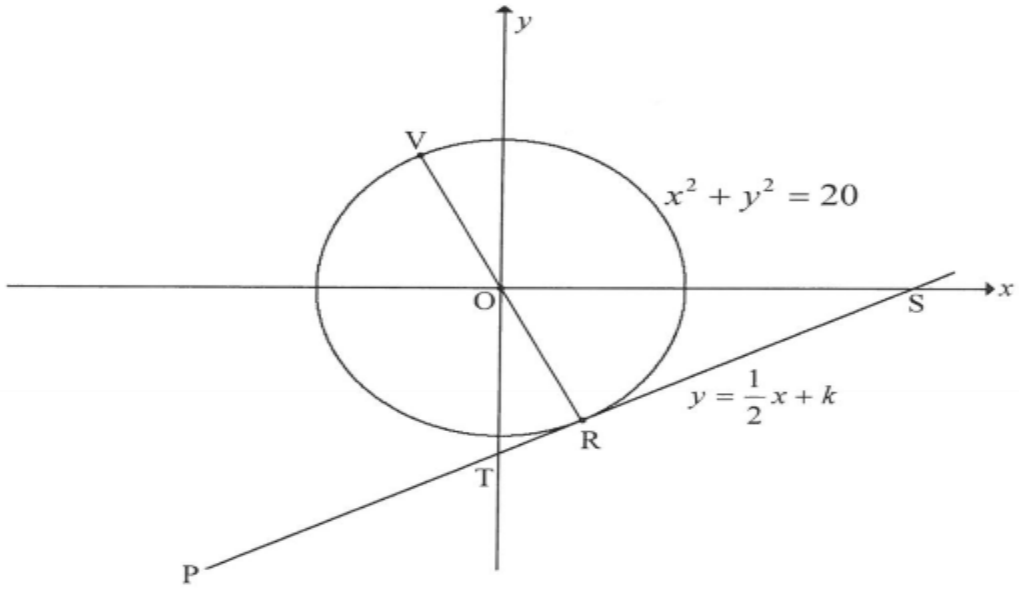
3.6 Determine the equation of a tangent to the circle passing through P, Q and R at point P in the form $y = mx + c$. (3)

3.7 Calculate the size of θ. (5)
[23]

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QUESTION 4

In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



4.1 Determine, giving reasons, the equation of OR in the form $y = mx + c$. (3)

4.2 Determine the coordinates of R. (4)

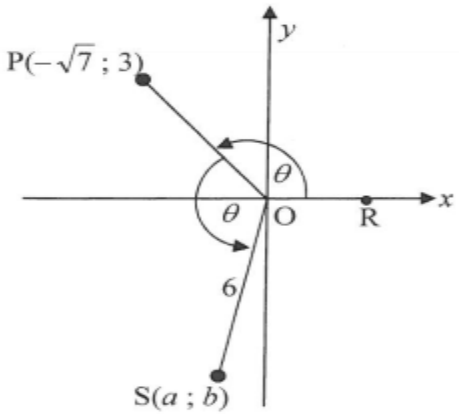
4.3 Determine the area of ΔOTS, given that R(2 ; –4). (6)

4.4 Calculate the length of VT. (4)
[17]

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QUESTION 5

5.1 $P(-\sqrt{7}\,;3)$ and $S(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below. $P\hat{O}R = P\hat{O}S = \theta$ and $OS = 6$.



Determine, WITHOUT using a calculator, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x-1}$ to a single trigonometric ratio. (3)

5.2.2 Hence, calculate the value of $\dfrac{4\sin 15°\cos 15°}{2\sin^2 15°-1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)

[13]

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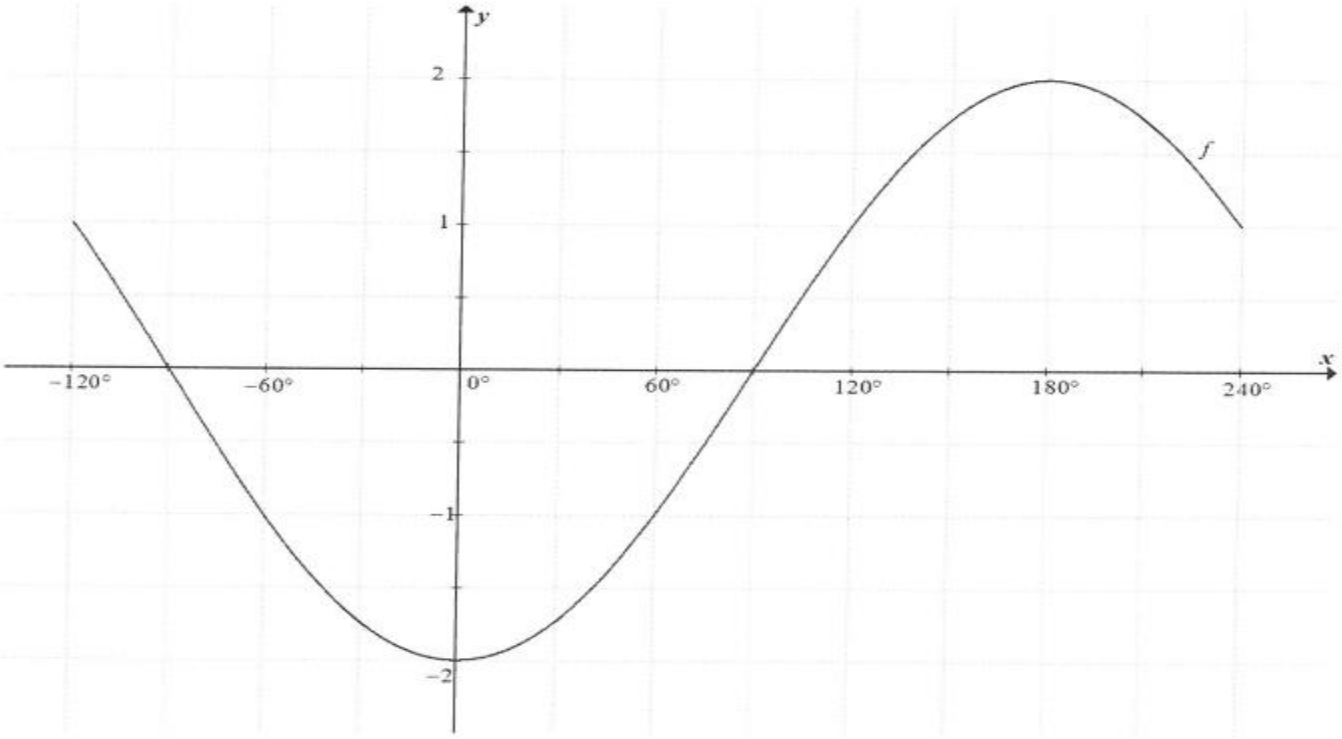
QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



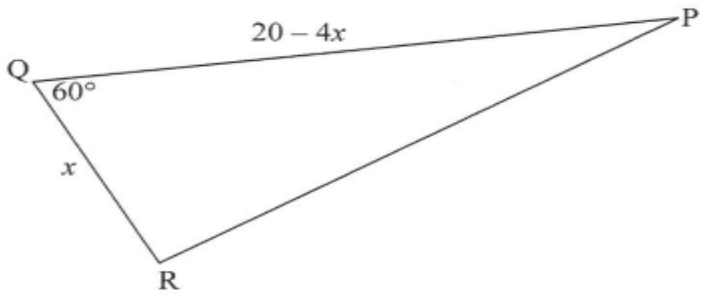
6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)
[13]

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QUESTION 7

7.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.

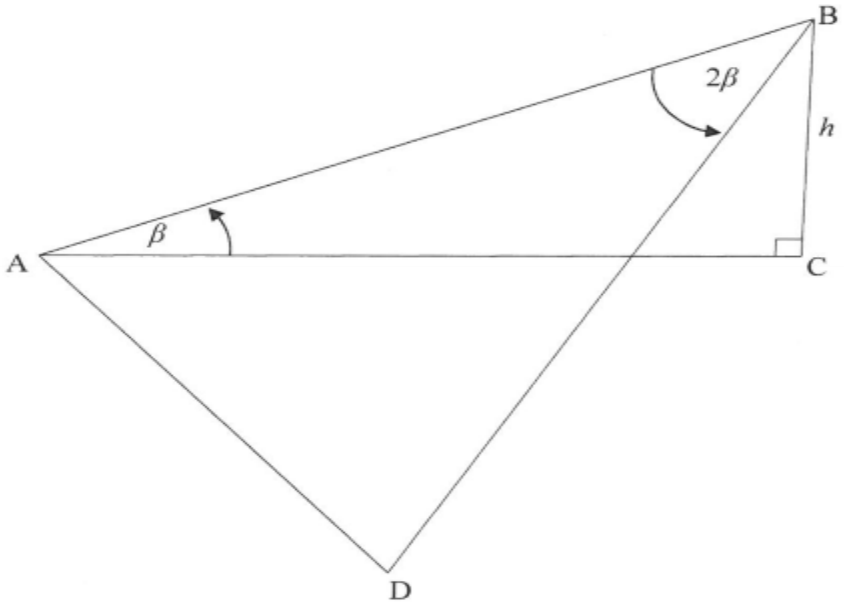


7.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and BA = BD.



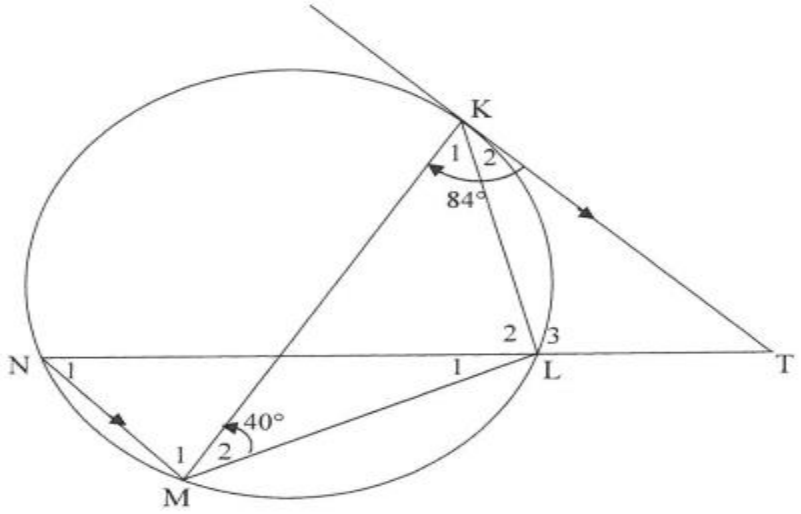
Determine the distance AD between the two anchors in terms of h. (7)
[15]

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Give reasons for ALL statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below, tangent KT to the circle at K is parallel to the chord NM. NT cuts the circle at L. ΔKML is drawn. $\hat{M}_2 = 40°$ and $M\hat{K}T = 84°$.



Determine, giving reasons, the size of:

8.1.1 $\hat{K}_2$ (2)

8.1.2 $\hat{N}_1$ (3)

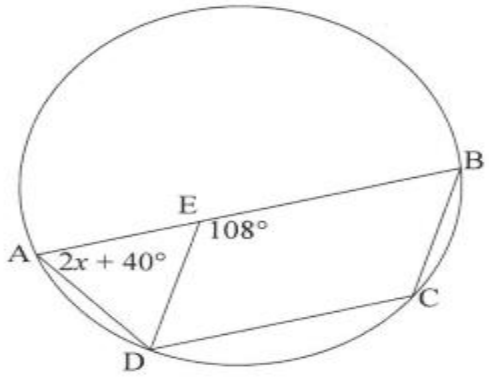
8.1.3 $\hat{T}$ (2)

8.1.4 $\hat{L}_2$ (2)

8.1.5 $\hat{L}_1$ (1)

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8.2 In the diagram below, AB and DC are chords of a circle. E is a point on AB such that BCDE is a parallelogram. $D\hat{E}B = 108°$ and $D\hat{A}E = 2x + 40°$.

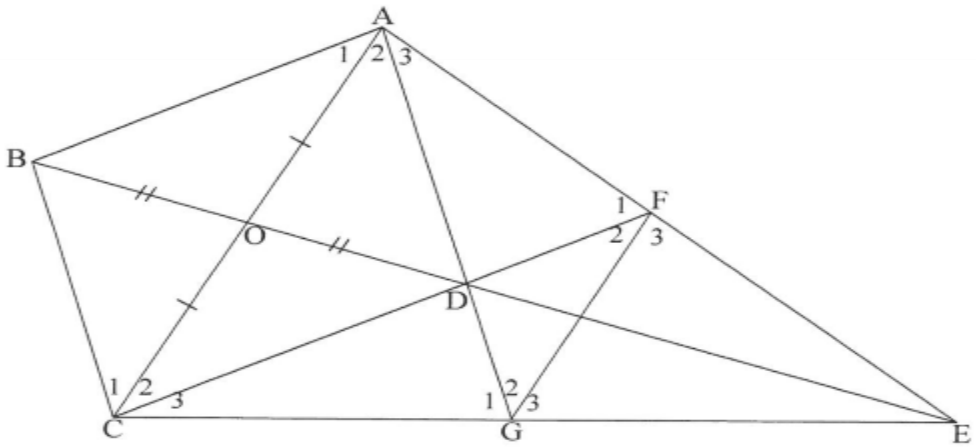


Calculate, giving reasons, the value of x. (5)
[15]

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QUESTION 9

In the diagram below, EO bisects side AC of ΔACE. EDO is produced to B such that BO = OD. AD and CD produced meet EC and EA at G and F respectively.



9.1 Give a reason why ABCD is a parallelogram. (1)

9.2 Write down, with reasons, TWO ratios each equal to $\frac{ED}{DB}$. (4)

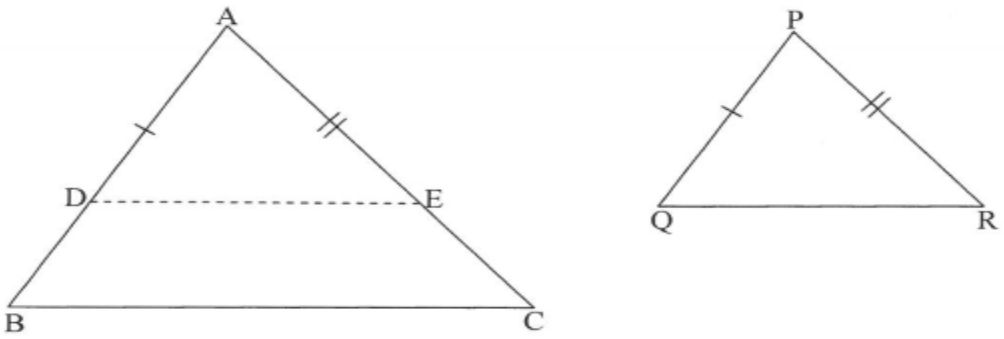
9.3 Prove that $\hat{A}_1 = \hat{F}_2$. (5)

9.4 It is further given that ABCD is a rhombus. Prove that ACGF is a cyclic quadrilateral. (3)
[13]

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QUESTION 10

10.1 In the diagram below, ΔABC and ΔPQR are given with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



DE is drawn such that $AD = PQ$ and $AE = PR$.

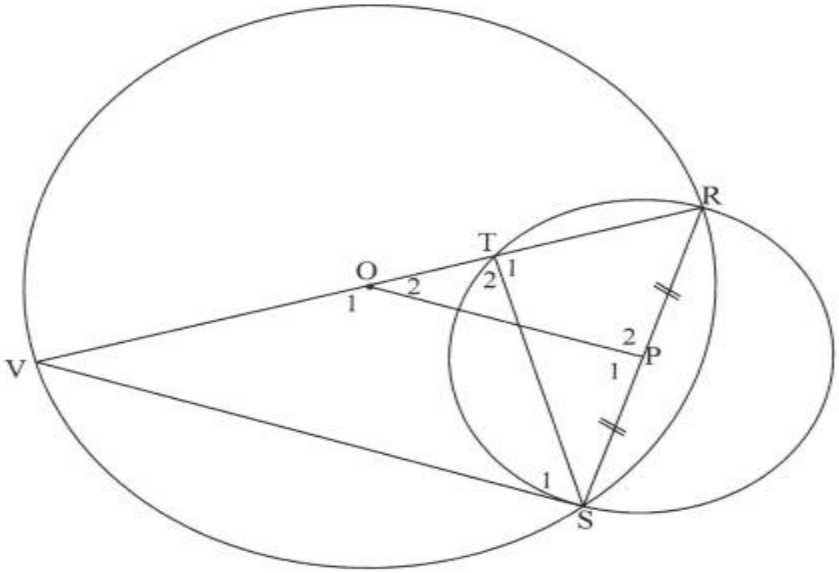
10.1.1 Prove that $\Delta ADE \equiv \Delta PQR$. (2)

10.1.2 Prove that $DE \parallel BC$. (3)

10.1.3 Hence, prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. (2)

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10.2 In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.



10.2.1 Why is OP ⊥ PS? (1)

10.2.2 Prove that ΔROP ||| ΔRVS. (4)

10.2.3 Prove that ΔRVS ||| ΔRST. (3)

10.2.4 Prove that $ST^2 = VT \cdot TR$. (6)

[21]

TOTAL: 150

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GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2016

MEMORANDUM

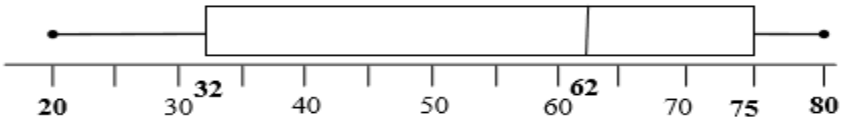
MARKS: 150
PUNTE: 150

This memorandum consists of 21 pages./
Hierdie memorandum bestaan uit 21 bladsye.

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Please turn over/*Blaai om asseblief*

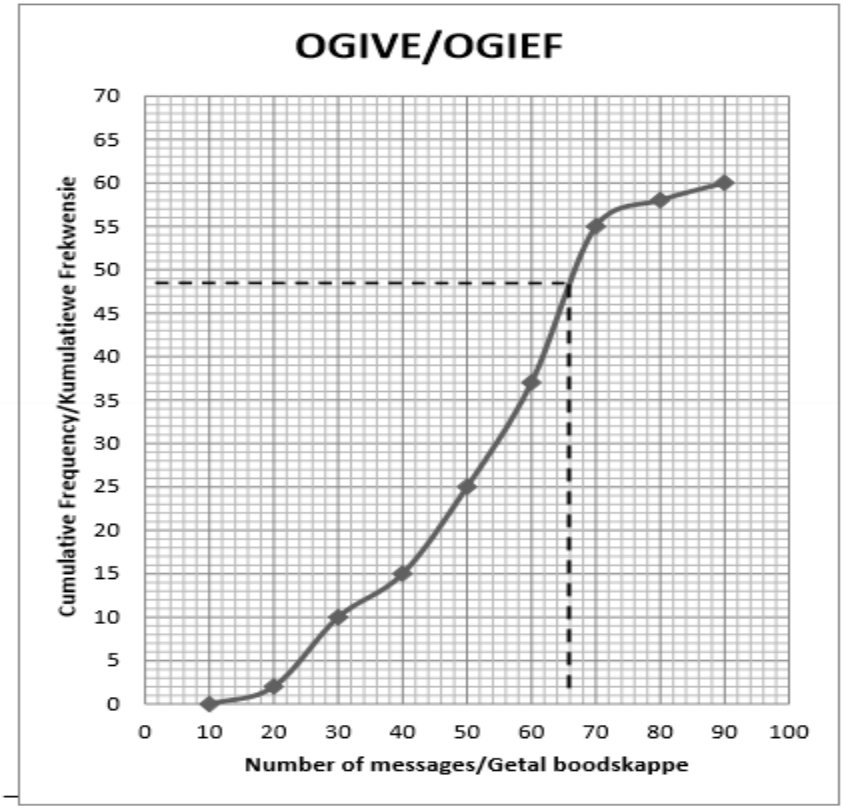
QUESTION/*VRAAG* 1



1.1	The data is skewed to the left/*Die data is skeef na links.* **OR/*OF*** The data is negatively skewed/*Die data is negatief skeef.*	✓answ/*antw* ✓answ/*antw* (1)
1.2	Range/*Omvang* = 80 – 20 = 60	✓ max. – min. ✓answ/*antw* (2)
1.3	25% of the learners failed/*van die leerders het gedruip*	✓ ✓answ/*antw* (2)
1.4	$54 = \frac{445 + T_4}{9}$ $T_4 = 41$ **20** 28 36 **41** **62** 69 **75** **75** **80**	✓ 20 ✓✓ 41 ✓ 62 ✓ 75 ✓ 80 (6) **[11]**

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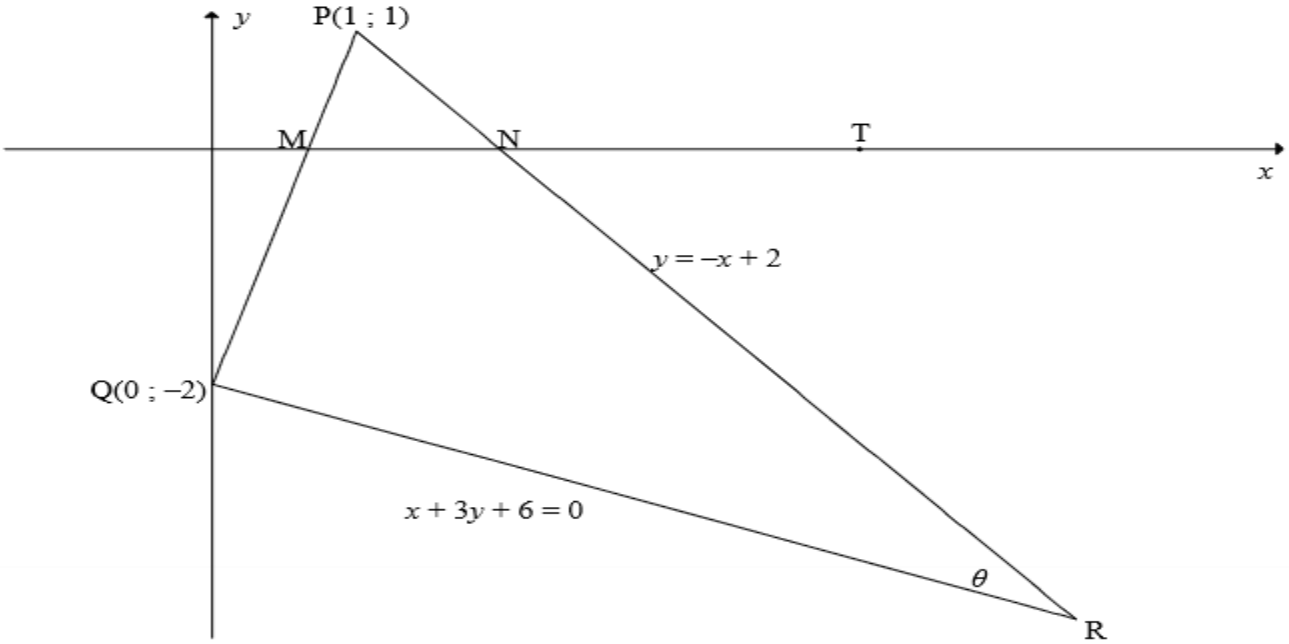
QUESTION/*VRAAG* 2

2.1	Mean/*Gemiddelde* $= \frac{2(15)+8(25)+...2(85)}{60} = \frac{3080}{60}$ $= 51{,}33$ messages per day/*boodskappe per dag*	✓ 3 080 ✓ $\frac{3080}{60}$ ✓answ/*antw* (3)
2.2		✓grounding at (10 ; 0) ✓plotting at upper limits ✓plotting. cumulative *f* ✓smooth shape of curve ✓*geanker by* (10 ; 0) ✓*stip by boonste limiete* ✓*plot kumulatiewe f* ✓*gladde vorm van kurwe* (4)
2.3	Number of days/*Getal dae* = 60 – 46 (see on graph above/*sien op grafiek hierbo*) = 14 days/*dae* **OR/*OF*** Number of days/*Getal dae* $= 2 + 3 + \frac{1}{2} \times 18 = 14$ days/*dae*	✓46 (accept 45 – 49) ✓answ/*antw* (accept 11 – 15) (2) ✓add correct values/*tel korrekte waardes by* ✓answ/*antw* (2) **[9]**

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QUESTION/*VRAAG* 3



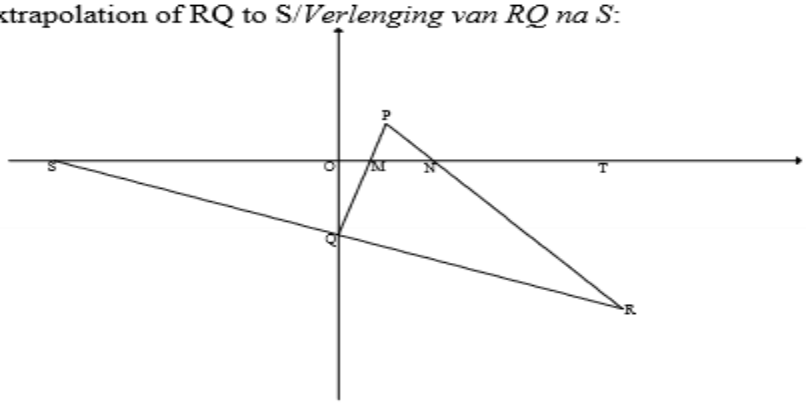
3.1	$m_{PQ} = \frac{1-(-2)}{1-0}$ $= 3$	✓subst (1 ; 1) & (0 ; –2) ✓answ/*antw* (2)
3.2	QR: $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore P\hat{Q}R = 90°$	✓ $m_{QR} = -\frac{1}{3}$ ✓ $m_{PQ} \times m_{QR} = -1$ (2)

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3.3	$-\frac{1}{3}x-2=-x+2$ $\frac{2}{3}x=4$ $x=6$ $y=-4$ $\therefore$ R(6 ; –4)	✓equating/*gelyk stel* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (3)
3.4	$PR=\sqrt{(1-6)^2+(1-(-4))^2}$ $=\sqrt{50}=5\sqrt{2}$ **OR/*OF*** $PR^2=(1-6)^2+(1-(-4))^2$ $=50$ $\therefore PR=\sqrt{50}=5\sqrt{2}$	✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2) ✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2)
3.5	PR is a diameter/*'n middellyn* [chord subtends/*kd onderspan* 90°] Centre of circle/*Midpt v sirkel*: $\left(\frac{1+6}{2};\frac{1-4}{2}\right)$ $=\left(3\frac{1}{2};-1\frac{1}{2}\right)$ $r=\frac{\sqrt{50}}{2}$ OR $\frac{5\sqrt{2}}{2}$ OR 3,54 $\therefore\left(x-\frac{7}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{50}{4}$ OR $\frac{25}{2}$ OR 12,5	✓✓S ✓✓ $\left(3\frac{1}{2};-1\frac{1}{2}\right)$ ✓*r*-value/*waarde* ✓answ/*antw* (6)
3.6	*m* of/*van* radius $=-1$ $\therefore$ *m* of/*van* tangent/*raaklyn* $=1$ Equation of tangent/*Vgl van raaklyn*: $y-y_1=(x-x_1)$ $y-1=x-1$ $\therefore y=x$ **OR/*OF*** $y=x+c$ $1=1+c$ $y=x$	✓*m* of tang/*rkl* ✓subst *m* & P(1 ; 1) into/*in* eq of line/*vgl v lyn* ✓answ/*antw* (3)
3.7	$\tan P\hat{N}T=m_{PR}=-1$ $\therefore P\hat{N}T=135°$ $\tan P\hat{M}T=m_{PQ}=3$ $\therefore P\hat{M}T=71{,}57°$ $\hat{P}=63{,}43°$ [ext ∠ of Δ/*buite ∠v* Δ] $\therefore\theta=26{,}57°$ [sum of ∠s in Δ/*som v ∠e in* Δ] **OR/*OF***	✓ $\tan P\hat{N}T=-1$ ✓ $P\hat{N}T=135°$ ✓ $P\hat{M}T=71{,}57°$ ✓ $\hat{P}=63{,}43°$ ✓answ/*antw* (5)

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Extrapolation of RQ to S/*Verlenging van RQ na S*:



$\tan \hat{PNT} = m_{PR} = -1$
$\therefore \hat{SNR} = 135°$
$\tan \hat{NSR} = m_{RS} = -\frac{1}{3}$
$\therefore \hat{NSR} = 18{,}43°$
$\theta = 180° - (135° + 18{,}43°)$ [sum of ∠s in Δ/*som v ∠e in Δ*]
$= 26{,}57°$

✓ $\tan \hat{PNT} = -1$
✓ $\hat{SNR} = 135°$
✓ $\tan \hat{NSR} = -\frac{1}{3}$
✓ $\hat{NSR} = 18{,}43°$
✓ answ/*antw*
(5)

OR/*OF*

$PQ^2 = 1^2 + 3^2 = 10$
$PQ = \sqrt{10}$
$\therefore \sin\theta = \frac{PQ}{PR} = \frac{\sqrt{10}}{\sqrt{50}} = \frac{1}{\sqrt{5}}$
$\therefore \theta = 26{,}57°$

✓ subst into/*in* distance formula/ *afstandsformule*
✓ distance/*afst* PQ
✓ correct trig ratio/ *korrekte trig vh*
✓ correct trig eq/ *korrekte trig vgl*
✓ answ/*antw*
(5)

OR/*OF*

$QR^2 = 6^2 + 2^2 = 40$
$QR = 2\sqrt{10}$
$\therefore \cos\theta = \frac{2\sqrt{10}}{\sqrt{50}} = \frac{2}{\sqrt{5}}$
$\therefore \theta = 26{,}57°$

✓ subst into/*in* distance formula/ *afstandsformule*
✓ distance/*afst* PQ
✓ correct trig ratio/ *korrekte trig vh*
✓ correct trig eq/ *korrekte trig vgl*
✓ answ/*antw*
(5)

OR/*OF*

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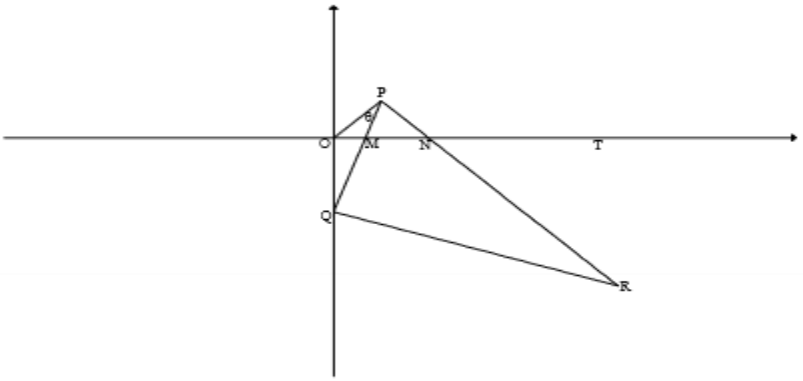
$$\tan\theta = \frac{m_{RQ} - m_{PR}}{1 + m_{RQ}.m_{PR}}$$

$$= \frac{-\frac{1}{3} - (-1)}{1 + (-\frac{1}{3})(-1)}$$

$$= \frac{1}{2}$$

$\therefore \theta = 26{,}57°$

✓correct formula/ *korrekte formule*
✓ $m_{RQ} = -\frac{1}{3}$
✓correct subst/ *subst korrek*
✓ $\tan\theta = \frac{1}{2}$
✓ $\theta = 26{,}57°$
(5)

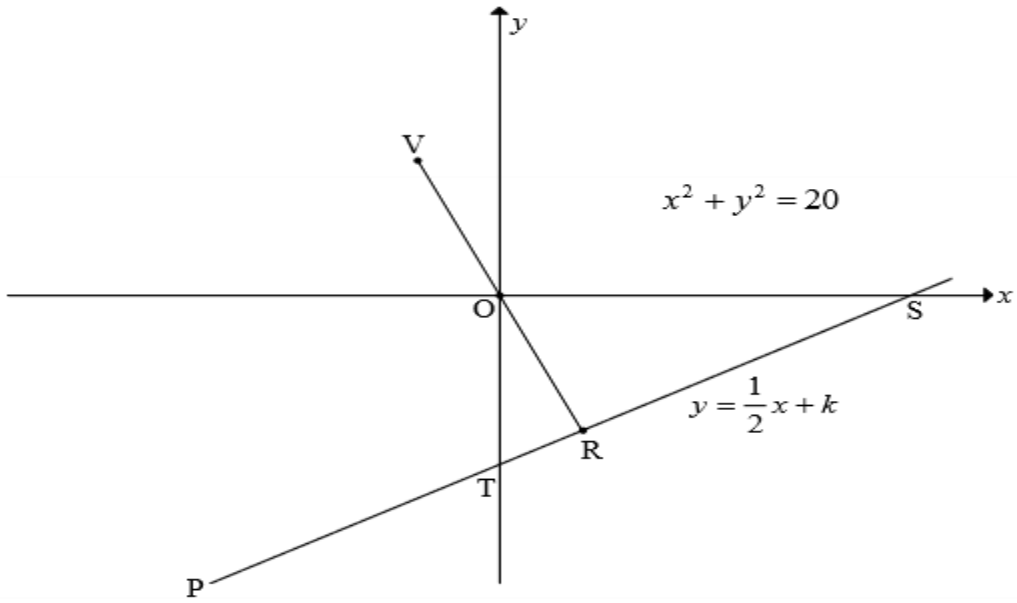


tangent OP goes through the origin/*raakl OP gaan deur oorsprong*
$P\hat{O}M = 45°$
$O\hat{P}M = \theta = \hat{P}$ [tan-chord theorem/*raakl-kdst*]
$\tan P\hat{M}T = m_{PQ} = 3$
$\therefore P\hat{M}T = 71{,}57°$
$\therefore \theta + 45° = 71{,}57°$ [ext ∠ of Δ/*buite-∠ v Δ*]
$\therefore \theta = 26{,}57°$

✓ $P\hat{O}M = 45°$
✓R
✓ $P\hat{M}T = 71{,}57°$
✓S
✓ $\theta = 26{,}57°$
(5)
[23]

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QUESTION/*VRAAG* 4



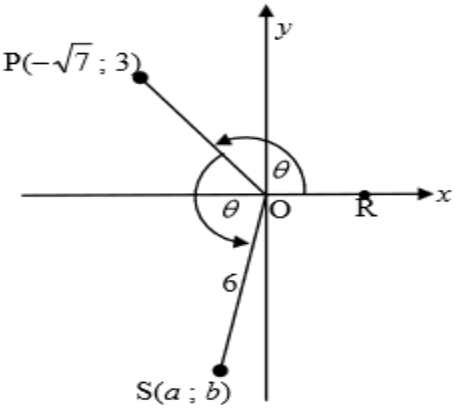
4.1	OR ⊥ TR [radius ⊥ tangent/*raakl*] $\therefore\ m_{TR} \times m_{OR} = -1$ $\therefore\ m_{OR} = -2$ $\therefore\ y = -2x$	✓S/R ✓*m* of/*van* OR ✓equation/*vgl* (3)
4.2	$x^2 + (-2x)^2 = 20$ $x^2 + 4x^2 = 20$ $5x^2 - 20 = 0$ $x^2 - 4 = 0$ $(x+2)(x-2) = 0$ $\therefore x = 2$ $y = -2(2) = -4$ $\therefore$ R(2 ; –4)	✓subst eq of OR into circle eq/ *subst vgl OR in sirkelvgl* ✓st. form/*st. vorm* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (4)

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4.3	Subst R(2 ; –4) into the equation of/*in vgl van* PRS: $-4 = \frac{1}{2}(2) + k$ $k = -5$ $\therefore$ OT = 5 $0 = \frac{1}{2}x - 5$ $x = 10$ $\therefore$ OS = 10 Area/*Oppervlakte* $= \frac{1}{2}$ OS . OT $= \frac{1}{2}(10)(5)$ = 25 sq units/*vk eenh*	✓correct subst/ *korrekte subst* ✓value of k ✓$y = 0$ ✓x-intercept/*afsnit* ✓correct subst into area form/ *subst korrek in opp-formule* ✓answ/*antw* (6)
4.4	$0 = \frac{x_V + 2}{2}$ and/*en* $0 = \frac{y_V - 4}{2}$ $\therefore$ V(–2 ; 4) T(0 ; –5) .... from/*van* 4.3 VT $= \sqrt{(-2-0)^2 + (4-(-5))^2}$ $= \sqrt{4+81}$ $= \sqrt{85}$	✓x-value/*waarde*V ✓y-value/*waarde*V ✓subst of points V and T into distance formula/ *subst punte V en T in afst-form* ✓answ/*antw* (4) **[17]**

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QUESTION/*VRAAG* 5



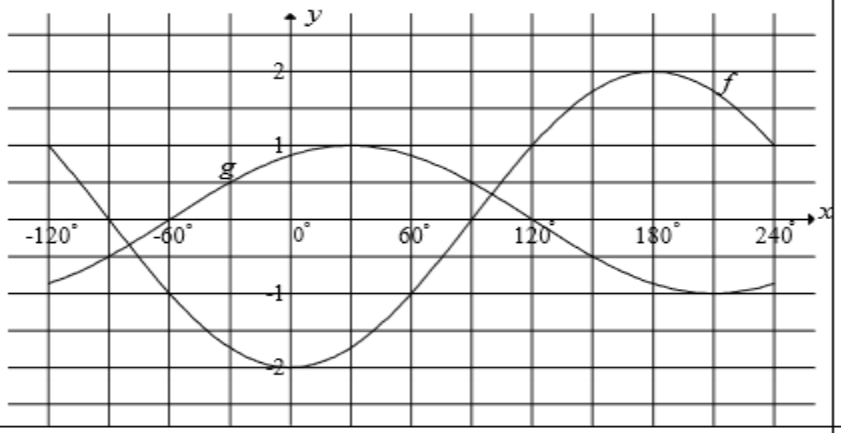
5.1.1	$\tan\theta = -\frac{3}{\sqrt{7}}$	✓answ/*antw* (1)
5.1.2	$\sin(-\theta) = -\sin\theta$ $OP^2 = \left(-\sqrt{7}\right)^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$	✓reduction/ *reduksie* ✓ $OP = 4$ ✓answ/*antw* (3)
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2\theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$ **OR/*OF*** $\frac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2\theta - 1)$ $= 12\left(\frac{-\sqrt{7}}{4}\right)^2 - 6$ $= \frac{21}{4} - \frac{24}{4}$ $= -\frac{3}{4}$ **OR/*OF***	✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4) ✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ ✓answ/*antw* (4)

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	$\frac{a}{6} = \cos 2\theta$ $a = 6(\cos^2\theta - \sin^2\theta)$ $= 6\left[\left(\frac{-\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$ $= 6\left(-\frac{2}{16}\right)$ $= -\frac{3}{4}$	✓ trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ & $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4)
5.2.1	$\frac{4\sin x.\cos x}{2\sin^2 x - 1} = \frac{2(2\sin x.\cos x)}{-(1-2\sin^2 x)}$ $= \frac{2\sin 2x}{-\cos 2x}$ $= -2\tan 2x$	✓2sin 2x ✓–cos 2x ✓answ/*antw* (3)
5.2.2	$\frac{4\sin 15°\cos 15°}{2\sin^2 15° - 1} = -2\tan 2(15°)$ $= -2\tan 30°$ $= -2\left(\frac{1}{\sqrt{3}}\right)$ $= -\frac{2}{\sqrt{3}}$ **OR/*OF*** $-\frac{2\sqrt{3}}{3}$	✓ $-2\tan 2(15°)$ ✓answ/*antw* (2) **[13]**

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QUESTION/*VRAAG* 6

6.1	$\sin(x+60°)+2\cos x=0$ $\sin x\cos 60°+\cos x\sin 60°+2\cos x=0$ $\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x+2\cos x=0$ $\frac{1}{2}\sin x=-2\cos x-\frac{\sqrt{3}}{2}\cos x$ $\sin x=-4\cos x-\sqrt{3}\cos x$ $\sin x=\cos x(-4-\sqrt{3})$ $\frac{\sin x}{\cos x}=\frac{\cos x(-4-\sqrt{3})}{\cos x}$ $\therefore \tan x=-4-\sqrt{3}$	✓expansion/*uitbreiding* ✓special angle values/ *spesiale ∠-waardes* ✓simpl/*vereenv* ✓ $\sin x=\cos x(-4-\sqrt{3})$ (4)
6.2	$\tan x=-4-\sqrt{3}$ $\tan x=-(4+\sqrt{3})$ *ref* $\angle=80{,}10°$ $x=-80{,}1°$ or/*of* $99{,}9°$	✓80,10° ✓99,90° ✓–80,1° (3)
6.3.1		✓(30° ; 1) ✓(–60° ; 0) ✓shape/*vorm* (3)
6.3.2	$\therefore \sin(x+60°)>-2\cos x$ $x\in(-80{,}10°\ ;\ 99{,}90°)$ **OR/*OF*** $-80{,}10°<x<99{,}90°$	✓✓critical values/ *kritiese waardes* ✓notation/*notasie* (3) **[13]**

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

QUESTION/*VRAAG* 7

7.1.1	Area of/*Oppervlakte van* $\Delta PQR = \frac{1}{2}PQ.QR.\sin\hat{Q}$ $= \frac{1}{2}x(20-4x)(\sin 60°)$ $= 10x - 2x^2\left(\frac{\sqrt{3}}{2}\right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$	✓subst into area rule/ *subst in opp-reël* ✓subst & simpl/ *subst en vereenv* (2)
7.1.2	For maximum area/*Vir maksimum opp:* $(\text{Area } \Delta PQR)' = 0$ $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore x_{max} = \frac{5}{2}$ or $2\frac{1}{2}$ or/*of* 2,5 **OR/*OF*** $x_{max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2}$ or $2\frac{1}{2}$ or 2,5 **OR/*OF*** $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5-x) = 0$ $\therefore x = 0$ or 5 $\therefore x_{max} = \frac{0+5}{2} = \frac{5}{2}$ or/*of* 2,5	✓$(\text{Area } \Delta PQR)' = 0$ ✓$5\sqrt{3} - 2\sqrt{3}x$ ✓answ/*antw* (3) ✓formula/*e* ✓ subst ✓answ/*antw* (3) ✓*x*-intercepts/ *x-afsnitte* ✓ subst ✓answ/*antw* (3)
7.1.3	$RP^2 = QP^2 + QR^2 - 2.QP.QR.\cos Q$ $= 10^2 + 2,5^2 - 2(10)(2,5)\cos 60°$ $= 81,25$ $\therefore RP = 9,01$	✓subst into cosine rule/*in cos-reël* ✓simpl/*vereenv* ✓answ/*antw* (3)

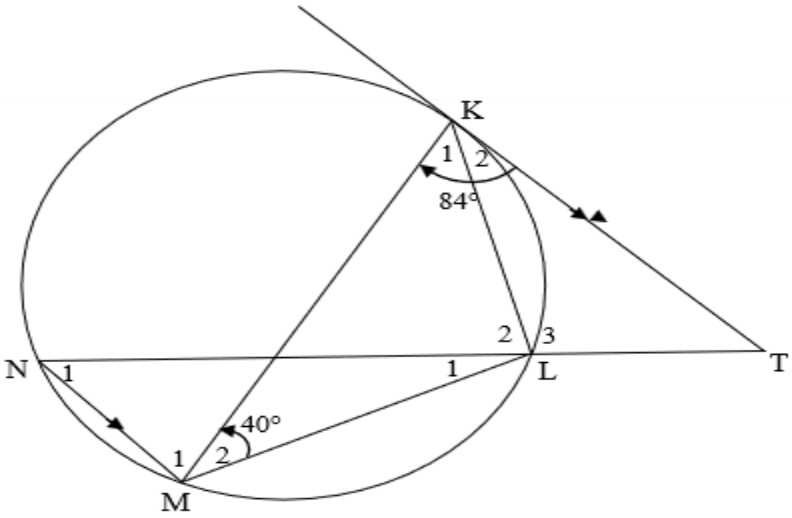
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7.2	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: $AB = BD$ and/*en* $A\hat{D}B = 90° - \beta$ [$\angle$s of/*v* $\Delta = 180°$] $\frac{\sin 2\beta}{AD} = \frac{\sin(90° - \beta)}{AB}$ $AD = \frac{AB.\sin 2\beta}{\sin(90° - \beta)}$ $= \frac{h}{\sin\beta} \times \frac{2\sin\beta.\cos\beta}{\cos\beta}$ $= 2h$	✓AB ito h and/*en* β ✓ $A\hat{D}B = 90° - \beta$ ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓AD as subject/ *onderwerp* ✓expansion/*uitbrei* ✓sin $(90° - \beta)$ $= \cos\beta$ ✓answer ito h (7)
	OR/*OF*	
	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: $AB = BD$ $AD^2 = AB^2 + AB^2 - 2AB.AB.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 (1 - 2\sin^2\beta)$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 + 4h^2$ $= 4h^2$ $\therefore AD = 2h$	✓AB ito h and/*en* β ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓expansion/*uitbrei* ✓multiplication/ *vermenigv* ✓simpl/*vereenv* ✓answer ito h (7)
	OR/*OF*	
	Split isosceles triangle ABQ into two congruent triangles AEB and DEB. Then $\Delta ABC \equiv \Delta BAE$ ($AB = AC$, $A\hat{B}E = B\hat{A}C = \beta$, h) $\therefore AE = ED = BC = h$ $\therefore AD = 2h$	(7)
		[15]

QUESTION/*VRAAG* 8

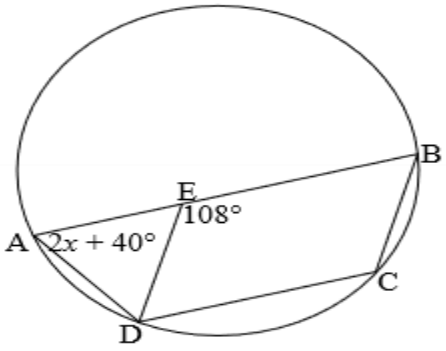
8.1



8.1.1	$\hat{K}_2 = \hat{M}_2 = 40°$ [tan chord theorem/*raakl-kdst*]	✓S ✓R (2)
8.1.2	$\hat{N}_1 = \hat{K}_1$ [∠s in the same seg/*∠e in dies segm*] $\hat{K}_1 = 84° - 40° = 44°$ $\therefore \hat{N}_1 = 44°$	✓S ✓R ✓S (3)
8.1.3	$\hat{T} = \hat{N}_1 = 44°$ [alt/*verw* ∠s/*e*; KT || NM]	✓S ✓R (2)
8.1.4	$\hat{L}_2 = \hat{K}_2 + \hat{T}$ [ext ∠ of Δ/*buite ∠ v Δ*] $= 40° + 44°$ $= 84°$	✓R ✓S (2)
8.1.5	In ΔKLM: $44° + 84° + 40° + \hat{L}_1 = 180°$ [∠s sum in Δ/*∠e som in Δ*] $\therefore \hat{L}_1 = 12°$	✓S (1)

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8.2

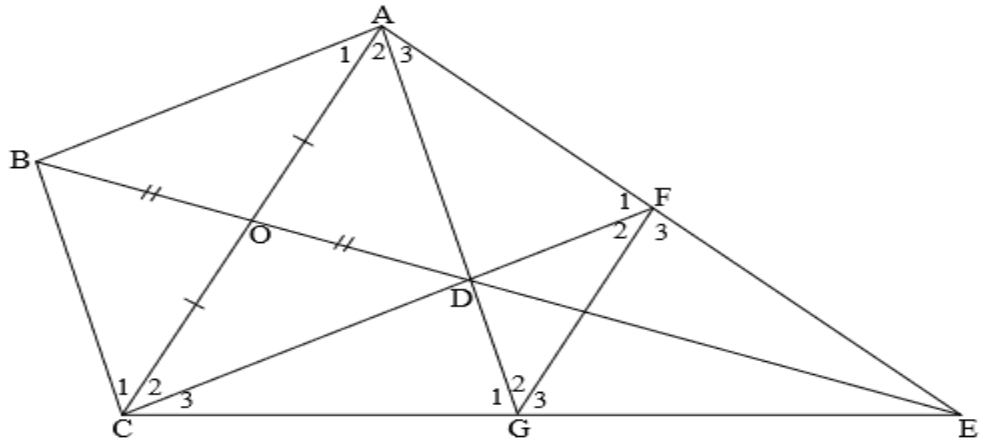


8.2	$\hat{C} = 108°$	[opp∠s of ‖m/*tos ∠e v ‖m*]	✓S ✓R
	$2x + 40° + 108° = 180°$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw* (5)
		OR/*OF*	
	$\hat{C} = 180° - (2x + 40°)$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$180° - (2x + 40°) = 108°$	[opp∠s of ‖m/*tos ∠e v ‖m*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw* (5) **[15]**

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QUESTION/*VRAAG* 9

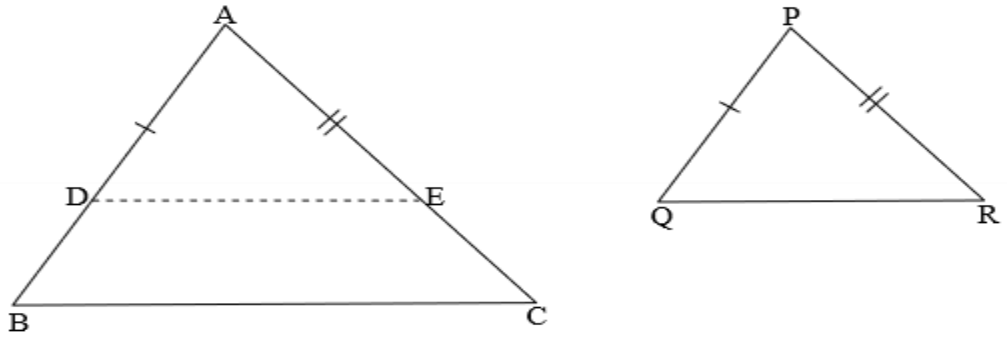


9.1	ABCD is a ‖m [diags of quad bisect each other/ *hoekl v vh halveer mekaar*]	✓ R (1)
9.2	$\frac{ED}{DB}=\frac{FE}{AF}$ [Prop Th/*Eweredigh st*; DF ‖ BA] $\frac{ED}{DB}=\frac{GE}{CG}$ [Prop Th/*Eweredigh st*; DG ‖ BC]	✓S ✓ R ✓S ✓ R (4)
9.3	$\frac{FE}{AF}=\frac{GE}{CG}$ [proved/*bewys*] ∴AC ‖ FG [line divides two sides of Δ in prop/ *lyn verdeel 2 sye van Δ eweredig*] $\hat{C}_2=\hat{F}_2$ [alt/*verw* ∠s/*e*; AC ‖ FG] $\hat{A}_1=\hat{C}_2$ [alt/*verw* ∠s/*e*; AB ‖ CD] ∴ $\hat{A}_1=\hat{F}_2$	✓S ✓ S ✓R ✓S ✓ S (5)
9.4	$\hat{A}_1=\hat{A}_2$ [diags of rhombus/*hoekl v ruit*] $\hat{A}_2=\hat{F}_2$ [$\hat{A}_1=\hat{F}_2$] ∴ ACGF = cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =] **OR/*OF*** $\hat{C}_2=\hat{A}_2$ [∠s opp equal sides of rhombus/ *∠e to gelyke sye v ruit*] $\hat{A}_2=\hat{G}_2$ [alt/*verw*-∠s/*e*; AC ‖ FG] ∴ $\hat{C}_2=\hat{G}_2$ ∴ ACGF is a cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =]	✓S ✓ S ✓R (3) ✓S ✓ S ✓R (3) **[13]**

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QUESTION/*VRAAG* 10

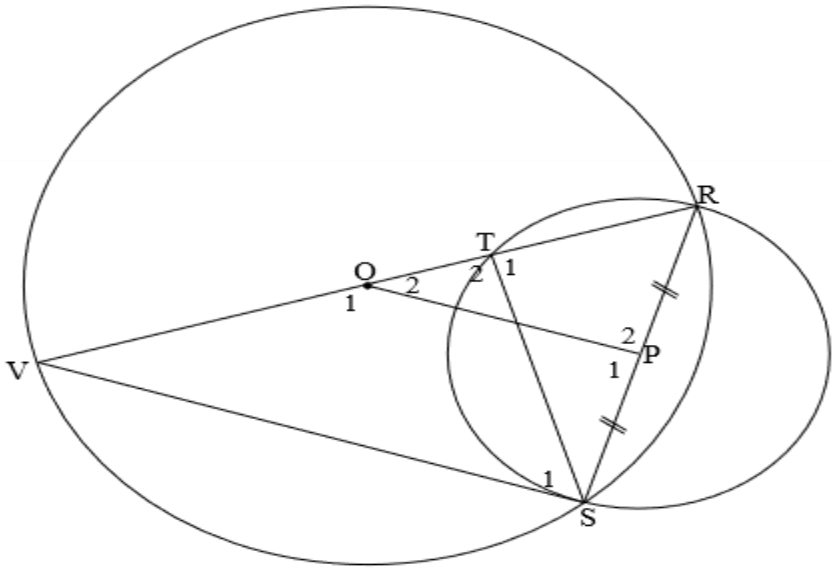
10.1



10.1.1	In ΔADE and/*en* ΔPQR: AD = PQ [construction/*konstr*] $\hat{A} = \hat{P}$ [given/*gegee*] AE = PR [construction/*konstr*] ∴ ΔADE ≡ ΔPQR [S∠S]	✓all/*al* 3 S's/*e* ✓reason/*rede* (2)
10.1.2	$A\hat{D}E = \hat{Q}$ [Δs ≡ ∴ corres/*ooreenk* ∠s/*e* =] But $\hat{B} = \hat{Q}$ [given/*gegee*] ∴ $A\hat{D}E = \hat{B}$ ∴ DE || BC [corres/*ooreenk* ∠s/*e* =]	✓ $A\hat{D}E = \hat{Q}$ ✓ $A\hat{D}E = \hat{B}$ ✓reason/*rede* (3)
10.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ [Prop Th/*Eweredigh st*; DE || BC] But/*Maar* AD = PQ and/*en* AE = PR [construction/*konstr*] ∴ $\frac{AB}{PQ} = \frac{AC}{PR}$	✓S/R ✓S (2)

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10.2



10.2.1	line from centre to midpt of chord/*lyn van midpt na midpt van koord*	✓answ/*antw* (1)
10.2.2	OP || VS [Midpt Theorem/*Midpt-stelling*] In ΔROP and/*en* ΔRVS: $\hat{R} = \hat{R}$ [common/*gemeen*] $\hat{O}_2 = \hat{V}$ [corresp/*ooreenk* ∠s/*e*; OP || VS] ∴ΔROP ||| ΔRVS [∠,∠,∠] **OR/*OF*** In ΔROP and/*en* ΔRVS: $\hat{P}_2 = V\hat{S}R$ [corresponding ∠s/ *ooreenkomstige* ∠'e] $\hat{R} = \hat{R}$ [common/*gemeen*] ∴ΔROP ||| ΔRVS [∠,∠,∠]	✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4) ✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4)

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10.2.3	In ΔRVS and/*en* ΔRST: $V\hat{S}R = S\hat{T}R = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{R}$ is common/*gemeen* $\hat{V} = T\hat{S}R$ ∴ ΔRVS ||| ΔRST [∠,∠,∠]	✓S ✓R ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (3)
10.2.4	In ΔRTS and/*en* ΔSTV: $R\hat{T}S = V\hat{T}S = 90°$ [∠ s on straight line/∠*e op rt lyn*] $\hat{R} = 90° - T\hat{S}R$ $= T\hat{S}V$ $T\hat{S}R = \hat{V}$ ∴ ΔRTS ||| ΔSTV [∠,∠,∠] $\therefore \frac{RT}{ST} = \frac{TS}{VT}$ $\therefore ST^2 = VT.TR$	✓ΔRTS & ΔSTV ✓S ✓S ✓S (with justification/*met motivering*) ✓ΔRTS ||| ΔSTV ✓ratio/*verh* (6)
		[21]

TOTAL/*TOTAAL*: 150

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basic education

Department:
Basic Education
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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2017

MARKS: 150

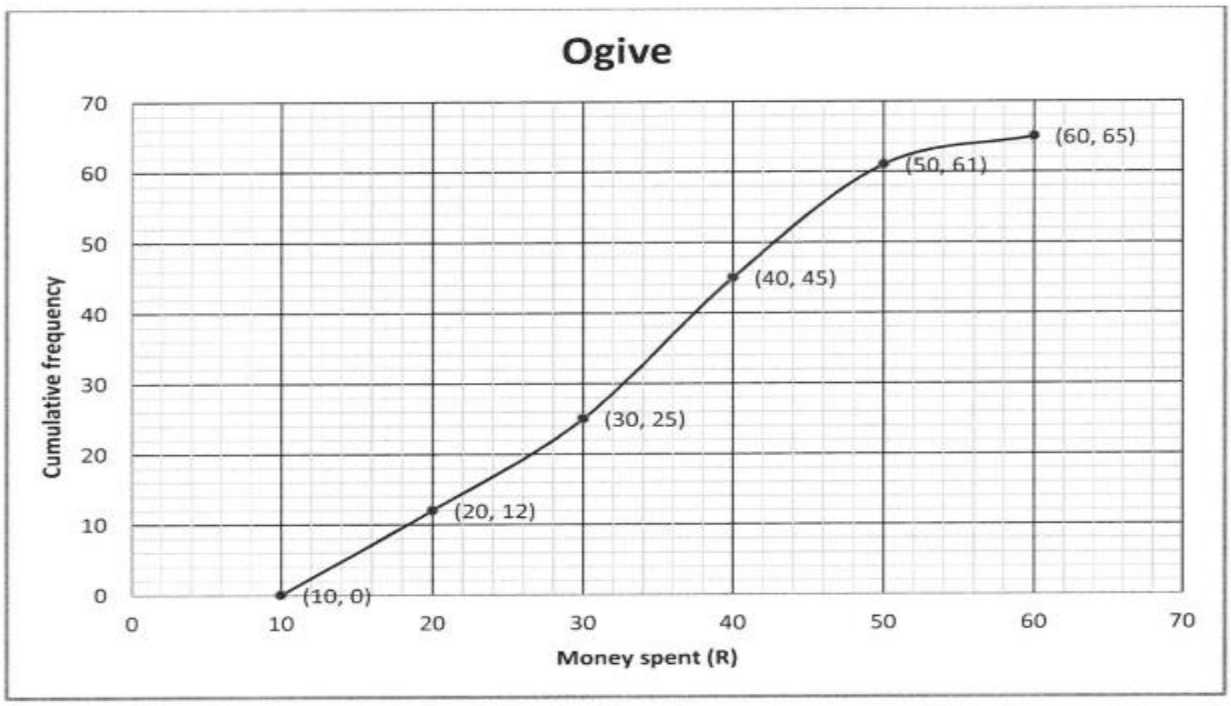
TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 28 pages.

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QUESTION 1

The amount of money, in rands, that learners spent while visiting a tuck shop at school on a specific day was recorded. The data is represented in the ogive below.



An incomplete frequency table is also given for the data.

Amount of money (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency	a	13	20	b	4

1.1 How many learners visited the tuck shop on that day? (1)

1.2 Write down the modal class of this data. (1)

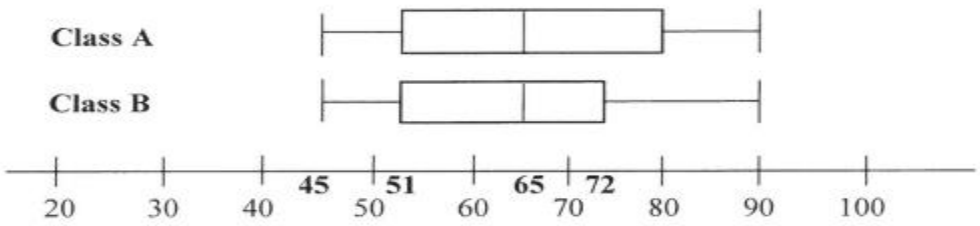
1.3 Determine the values of a and b in the frequency table. (2)

1.4 Use the ogive to estimate the number of learners who spent at least R45 on the day the data was recorded at the tuck shop. (2)
[6]

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QUESTION 2

2.1 Mrs Smith has two classes, each having 30 learners. Their final marks (out of 100) for the year are represented in the box and whisker diagram below.



2.1.1 Determine the interquartile range of Class B. (2)

2.1.2 Explain the significance in the difference of the length of the boxes in the diagram. (2)

2.1.3 Mrs Smith studied the results and made the comment that there was no significant difference in the performance of the two classes. Give TWO reasons you think Mrs Smith will use to prove her statement. (2)

2.2 Eight couples entered a dance competition. Their performances were scored by two judges. The scores (out of 20) are given in the table below.

COUPLE	1	2	3	4	5	6	7	8
JUDGE 1	18	4	6	8	5	12	10	14
JUDGE 2	15	6	3	5	5	14	8	15

2.2.1 Determine the equation of the least squares regression line of the scores given by the two judges. (3)

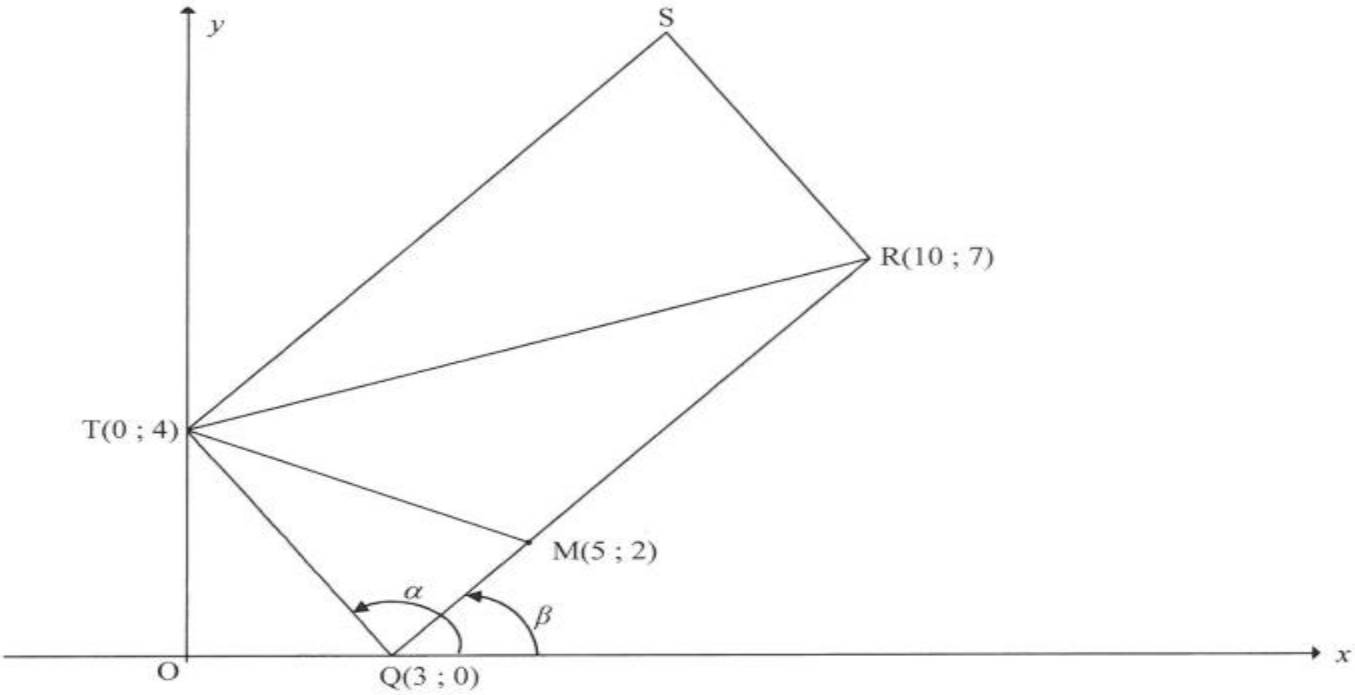
2.2.2 A ninth couple entered late for the competition and received a score of 15 from JUDGE 1. Estimate the score that might have been assigned by JUDGE 2 to the nearest integral value. (2)

2.2.3 Are the judges consistent in assigning scores to the performance of the couples? Prove your answer and support it with relevant statistics. (2)
[13]

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QUESTION 3

In the diagram, Q(3 ; 0), R(10 ; 7), S and T(0 ; 4) are the vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M(5 ; 2). The angles of inclination of TQ and RQ are α and β respectively.



3.1 Calculate the gradient of TQ. (1)

3.2 Calculate the length of RQ. Leave your answer in surd form. (2)

3.3 F(k ; –8) is a point in the Cartesian plane such that T, Q and F are collinear. Calculate the value of k. (4)

3.4 Calculate the coordinates of S. (4)

3.5 Calculate the size of $T\hat{S}R$. (6)

3.6 Calculate, in the simplest form, the ratio of:

3.6.1 $\frac{MQ}{RQ}$ (3)

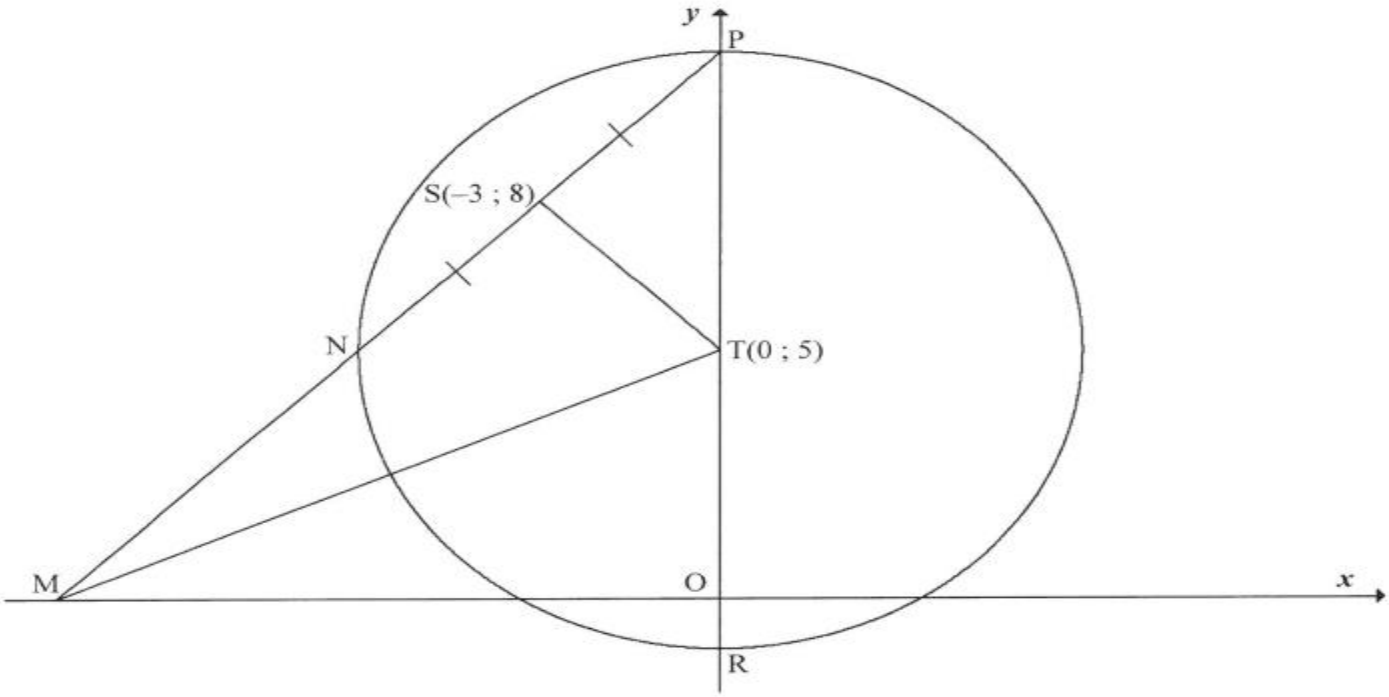
3.6.2 $\frac{\text{area of } \Delta TQM}{\text{area of parallelogram } RQTS}$ (3)

[23]

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QUESTION 4

In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



4.1 Give a reason why TS ⊥ NP. (1)

4.2 Determine the equation of the line passing through N and P in the form $y = mx + c$. (5)

4.3 Determine the equations of the tangents to the circle that are parallel to the x-axis. (4)

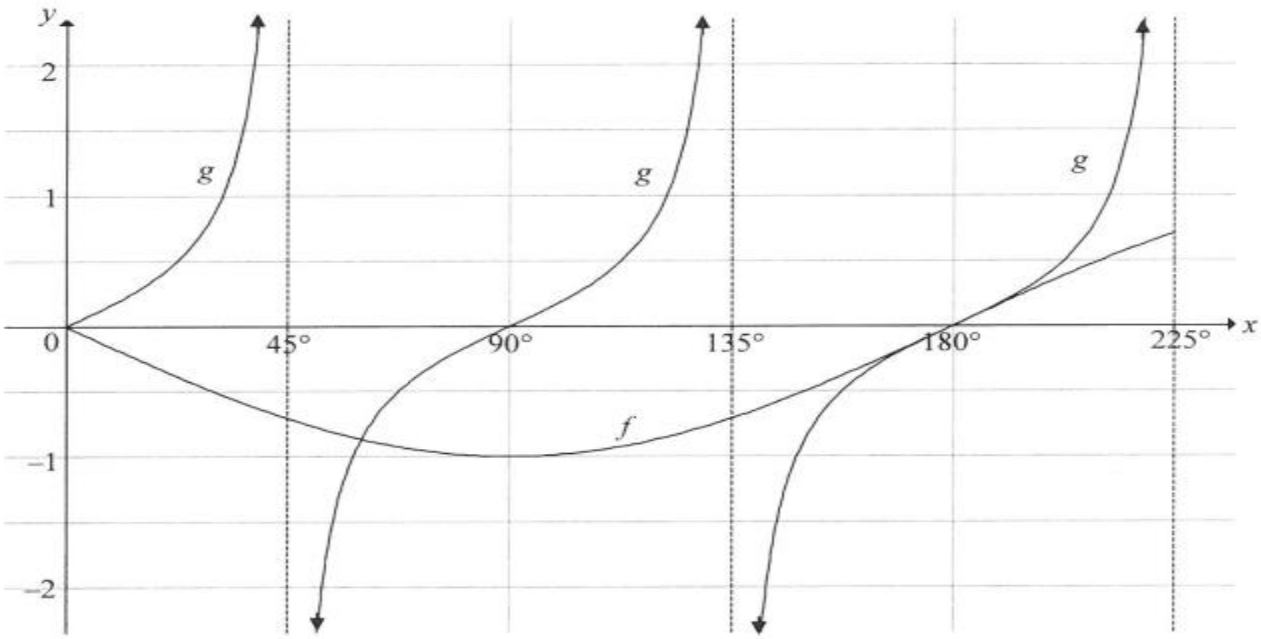
4.4 Determine the length of MT. (4)

4.5 Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle STM in the form $(x-a)^2 + (y-b)^2 = r^2$. (5)
[19]

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QUESTION 5

In the diagram, the graphs of the functions $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0° \le x \le 225°$.



5.1 Write down the values of a and b. (2)

5.2 Write down the period of $f(3x)$. (2)

5.3 Determine the values of x in the interval $90° \le x \le 225°$ for which $f(x).g(x) \le 0$. (3)
[7]

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QUESTION 6

6.1 **Without using a calculator**, determine the following in terms of sin 36°:

6.1.1 sin 324° (1)

6.1.2 cos 72° (2)

6.2 Prove the identity: $1-\frac{\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta$ (4)

6.3 Use QUESTION 6.2 to determine the general solution of:

$$1-\frac{\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}=\frac{1}{4}$$ (6)

6.4 Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$

6.4.1 Use the formula for $\cos(A-B)$ to derive a formula for $\sin(A-B)$. (4)

6.4.2 **Without using a calculator**, show that

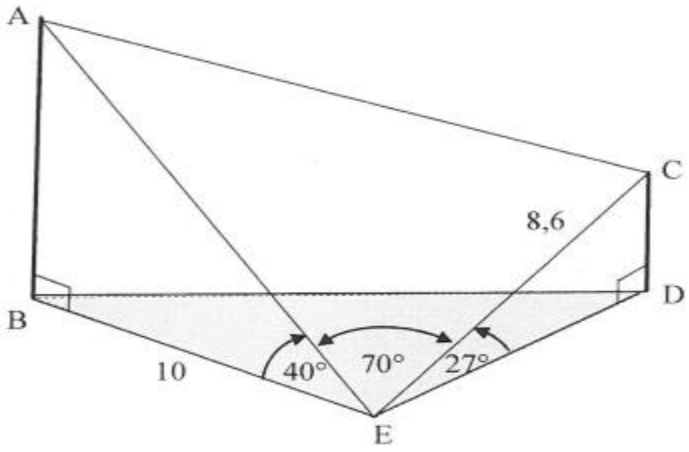
$$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)=\frac{1}{\sqrt{2}}$$

for all values of x. (6)
[23]

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QUESTION 7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cables AE and CE anchor the poles at E. Another steel cable connects A and C. CE = 8,6 m, BE = 10 m, $A\hat{E}B = 40°$, $A\hat{E}C = 70°$ and $C\hat{E}D = 27°$.



Calculate the:

7.1 Height of pole CD (2)

7.2 Length of cable AE (2)

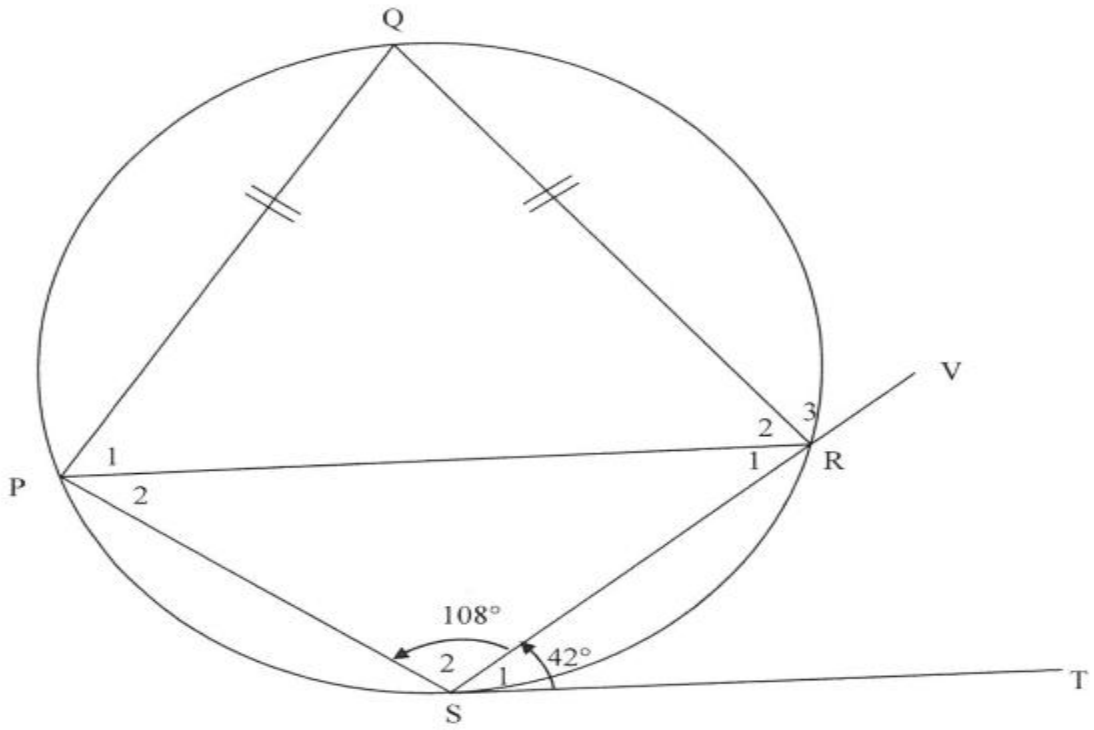
7.3 Length of cable AC (4)
[8]

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Give reasons for ALL statements and calculations in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. PQ = QR, $\hat{S}_1 = 42°$ and $\hat{S}_2 = 108°$.



Determine, with reasons, the size of the following angles:

8.1 $\hat{Q}$ (2)

8.2 $\hat{R}_2$ (2)

8.3 $\hat{P}_2$ (2)

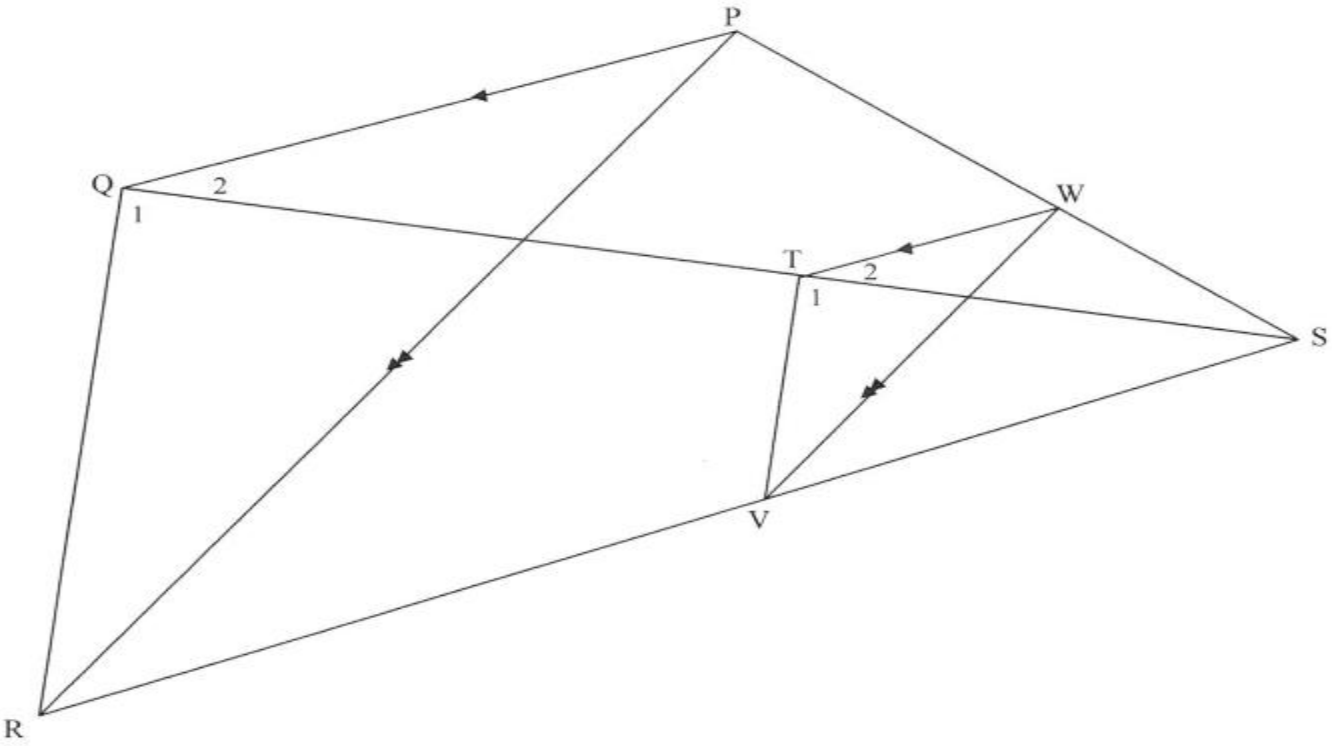
8.4 $\hat{R}_3$ (2)

[8]

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QUESTION 9

In the diagram, PQRS is a quadrilateral with diagonals PR and QS drawn. W is a point on PS. WT is parallel to PQ with T on QS. WV is parallel to PR with V on RS. TV is drawn. PW : WS = 3 : 2.



9.1 Write down the value of the following ratios:

9.1.1 $\frac{ST}{TQ}$ (2)

9.1.2 $\frac{SV}{VR}$ (1)

9.2 Prove that $\hat{T}_1 = \hat{Q}_1$. (4)

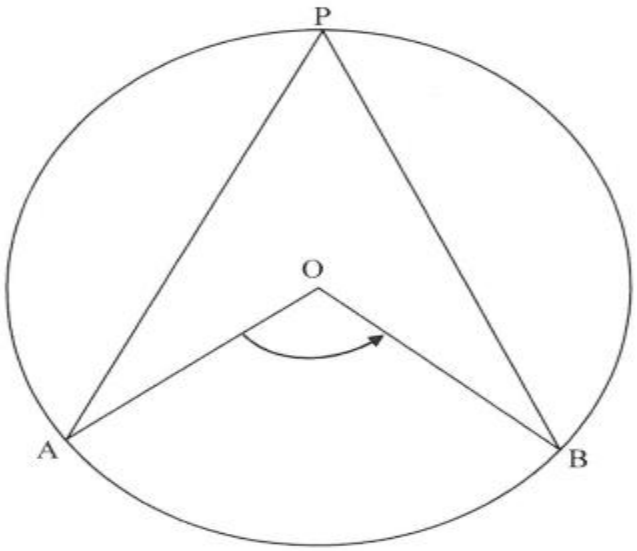
9.3 Complete the following statement: $\Delta VWS \;|||\; \Delta\ldots$ (1)

9.4 Determine WV : PR. (2)
[10]

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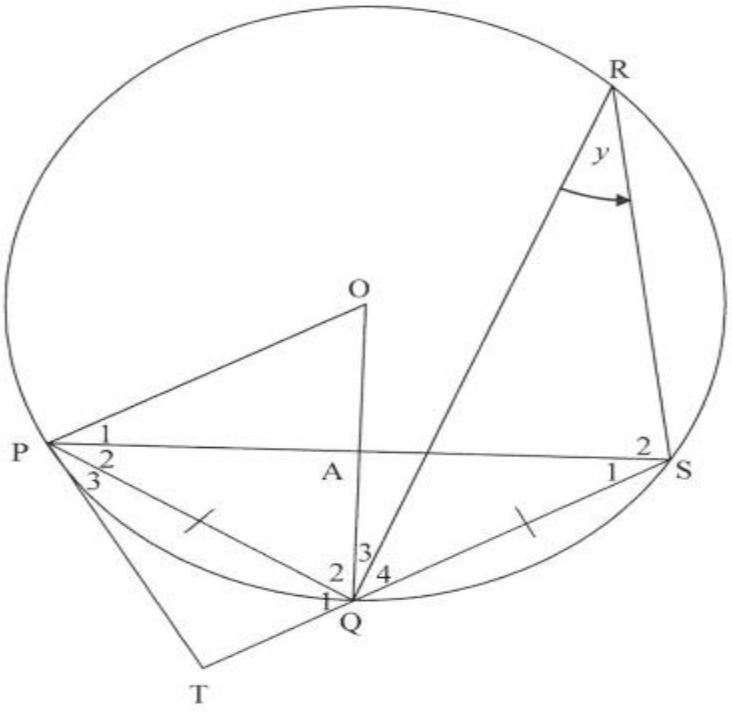
QUESTION 10

10.1 In the diagram, O is the centre of the circle and P is a point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.



Use the diagram to prove the theorem that states that $A\hat{O}B = 2A\hat{P}B$. (5)

10.2 In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle. PQ = QS and $Q\hat{R}S = y$. The tangent at P meets SQ produced at T. OQ intersects PS at A.



10.2.1 Give a reason why $\hat{P}_2 = y$. (1)

10.2.2 Prove that PQ bisects $T\hat{P}S$. (4)

10.2.3 Determine $P\hat{O}Q$ in terms of y. (2)

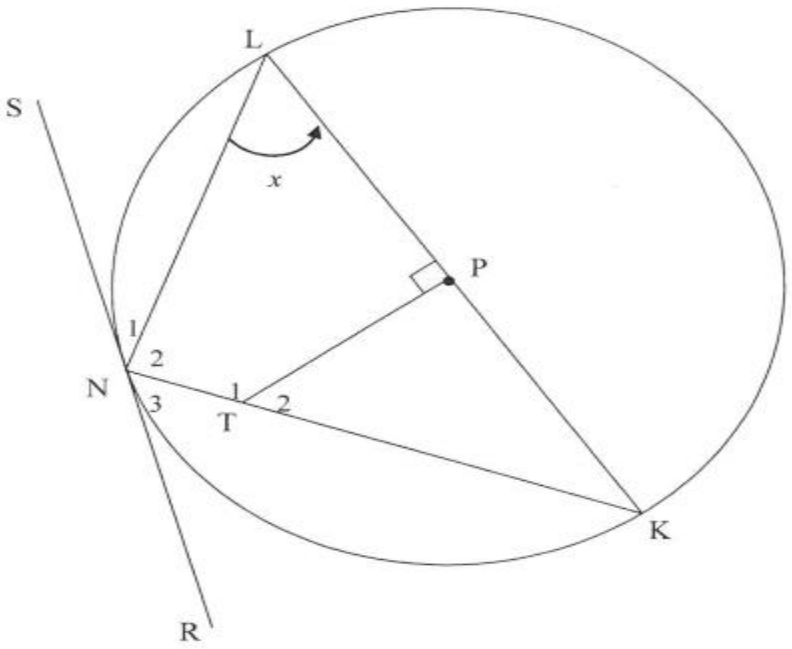
10.2.4 Prove that PT is a tangent to the circle that passes through points P, O and A. (2)

10.2.5 Prove that $O\hat{A}P = 90°$. (5)
[19]

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QUESTION 11

In the diagram, LK is a diameter of the circle with centre P. RNS is a tangent to the circle at N. T is a point on NK and TP ⊥ KL. $P\hat{L}N = x$.



11.1 Prove that TPLN is a cyclic quadrilateral. (3)

11.2 Determine, giving reasons, the size of $\hat{N}_1$ in terms of x. (3)

11.3 Prove that:

11.3.1 $\Delta KTP \,|||\, \Delta KLN$ (3)

11.3.2 $KT \cdot KN = 2KT^2 - 2TP^2$ (5)

[14]

TOTAL: 150

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MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2017

MEMORANDUM

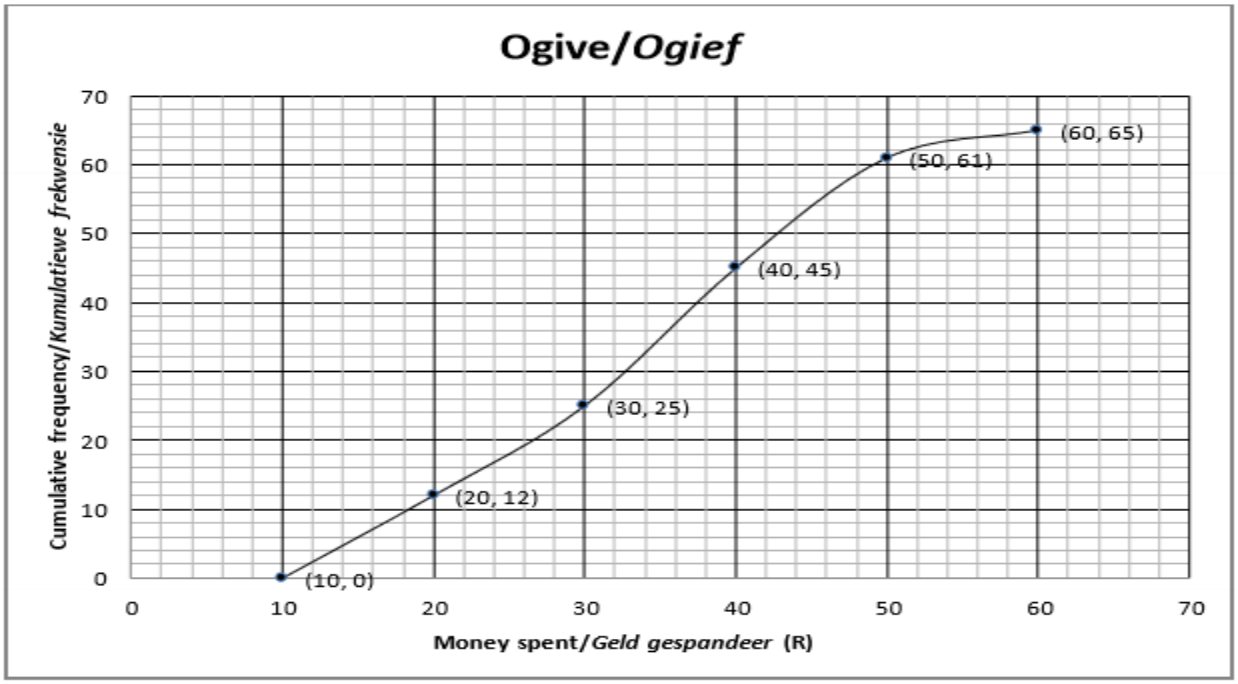
MARKS / PUNTE: 150

This memorandum consists of 21 pages.
Hierdie memorandum bestaan uit 21 bladsye.

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QUESTION/*VRAAG* 1



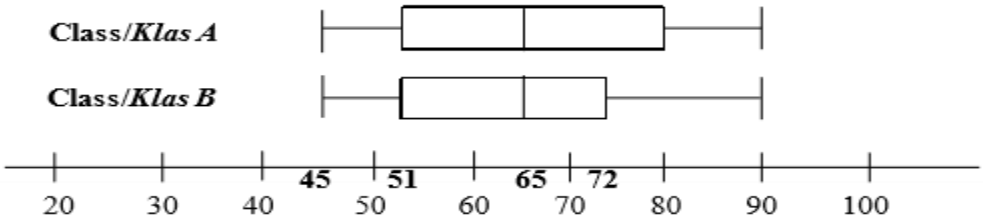
Amount of money/ *Bedrag geld* (in R)	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$	$50 \le x < 60$
Frequency *Frekwensie*	a	13	20	b	4

1.1	65 learners/*leerders*	✓ answer (1)
1.2	Modal class/*Modale klas*: $30 \le x < 40$	✓ answer (1)
1.3	$a = 12$ $b = 61 - 45$ $= 16$	✓ answer ✓ answer (2)
1.4	No. of learners/*Aantal leerders* $= 65 - 54$ **OR/*OF*** $65 - 55$ $= 11$ $= 10$ Answer only: full marks	✓ 54 or 55 ✓ 11 or 10 (2) **[6]**

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QUESTION/*VRAAG* 2

2.1



2.1.1	IQR of Class B/*IKV van Klas B* = $Q_3 - Q_1$ = 72 – 51 = 21 marks/*punte*	✓72 and 51 ✓ 21 only (2)
2.1.2	Although the boxes contain the same number of data points, the marks for Class A are more widely spread./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas A meer verspreid.* **OR/OF** Although the boxes contain the same number of data points, the marks for Class B are more clustered./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas B nader aan mekaar.*	✓ ✓ Class A is more widely spread (2) ✓ ✓ Class B is more clustered (2)
2.1.3	Medians are the same/*Mediane is dieselfde* Ranges are the same **OR** Maximum and minimum values are the same/*Variasiewydtes is dieselfde **OF** die maksimum en minimum waarde is dieselfde* 75% of both classes obtained 51 and above/*75% van albei klasse behaal 51 en meer.*	✓ ✓ any TWO of the 3 reasons mentioned (2)

2.2

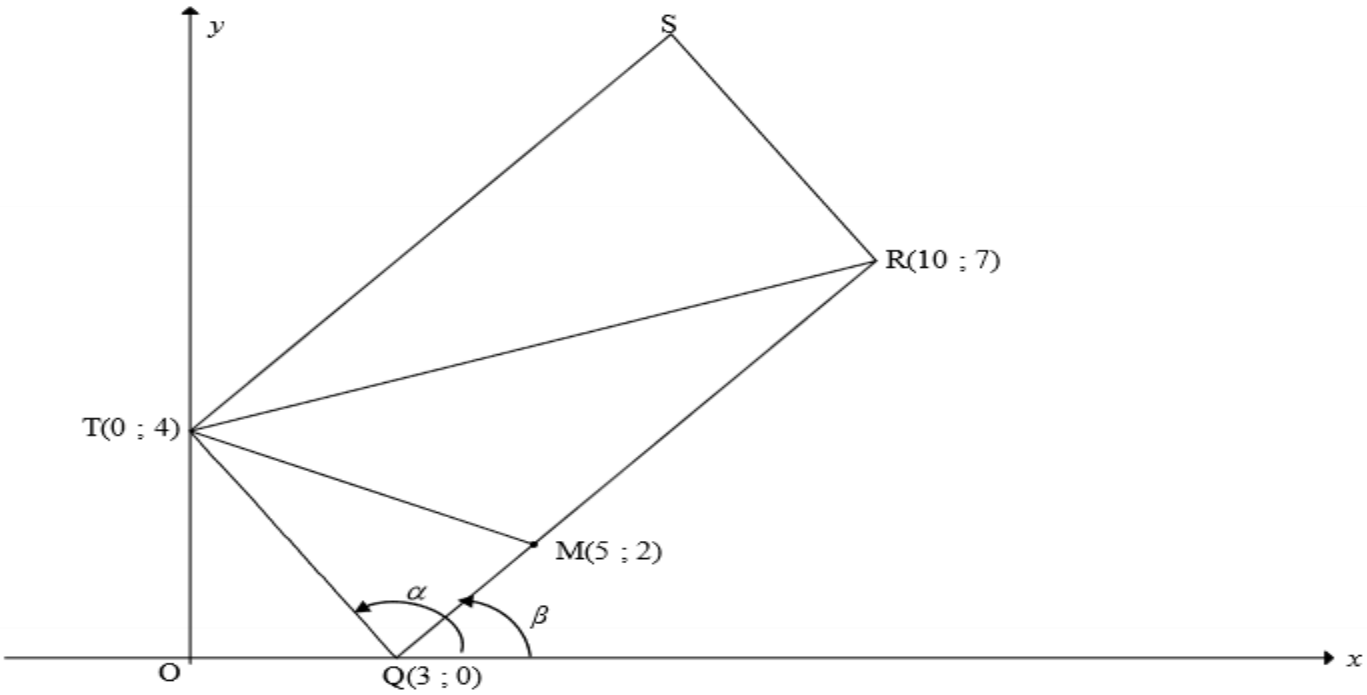
COUPLE/*PAAR*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
JUDGE 1/ *BEOORDELAAR 1*	18	4	6	8	5	12	10	14
JUDGE 2/ *BEOORDELAAR 2*	15	6	3	5	5	14	8	15

2.2.1	$a = -0,03$ $b = 0,93$ $\hat{y} = -0,03 + 0,93x$	✓ value a ✓ value b ✓ equation (3)
2.2.2	$\hat{y} = -0,03 + 0,93(15)$ = 13,92 **OR/*OF*** 13,85 ≈ 14	✓ substitution ✓ answer (2)
2.2.3	Yes **OR** they are consistent, because $r = 0,9$. ($r = 0,89567\ldots$)/*Ja OF hulle is konsekwent, want $r = 0,9$. ($r = 0,89567\ldots$)*	✓ statement ✓ $r = 0,9$ (2) **[13]**

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QUESTION/*VRAAG* 3



3.1	$m_{TQ} = \frac{4-0}{0-3}$ $= -\frac{4}{3}$	✓ answer (1)
3.2	$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $RQ = \sqrt{(10-3)^2+(7-0)^2}$ $RQ = \sqrt{98} = 7\sqrt{2}$	✓ substitution/*substitusie* ✓ answer in surd form (2)
3.3	$m_{FQ} = m_{TQ}$ $\frac{-8}{k-3} = -\frac{4}{3}$ $4k-12 = 24$ $k = 9$ **OR/*OF*** $m_{FT} = m_{QT}$ $\frac{-8-4}{k-0} = -\frac{4}{3}$ $-36 = -4k$ $k = 9$ **OR/*OF*** Equation of TQ: $y = -\frac{4}{3}x+4$ $-8 = -\frac{4}{3}k+4$ $k = 9$	✓ equating gradients/*stel gradient gelyk* ✓ $m_{FQ} = \frac{-8}{k-3}$ ✓ simplification/*vereenvoudig* ✓ answer (4) ✓ gradient ✓ equation of TQ/*vgl van TQ* ✓ substitution of $(k\ ;-8)$ /*substitusie van* $(k\ ;-8)$ ✓ answer (4)

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3.4	Using transformation/*Gebruik transformasie*: $\therefore$ S(7 ; 11) **OR/*OF*** Midpoint of TR = midpoint of SQ [diag ||m/*hkle*||*m*] Midpoint of TR = $(5 ; \frac{11}{2})$ $\frac{x_S+3}{2}=5$ and $\frac{y_S+0}{2}=\frac{11}{2}$ $\therefore x_S=7$ and $y_S=11$ $\therefore$ S(7 ; 11) **OR/*OF*** Equation of TS: $y=\left(\frac{7-2}{10-5}\right)x+4=x+4$ Equation of RS: $y-7=-\frac{4}{3}(x-10)$ $y=-\frac{4}{3}x+\frac{61}{3}$ $x+4=-\frac{4}{3}x+\frac{61}{3}$ $7x=49$ $x=7$ $\therefore y=11$ $\therefore$ S(7 ; 11)	✓ ✓ *x*–value/*waarde* ✓ ✓ *y*–value/*waarde* (4) ✓ *x*–value/*waarde* of/*van* T ✓ *y*–value/*waarde* of/*van* T ✓ *x*–value/*waarde* of/*van* S ✓ *y*–value/*waarde* of/*van* S (4) ✓ equations of TS and RS/*vgls van TS en RS* ✓ equating / *gelykstel* ✓ *x*–value/*waarde* ✓ *y*–value/*waarde* (4)
3.5	$T\hat{S}R = T\hat{Q}R$ [opp $\angle$s of ||m/*teenoorst* $\angle e$ ||*m*] $T\hat{Q}R = \alpha - \beta$ $\tan\alpha = m_{TQ} = -\frac{4}{3}$ $\therefore \alpha = 180° - 53{,}13° = 126{,}87°$ $\tan\beta = m_{RQ} = \frac{7}{7} = 1$ $\therefore \beta = 45°$ $T\hat{Q}R = 126{,}87° - 45°$ $= 81{,}87°$ $T\hat{S}R = 81{,}87°$ **OR/*OF***	✓ $T\hat{Q}R = \alpha - \beta$ ✓ $\tan\alpha = m_{TQ}$ ✓ α ✓ $\tan\beta = m_{RQ}$ ✓ β ✓ answer (6)

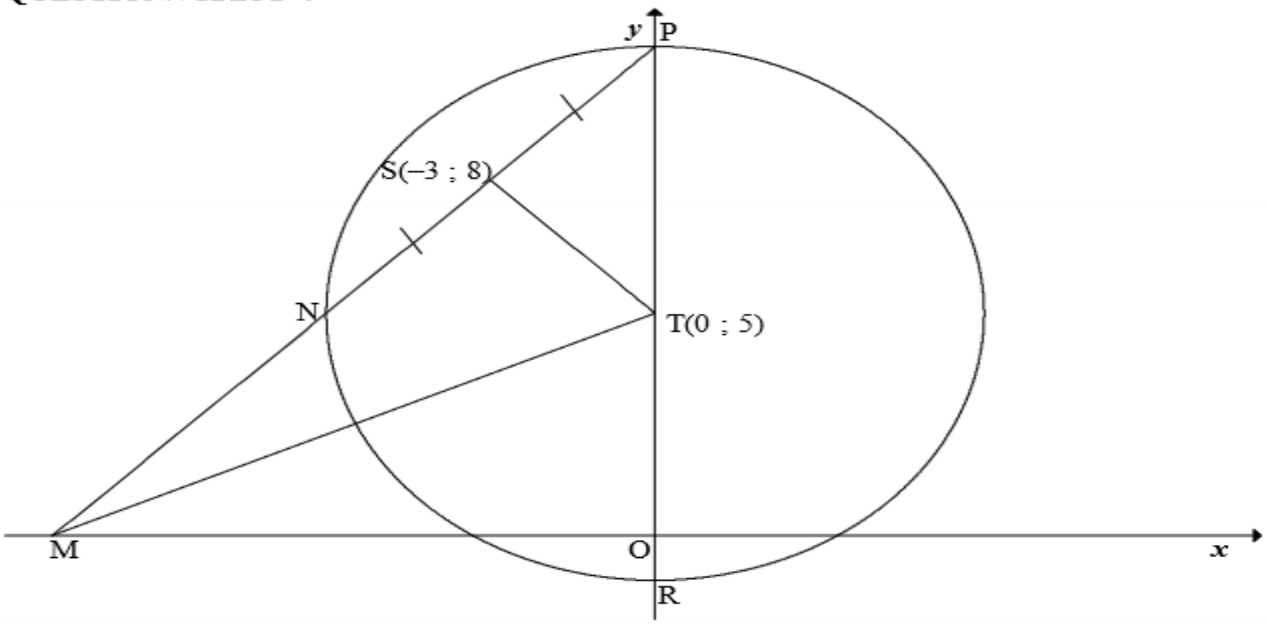
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	$TQ = SR = 5$ $TR = \sqrt{100+9} = \sqrt{109}$ $RQ = TS = \sqrt{49+49} = \sqrt{98}$ $\cos R\hat{Q}T = \cos T\hat{S}R = \frac{TQ^2 + RQ^2 - TR^2}{2.TQ.RQ}$ $= \frac{25+98-109}{2(5)(\sqrt{98})}$ $= 0{,}141\ldots$ $R\hat{Q}T = T\hat{S}R = 81{,}87°$	✓ length of TQ **OR** SR ✓ length of TR ✓ length of RQ **OR** TS ✓ correct subst into cosine rule ✓ simplification ✓ answer (6)
3.6.1	$MQ = \sqrt{(5-3)^2 + (2-0)^2}$ $MQ = \sqrt{8}$ $\frac{MQ}{RQ} = \frac{\sqrt{8}}{\sqrt{98}}$ Answer only: full marks $= \frac{2}{7}$ or 0,29	✓substitution/*substitusie* ✓ $MQ = \sqrt{8} = 2\sqrt{2}$ ✓ answer (3)
3.6.2	$\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{\frac{1}{2}.QM.\perp h}{\frac{1}{2}.QR.\perp h}$ [$\perp$h same/*dieselfde*] $= \frac{QM}{QR} = \frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\text{area of } \Delta TQM}{2 \times \text{area of } \Delta TQR}$ $= \frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{7}$ **OR/*OF*** $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{QM}{QR}$ $= \frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\text{area of } \Delta TQM}{2\text{area of } \Delta TQR}$ $= \frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{7}$ **OR/*OF***	✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{2}{7}$ ✓ area parm $RQTS = 2$area ΔTQR ✓ answer (3) ✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{2}{7}$ ✓ area parm $RQTS = 2$area ΔTQR ✓ answer (3)

	$\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$ $= \frac{1}{2}\left(\frac{2}{7}\right)$ $= \frac{1}{7}$ **OR/*OF*** $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QT.QM\sin(\alpha-\beta)}{2\text{area of } \Delta QTR}$ $= \frac{\frac{1}{2}QT.QM\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR\sin(\alpha-\beta)]}$ $= \frac{1}{2}\left(\frac{2}{7}\right)$ $= \frac{1}{7}$	✓ $\frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$ ✓ $\frac{1}{2}\left(\frac{2}{7}\right)$ ✓ answer (3) ✓ area parm $RQTS = 2$area ΔTQR ✓ $\frac{\frac{1}{2}QT.QM\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR\sin(\alpha-\beta)]}$ ✓ answer (3) **[23]**

QUESTION/*VRAAG* 4



4.1	line from centre to **midpt of chord** / *lyn vanaf midpt na midpt van koord*	✓ answer (1)
4.2	$m_{ST} = \frac{8-5}{-3-0}$ $= -1$ $m_{ST} \times m_{NP} = -1$ [TS ⊥ NP] $\therefore m_{NP} = 1$ $\therefore y = x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = x + 11$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 8 = 1(x + 3)$ $y = x + 11$	✓ subst (–3 ; 8) and (0 ; 5) into gradient formula ✓ m_{ST} ✓ m_{NP} ✓ subst (–3 ; 8) into equation of a line ✓ equation (5)
4.3	P(0 ; 11) [y-intercept of chord NP] ∴ radius is 6 units R(0 ; –1) Equations of the tangents to the circle parallel to the x-axis/ *Vgls van die raaklyne aan die sirkel || aan die x-as*: $y = 11$ and $y = -1$	✓ coordinates of P/ koördinate v P ✓ coordinates of R koördinate van R ✓✓ answers (4)
4.4	M(–11 ; 0) [x–intercept of/*x-afsnit van* NP] $MT = \sqrt{(0-11)^2 + (5-0)^2}$ $MT = \sqrt{146} = 12{,}08$	✓✓ coordinates of M ✓ substitution ✓ answer (4)

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4.5	MT = diameter/*middellyn* [conv∠ in $\frac{1}{2}$ circle/*omgek* ∠*in* $\frac{1}{2}$ *sirkel*] radius $= \frac{\sqrt{146}}{2}$ units Centre of circle/*Middelpunt v sirkel* = Midpoint MT /*Middelpunt MT* $= \left(\frac{-11}{2}; \frac{5}{2}\right)$ Equation of circle through S, T and M: $\left(x+\frac{11}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\frac{146}{4}$ **OR/*OF*** $\left(x+5\frac{1}{2}\right)^2+\left(y-2\frac{1}{2}\right)^2=\frac{73}{2}=6{,}04$	✓ radius of circle ✓ *x* value of M ✓ *y* value of M ✓ LHS of equation ✓ RHS of equation (5) **[19]**

QUESTION/*VRAAG* 5

5.1	$a=-1$ $b=2$	✓ answer ✓ answer (2)
5.2	$f(3x)=-\sin 3x$ Period of $f(3x)=\frac{360°}{3}$ $=120°$ Answer only: Full marks	✓ $\frac{360°}{3}$ ✓ answer (2)
5.3	$x \in [90°; 135°) \cup \{180°\}$ **OR/*OF*** $90° \le x < 135°$ or $x = 180°$	✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct brackets (3) ✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct inequalities (3) **[7]**

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QUESTION/*VRAAG 6*

6.1.1	$\sin(360° - 36°) = -\sin 36°$	✓ answer (1)
6.1.2	$\cos 72° = \cos(2 \times 36°)$ $= 1 - 2\sin^2 36°$ Answer only: Full marks	✓ double angle/*dubbelhoek* ✓answer (2)
6.2	R.T.P.: $1 - \frac{\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta$ $\text{LHS} = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{1}{\cos^2\theta}}$ $= \cos^2\theta$ $= \text{RHS}$ **OR/*OF*** $\text{LHS} = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}} \times \frac{\cos^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$ $= \frac{\cos^2\theta}{1}$ $= \cos^2\theta$ $= \text{RHS}$ **OR/*OF***	✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity/ *kwosiëntidentiteit* ✓ denominator as a single fraction / *Noemer as enkelbreuk* ✓ square identity/*vierkantidentiteit* (4) ✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity / *kwosiëntidentiteit* ✓ $\times \frac{\cos^2\theta}{\cos^2\theta}$ ✓ square identity/*vierkantidentiteit* (4) ✓ quotient identity/

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	$\text{LHS} = 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \div \left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\right)$ $= 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}\right)$ $= 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1}\right)$ $= 1-\sin^2\theta$ $= \cos^2\theta$ $= \text{RHS}$	*kwosiëntidentiteit* ✓ writing as a single fraction/ *skryf as enkelbreuk* ✓ square identity/*vierkantidentiteit* ✓ simplification/*vereenvoudiging* (4)
6.3	$\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = 60°+k.360°$ or $\frac{1}{2}x = 300°+k.360°$ or $\frac{1}{2}x = 120°+k.360°$ or $\frac{1}{2}x = 240°+k.360°$ $x = 120°+k.720°$ or $x = 600°+k.720°$ or $x = 240°+k.720°$ or $x = 480°+k.720°;\ k \in Z$ **OR/*OF*** $\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = \pm 60°+k.360°$ or $\frac{1}{2}x = \pm 120°+k.360°$ $x = \pm 120°+k.720°$ or $x = \pm 240°+k.720°;\ k \in Z$	✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ 60° and 300° ✓ 120° and 240° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°;\ k \in Z$ (6) ✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ $\pm 60°$ ✓ $\pm 120°$ ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°\ k \in Z$ (6)

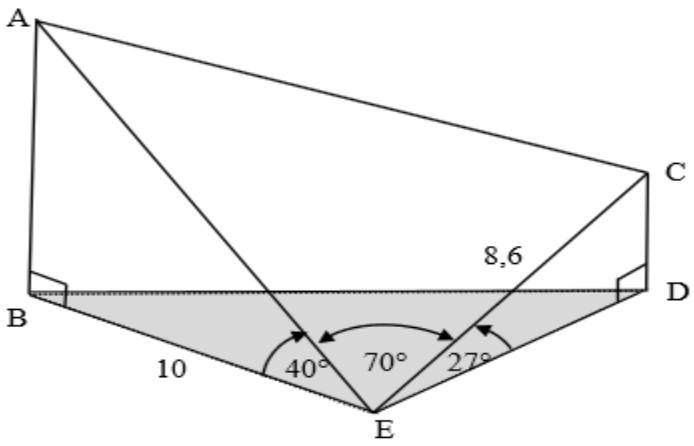
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6.4.1	$\sin(A-B) = \cos[90° - (A-B)]$ $= \cos[(90° - A) - (-B)]$ $= \cos(90° - A)\cos(-B) + \sin(90° - A)\sin(-B)$ $= \sin A\cos B + \cos A(-\sin B)$ $= \sin A\cos B - \cos A\sin B$ **OR/*OF*** $\sin(A-B) = \cos[90° - (A-B)]$ $= \cos[(90° + B) - A]$ $= \cos(90° + B)\cos A + \sin(90° + B)\sin A$ $= -\sin B\cos A + \cos B\sin A$ $= \sin A\cos B - \cos A\sin B$	✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4) ✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4)
6.4.2	$\sin(x+64°)\cos(x+379°) + \sin(x+19°)\cos(x+244°)$ $= \sin(x+64°)\cos(x+19°) + \sin(x+19°)[-\cos(x+64°)]$ $= \sin(x+64°)\cos(x+19°) - \cos(x+64°)\sin(x+19°)$ $= \sin[x+64° - (x+19°)]$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$	✓ $\cos(x+379°) = \cos(x+19°)$ ✓✓ $\cos(x+244°) = -\cos(x+64°)$ ✓✓compound formula identity/ *saamgestelde identiteit* ✓ $\sin 45°$ (6) **[23]**

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QUESTION/*VRAAG* 7

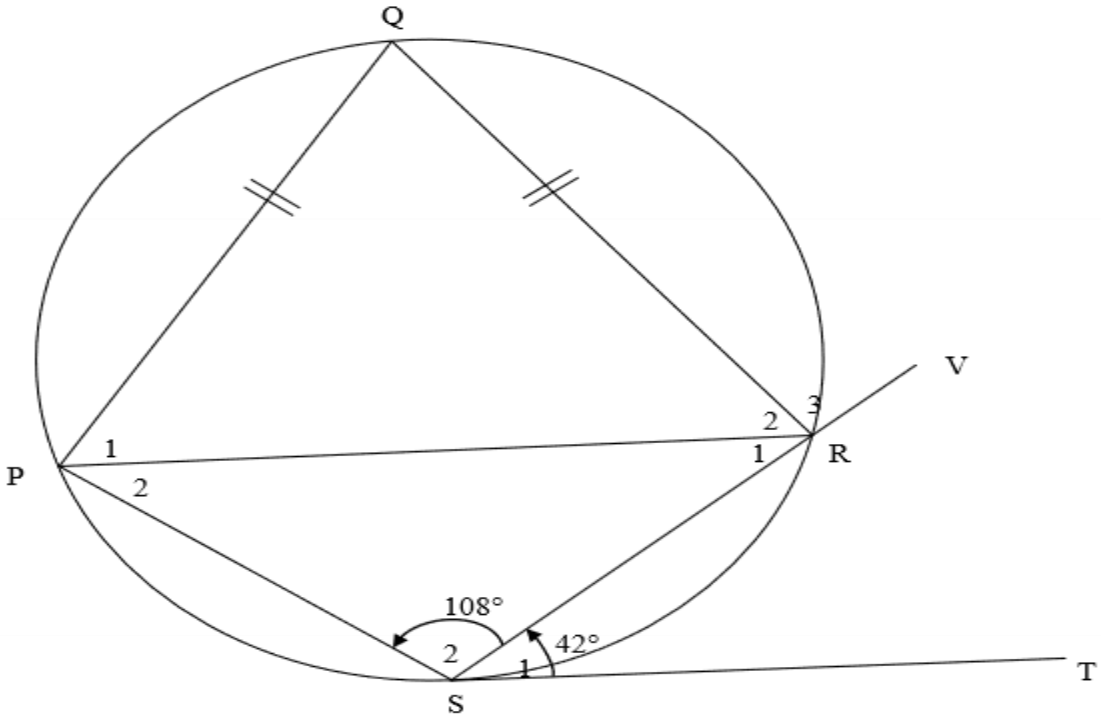


7.1	$\sin 27° = \frac{CD}{8,6}$ $CD = 8,6\sin 27°$ $CD = 3,90\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.2	$\cos 40° = \frac{10}{AE}$ $AE = \frac{10}{\cos 40°}$ $AE = 13,05\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.3	$AC^2 = CE^2 + AE^2 - 2\,CE.AE(\cos A\hat{E}C)$ $= (8,6)^2 + (13,05)^2 - 2(8,6)(13,05)(\cos 70°)$ $= 167,49$ $AC = 12,94\text{ m}$	✓ correct use of cosine rule in ΔACE/ *korrekte gebruik van reel in* ΔACE ✓ correct subst into cosine rule ✓ AC^2 ✓ answer (4) **[8]**

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KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

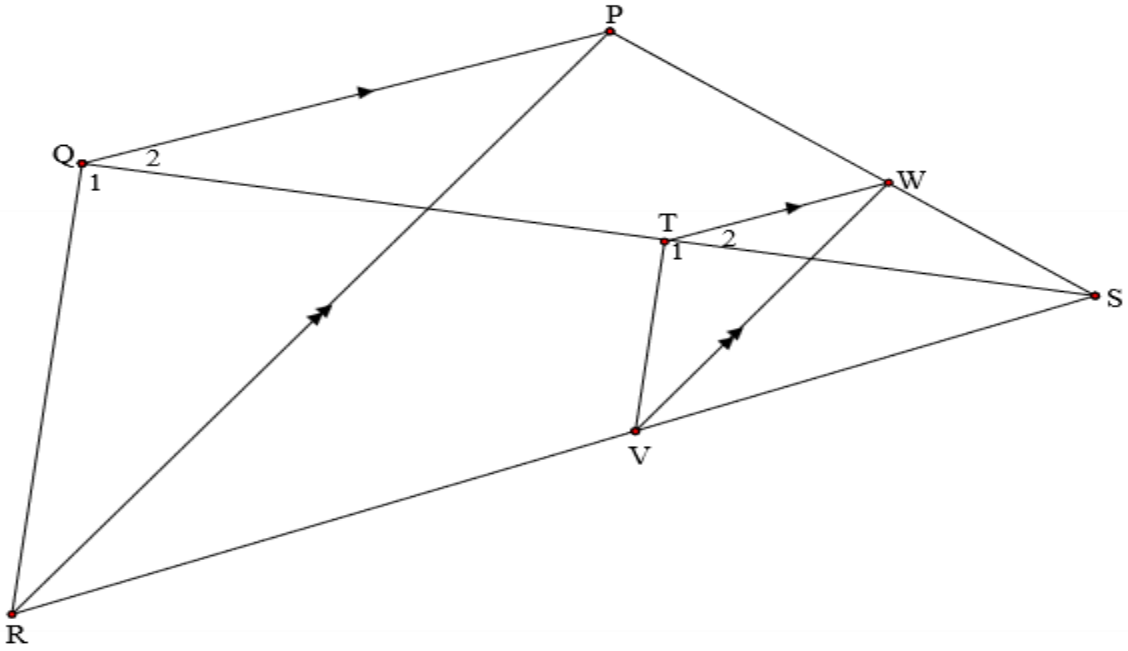
QUESTION/*VRAAG* 8



8.1	$\hat{Q} = 72°$ [opp ∠s of cyclic quad/*teenoorst ∠e koordevh*]	✓ S ✓ R (2)
8.2	$\hat{R}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{R}_2 = \frac{180° - 72°}{2}$ [sum of ∠s in Δ/*som v ∠e in Δ*] $= 54°$	✓ S/R ✓ answer (2)
8.3	$\hat{P}_2 = 42°$ [tan chord theorem/*raakl-koordst*]	✓ S ✓ R (2)
8.4	$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= 54° + 42°$ $= 96°$ **OR/*OF*** $\hat{R}_1 = 180° - 108° - 42° = 30°$ [sum of/*som van*∠s/e in Δ] $\hat{R}_3 = 180° - \hat{R}_1 - \hat{R}_2$ [∠s on str line/*∠e op reguitlyn*] $= 180° - 30° - 54°$ [sum of/*som van*∠s/e in Δ] $= 96°$	✓ R ✓ S (2) ✓ $\hat{R}_1 = 30°$ ✓ S (2) **[8]**

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QUESTION/*VRAAG* 9

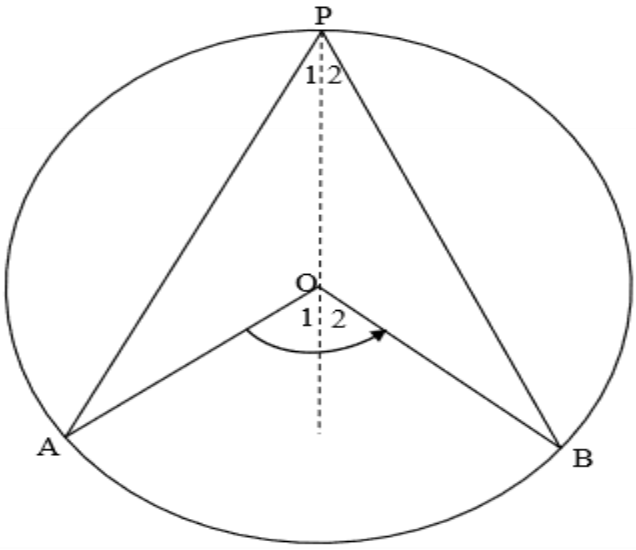


9.1.1	$\frac{ST}{TQ}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; TW ‖ QP] $=\frac{2}{3}$	✓ S ✓ S (2)
9.1.2	$\frac{SV}{VR}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; VW ‖ RP] $=\frac{2}{3}$	✓ answer (1)
9.2	$\frac{ST}{TQ}=\frac{SV}{VR}$ [both equal/*beide gelyk* $\frac{WS}{PW}$] ∴ TV ‖ QR [line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*] $\therefore \hat{T}_1=\hat{Q}_1$ [corresp/*ooreenkomst* ∠s/e; TV ‖ QR]	✓ S ✓ S ✓ R ✓ R (4)
9.3	ΔVWS ||| ΔRPS	✓ ΔRPS (any order) (1)
9.4	$\frac{WV}{PR}=\frac{SW}{SP}$ [ΔVWS ||| ΔRPS] $=\frac{2}{5}$ **OR/*OF*** $\frac{WV}{PR}=\frac{SV}{SR}$ [ΔVWS ||| Δ RPS] $=\frac{2}{5}$	✓ ratio ✓ answer (2) **[10]**

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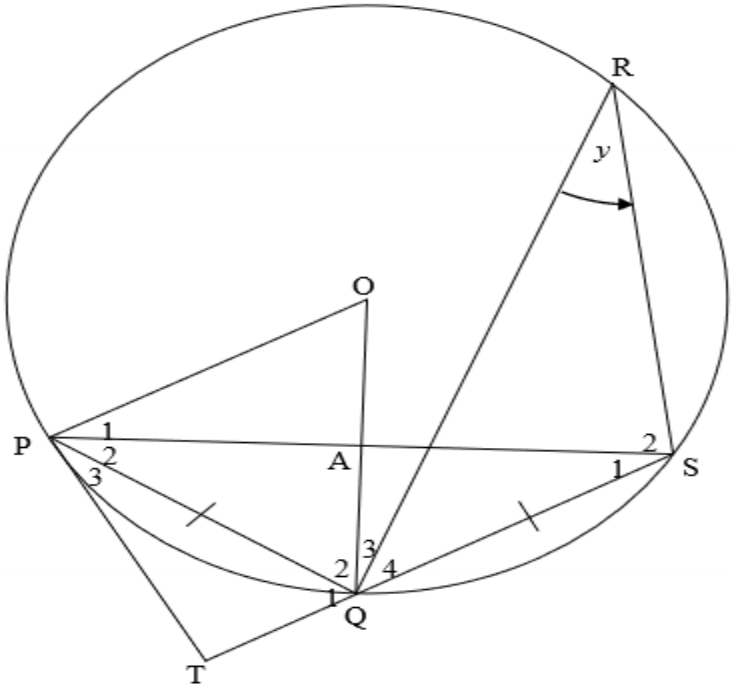
QUESTION/*VRAAG* 10

10.1



	Constr/*Konst* : Draw line PO and extend /*Trek lyn PO en verleng* Proof/*Bewys* : $OP = OA$ [radii] $\therefore \hat{P}_1 = \hat{A}$ [$\angle$s opp/*teenoor* = sides/*sye*] but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext $\angle$ of Δ] $\therefore \hat{O}_1 = 2\hat{P}_1$ Similarly/*Netso*, $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ i.e. $A\hat{O}B = 2A\hat{P}B$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S (5)

10.2



10.2.1	∠s in the same segment/ *∠e in dieselfde sirkelsegment*	✓ R (1)
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ [∠s opp equal sides/*∠e teenoor = sye*] $\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/*raakl-koordst*] $\therefore \hat{P}_2 = \hat{P}_3$ $\therefore$ PQ bisects $T\hat{P}S$	✓ S ✓ R ✓ S ✓ R (4)
10.2.3	$P\hat{O}Q = 2\hat{S}_1 = 2y$ [∠at centre =2×∠at circ/*midpts∠= 2×omtreks∠*]	✓ S ✓ R (2)
10.2.4	$T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/*bewys* in 11.2.2] $\therefore T\hat{P}A = P\hat{O}Q$ [proved/*bewys* in 11.2.3] $\therefore$ PT = tangent [converse tan chord theorem/*omgek raakl-koordst*]	✓ $T\hat{P}A = P\hat{O}Q$ ✓ R (2)

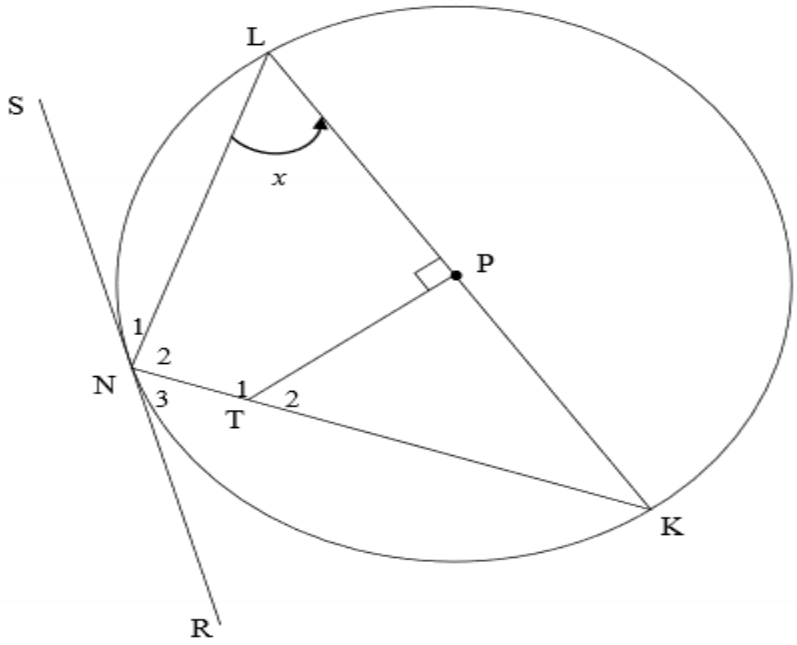
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10.2.5	$O\hat{P}Q + O\hat{Q}P = 180° - 2y$ [sum of/*sum v* ∠s/*e* in Δ]	✓ S
	$\therefore O\hat{Q}P = 90° - y$ [∠s opp equal sides/*∠e to = sye;* OP = OQ]	✓ S ✓ R
	In ΔPAQ:	
	$O\hat{Q}P + \hat{P}_2 + Q\hat{A}P = 180°$	
	$90° - y + y + Q\hat{A}P = 180°$ [sum of/*sum v* ∠s/*e* in Δ]	✓ S
	$Q\hat{A}P = 90°$	✓ S
	$\therefore O\hat{A}P = 90°$ [∠s/*e* on straight line/*op reguitlyn*]	(5)
	OR/*OF*	
	$O\hat{P}T = 90°$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$\therefore \hat{P}_1 = 90° - 2y$	✓ S
	$\hat{P}_1 + \hat{O} + O\hat{A}P = 180°$ [sum of/*sum v* ∠s/*e* in Δ]	
	$(90° - 2y) + 2y + O\hat{A}P = 180°$	✓ S
	$\therefore O\hat{A}P = 90°$	✓ S (5)
	OR/*OF*	
	POSQ is a kite/*'n vlieër*	
	$\therefore OQ \perp PS$ [diag of a kite/*hoeklyne v vlieër*]	✓✓✓ S
	$\therefore O\hat{A}P = 90°$	✓✓ R (5)
	OR/*OF*	
	In ΔOAP and ΔOAS	
	OP = OS (radii)	✓ S
	OA is common	✓ S
	$P\hat{O}A = 2y$	
	$= 2\hat{P}_2$	
	$= Q\hat{O}S$	✓ S
	ΔOAP ≡ ΔOAS (SAS)	✓ R
	$O\hat{A}P = O\hat{A}S$ (≡Δs)	
	$O\hat{A}P = O\hat{A}S = 90°$ (∠s on str line)	✓ S (5) **[19]**

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QUESTION/*VRAAG* 11



11.1	$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad/ *'n koordevh* [opp ∠s of quad is suppl/ *teenoor∠e v vh is suppl*] **OR** $\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad [ext ∠ = int opp ∠/*buite∠ = to binne ∠*]	✓ S ✓ R ✓ R (3) ✓ S ✓ R ✓ R (3)
11.2	$\hat{T}_2 = P\hat{L}N = x$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{N}_3 = x$ [tan chord theorem/*raakl-koordst*] $\hat{N}_2 = 90°$ [∠in semi circle/ *halfsirkel*] $\hat{N}_1 = 90° - x$ [straight line/*reguitlyn*]	✓ R ✓ S✓ R (3) ✓ R ✓ S✓ R (3) ✓ R ✓ S ✓ S (3)

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11.3.1	In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$ [$\angle\angle\angle$] **OR/*OF*** In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\hat{T}_2 = P\hat{L}N = x$ [proved in 11.2 **OR** sum of $\angle$s in Δ] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
11.3.2	$\frac{KT}{KL} = \frac{KP}{KN}$ [$\mid\mid\mid$ Δs] $\therefore KT \cdot KN = KP \cdot KL$ But $KL = 2KP$ [radii: PK = LP] $\therefore KT \cdot KN = KP \cdot 2KP$ $= 2KP^2$ $= 2(KT^2 - TP^2)$ [Theorem of Pythagoras] $= 2KT^2 - 2TP^2$	✓ S/R ✓ S ✓ S ✓ S ✓ S (5) **[14]**

TOTAL/*TOTAAL*: 150

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 27 pages.

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QUESTION 1

An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

DAYS	1	2	3	4	5	6	7	8	9	10
UNITS OF BLOOD	45	59	65	73	79	82	91	99	101	106

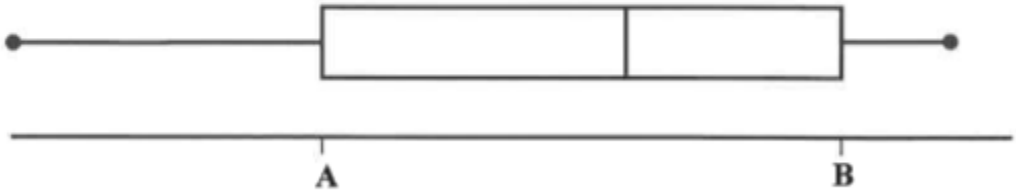
1.1 Calculate:

1.1.1 The mean of the units of blood donated per day over the period of 10 days (2)

1.1.2 The standard deviation of the data (2)

1.1.3 How many days is the number of units of blood donated at college X outside one standard deviation from the mean? (3)

1.2 The number of units of blood donated by the students of college X is represented in the box and whisker diagram below.



1.2.1 Describe the skewness of the data. (1)

1.2.2 Write down the values of **A** and **B**, the lower quartile and the upper quartile respectively, of the data set. (2)

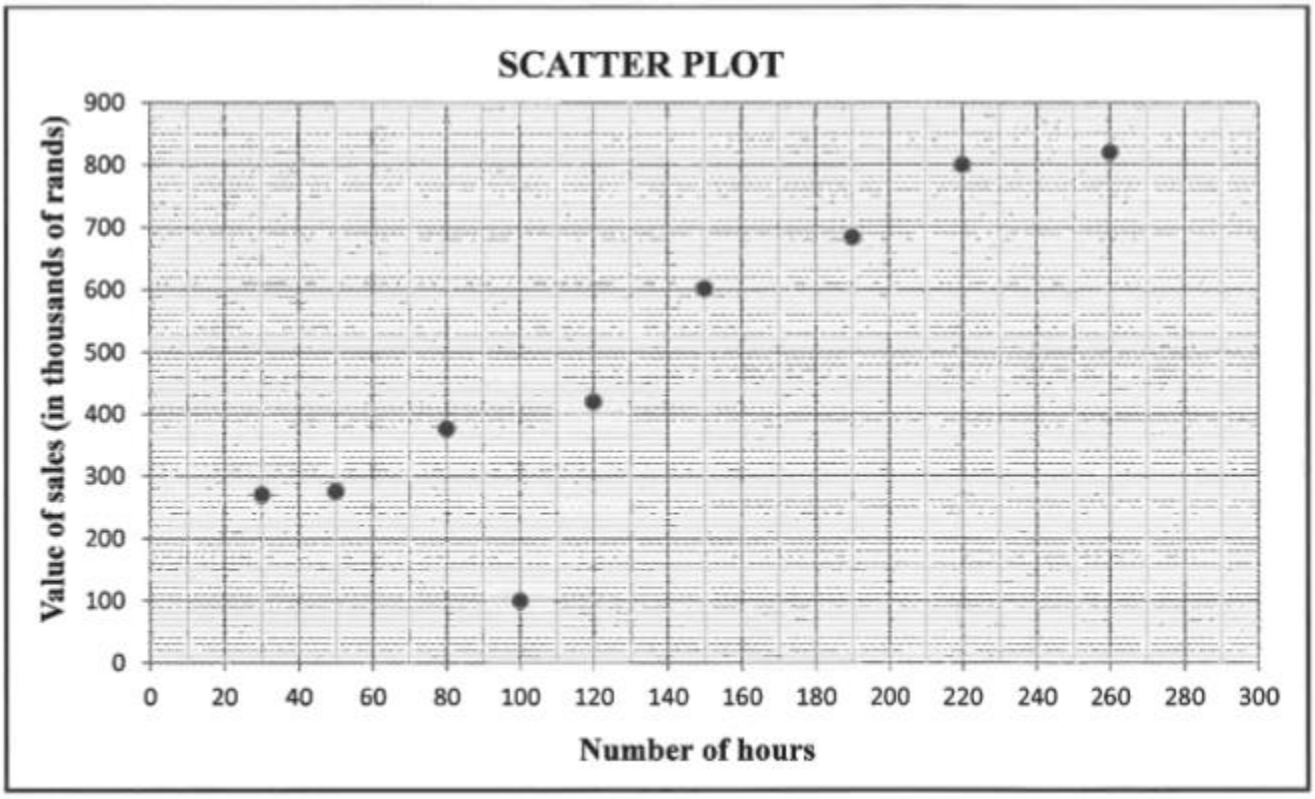
1.3 It was discovered that there was an error in counting the number of units of blood donated by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were NOT counted over the ten days? (1)

[11]

QUESTION 2

The table below shows the number of hours that a sales representative of a company spent with each of his nine clients in one year and the value of the sales (in thousands of rands) for that client.

NUMBER OF HOURS	30	50	80	100	120	150	190	220	260
VALUE OF SALES (IN THOUSANDS OF RANDS)	270	275	376	100	420	602	684	800	820



2.1 Identify an outlier in the data above. (1)

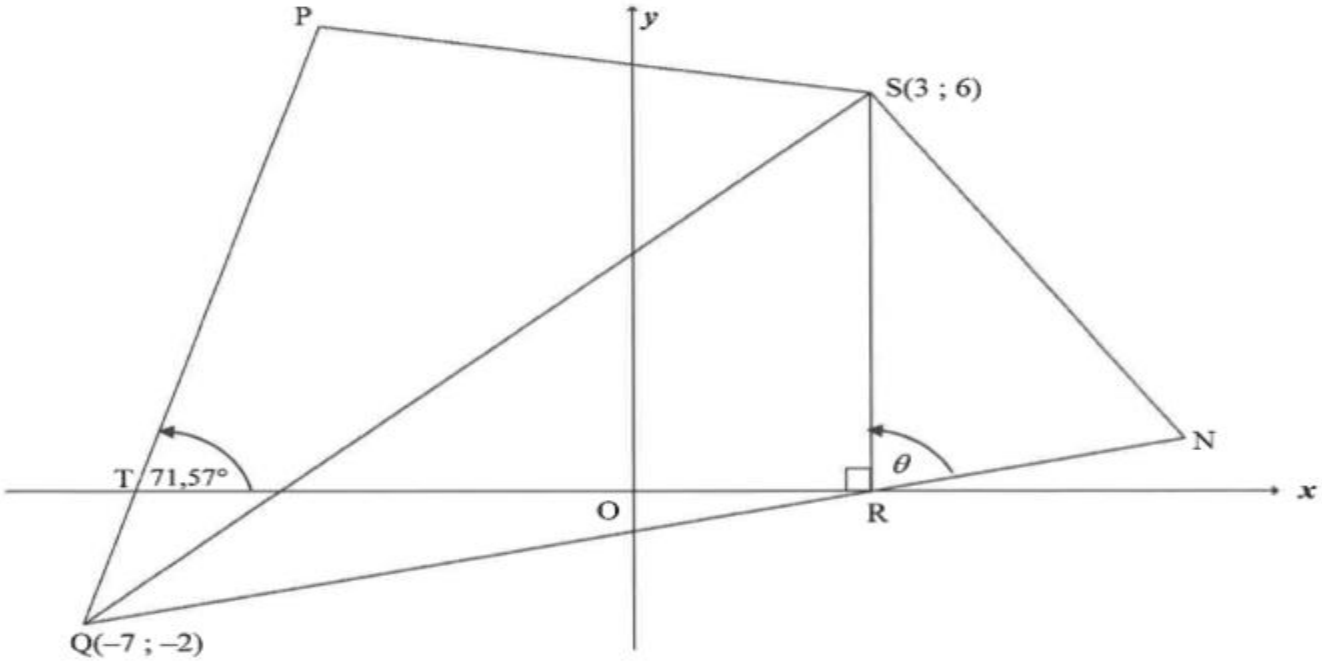
2.2 Calculate the equation of the least squares regression line of the data. (3)

2.3 The sales representative forgot to record the sales of one of his clients. Predict the value of this client's sales (in thousands of rands) if he spent 240 hours with him during the year. (2)

2.4 What is the expected increase in sales for EACH additional hour spent with a client? (2)
[8]

QUESTION 3

In the diagram, P, Q(–7 ; –2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. $P\hat{T}O = 71{,}57°$ and $S\hat{R}N = \theta$.



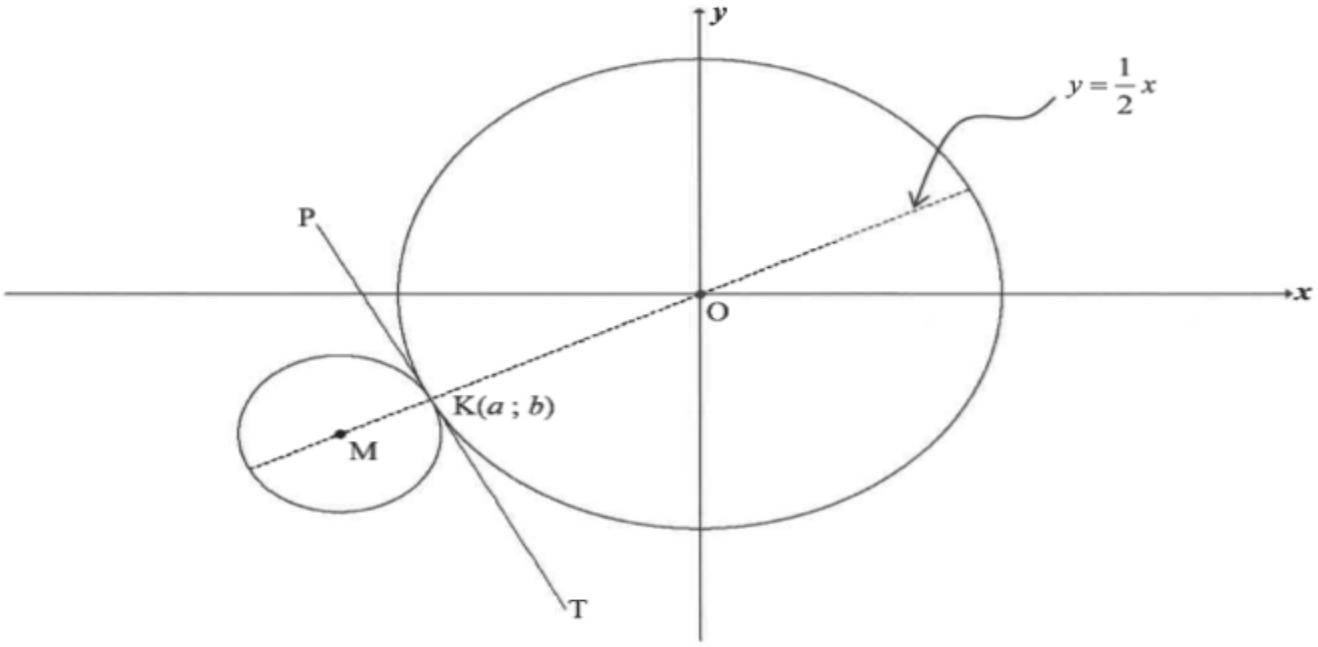
Determine:

3.1	The equation of SR	(1)
3.2	The gradient of QP to the nearest integer	(2)
3.3	The equation of QP in the form $y = mx + c$	(2)
3.4	The length of QR. Leave your answer **in surd form**.	(2)
3.5	$\tan(90° - \theta)$	(3)
3.6	The area of ΔRSN, **without using a calculator**	(6)
		[16]

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QUESTION 4

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



4.1 Write down the length of OK **in surd form**. (1)

4.2 Show that K is the point (–12 ; –6). (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3)

4.3.2 The coordinates of M (6)

4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M(–16 ; –8). (5)

[24]

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QUESTION 5

5.1 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°; 270°]$, calculate, **without using a calculator**, the values in simplest form of:

5.1.1 $\sin 2\theta$ (4)

5.1.2 $\sin^2 \theta$ (3)

5.2 Simplify $\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$ to a single trigonometric ratio. (6)

5.3 Determine the value of $\sin 3x.\cos y + \cos 3x.\sin y$ if $3x + y = 270°$. (2)

5.4 Given: $2\cos x = 3\tan x$

5.4.1 Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$. (3)

5.4.2 Determine the general solution of x if $2\cos x = 3\tan x$. (5)

5.4.3 Hence, determine two values of y, $144° \leq y \leq 216°$, that are solutions of $2\cos 5y = 3\tan 5y$. (4)

5.5 Consider: $g(x) = -4\cos(x + 30°)$

5.5.1 Write down the maximum value of $g(x)$. (1)

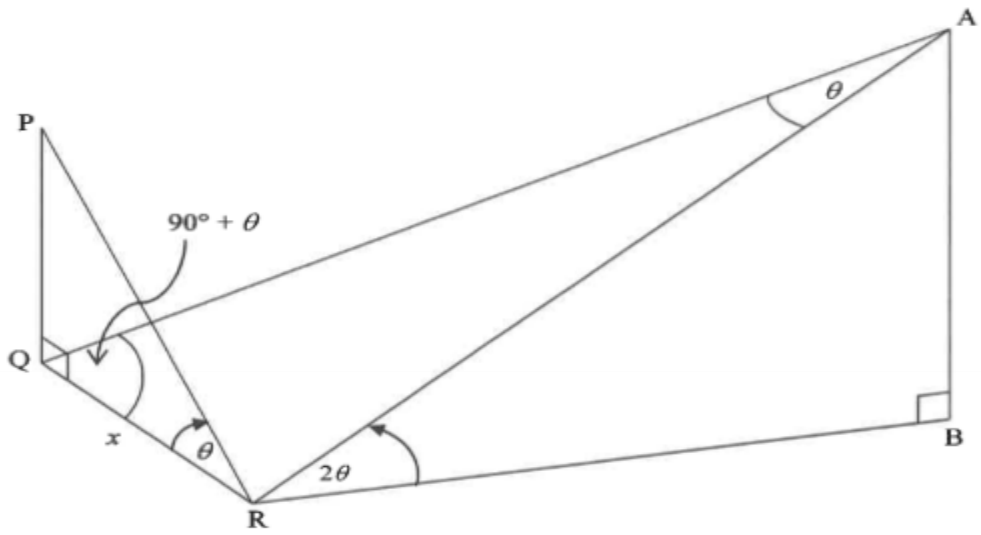
5.5.2 Determine the range of $g(x) + 1$. (2)

5.5.3 The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form. (3)
[33]

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QUESTION 6

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



6.1 Determine in terms of x and θ:

6.1.1 QP (2)

6.1.2 AR (2)

6.2 Show that $AB = 2x\cos^2\theta$ (4)

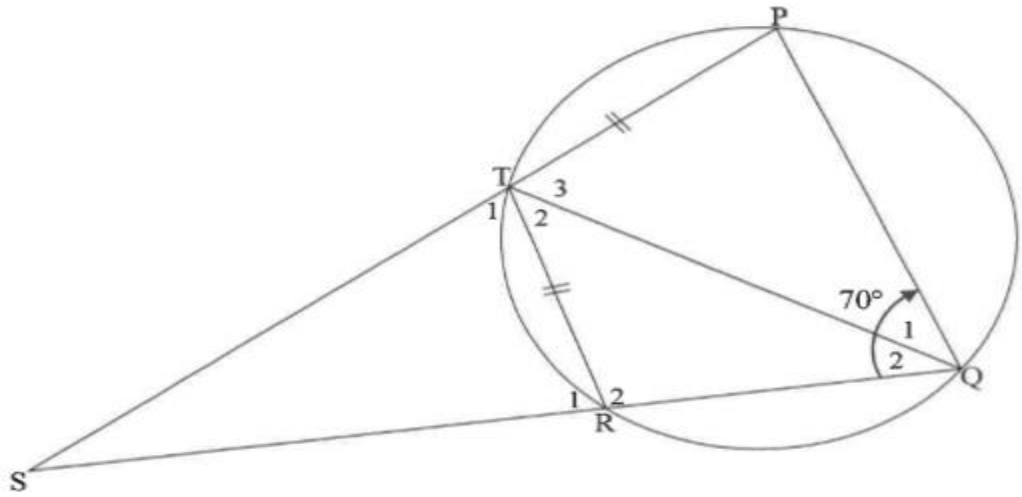
6.3 Determine $\frac{AB}{QP}$ if $\theta = 12°$. (2)

[10]

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QUESTION 7

In the diagram, PQRT is a cyclic quadrilateral in a circle such that PT = TR. PT and QR are produced to meet in S. TQ is drawn. $S\hat{Q}P = 70°$



7.1 Calculate, with reasons, the size of:

7.1.1 $\hat{T}_1$ (2)

7.1.2 $\hat{Q}_1$ (2)

7.2 If it is further given that PQ || TR:

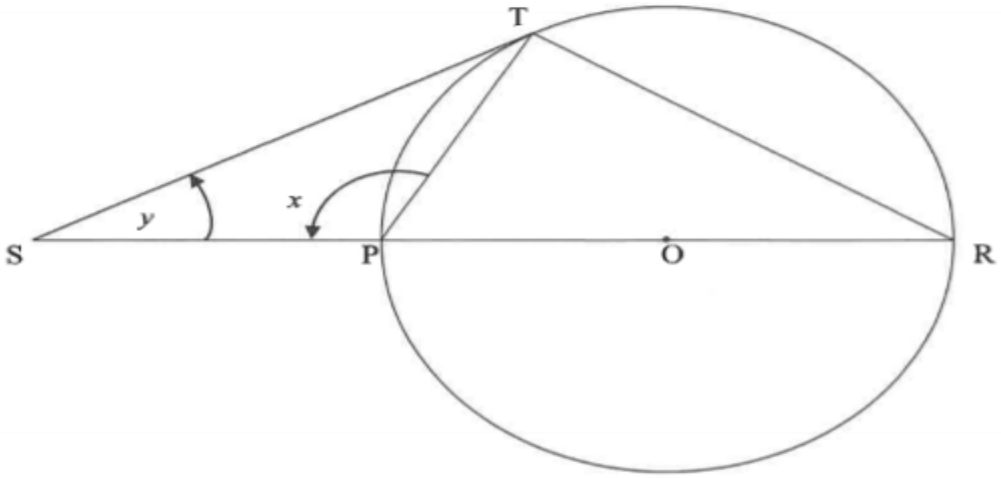
7.2.1 Calculate, with reasons, the size of $\hat{T}_2$ (2)

7.2.2 Prove that $\frac{TR}{TS} = \frac{RQ}{RS}$ (2)
[8]

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QUESTION 8

In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. $S\hat{P}T = x$ and $\hat{S} = y$.

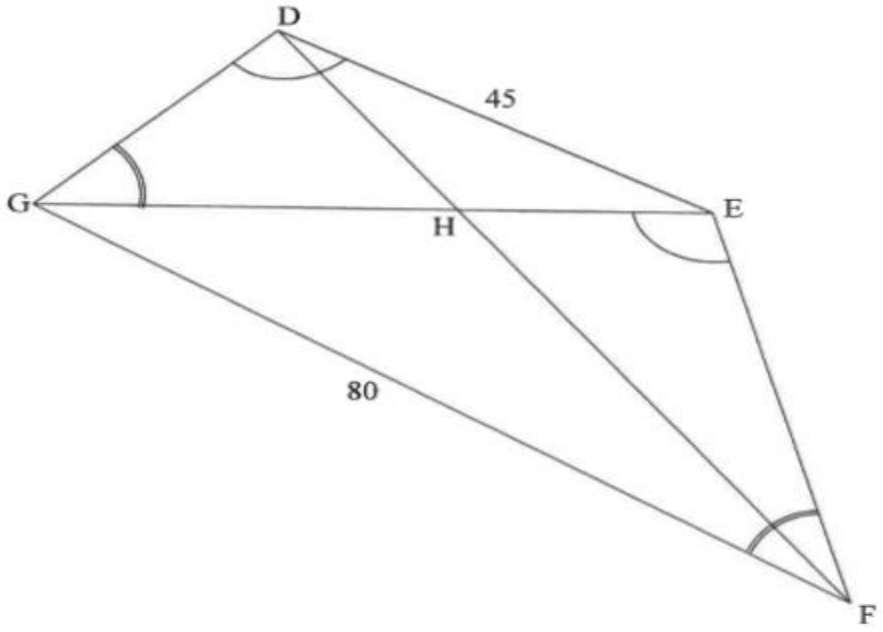


Determine, with reasons, y in terms of x. **[6]**

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QUESTION 9

In the diagram, DEFG is a quadrilateral with DE = 45 and GF = 80. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.

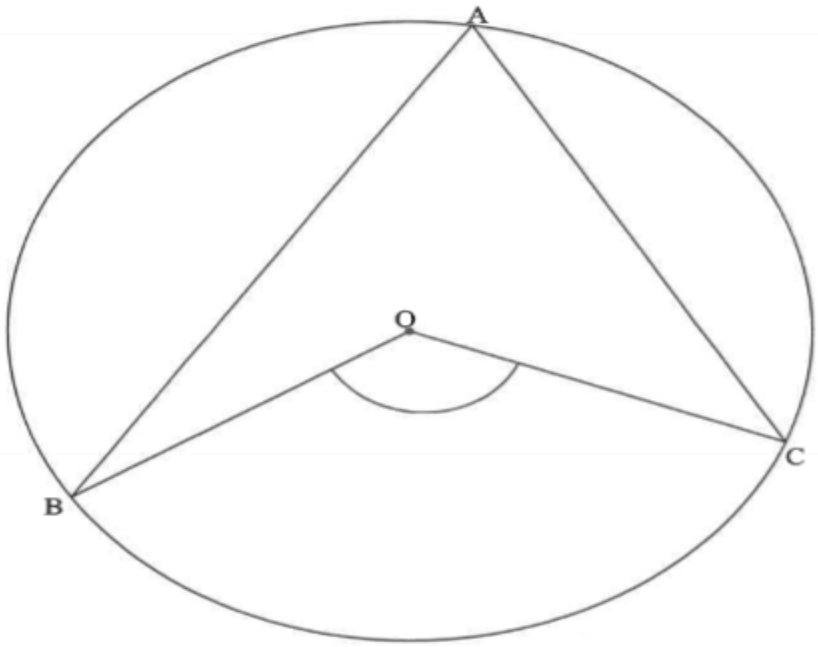


9.1	Give a reason why $\Delta DEG \mid\mid\mid \Delta EGF$.	(1)
9.2	Calculate the length of GE.	(3)
9.3	Prove that $\Delta DEH \mid\mid\mid \Delta FGH$.	(3)
9.4	Hence, calculate the length of GH.	(3)
		[10]

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QUESTION 10

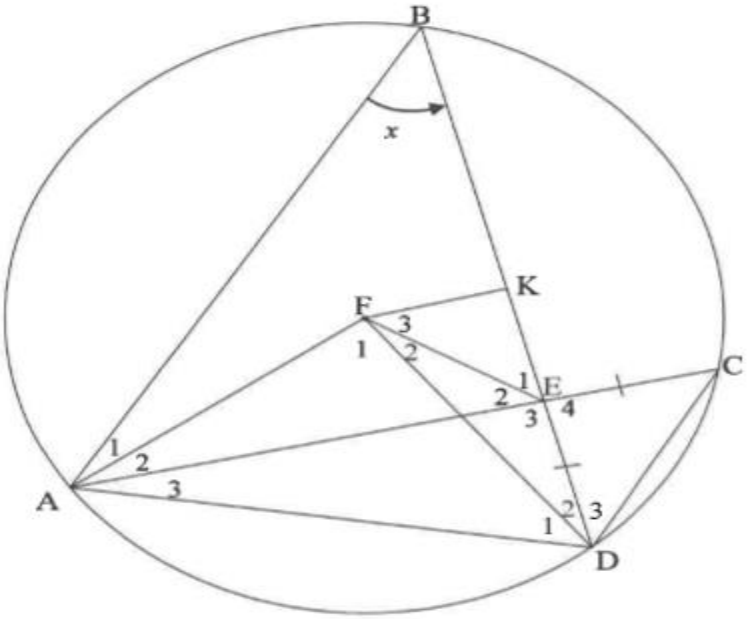
10.1 In the diagram, O is the centre of the circle with A, B and C drawn on the circle.



Prove the theorem which states that $B\hat{O}C = 2\hat{A}$. (5)

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10.2 In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



10.2.1 Determine, with reasons, the size of EACH of the following in terms of x:

(a) $\hat{F}_1$ (2)

(b) $\hat{C}$ (2)

10.2.2 Prove, with reasons, that AFED is a cyclic quadrilateral. (4)

10.2.3 Prove, with reasons, that $\hat{F}_3 = x$. (6)

10.2.4 If area $\Delta AEB = 6{,}25 \times$ area ΔDEC, calculate $\frac{AE}{ED}$. (5)

[24]

TOTAL: 150

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NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 22 pages./
Hierdie nasienriglyne bestaan uit 20 bladsye.

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QUESTION/*VRAAG* 1

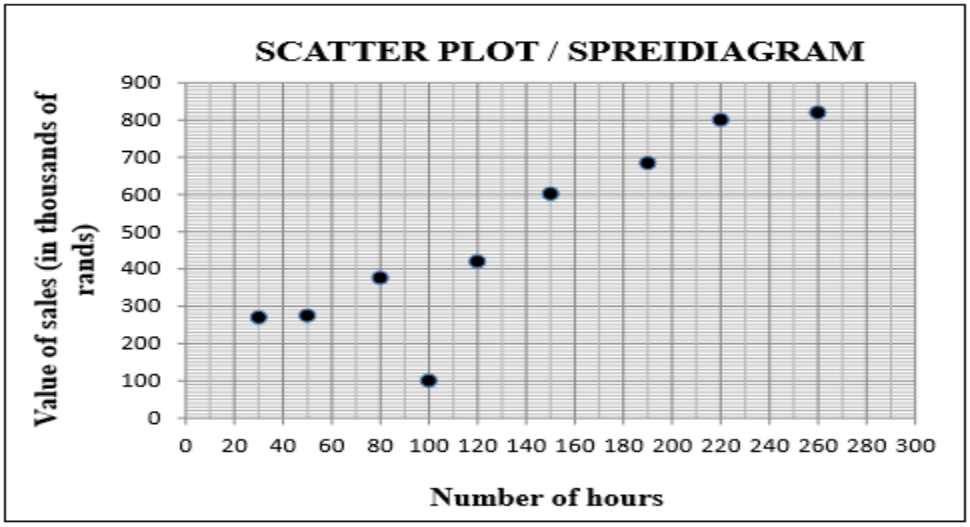
Days/*Dae*	1	2	3	4	5	6	7	8	9	10
Units of blood/ *Eenhede bloed*	45	59	65	73	79	82	91	99	101	106

1.1.1	$\bar{x} = \frac{800}{10}$ $= 80$ Answer only: full marks	✓ 800 (addition of units) ✓ answer (CA if ÷ 10) (2)
1.1.2	$\sigma = 18{,}83$ No penalty for rounding	✓✓ answer (A) (2)
1.1.3	(61,17 ; 98,83) Days 1, 2, 8, 9 and 10 lie outside 1 standard deviation from the mean ∴5 days Correct answer only: full marks provided that 1.1.1. & 1.1.2 both correct	✓ mean – 1 SD ✓ mean + 1 SD ✓ answer (3)
1.2.1	Skewed to the left or negatively skewed/ *Skeef na links of negatief skeef*	✓ answer (1)
1.2.2	A = 65 B = 99 Answers without labelling: 1/2	✓ answer ✓ answer (2)
1.3	New total = 95 × 10 = 950 ∴Units not counted = 950 – 800 = 150	✓ answer (CA from 1.1.1) (1) **[11]**

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QUESTION/*VRAAG* 2

Number of hours **Aantal uur**	30	50	80	100	120	150	190	220	260
Value of sales (in thousands of rands) **Waarde van verkope (in duisend rand)**	270	275	376	100	420	602	684	800	820

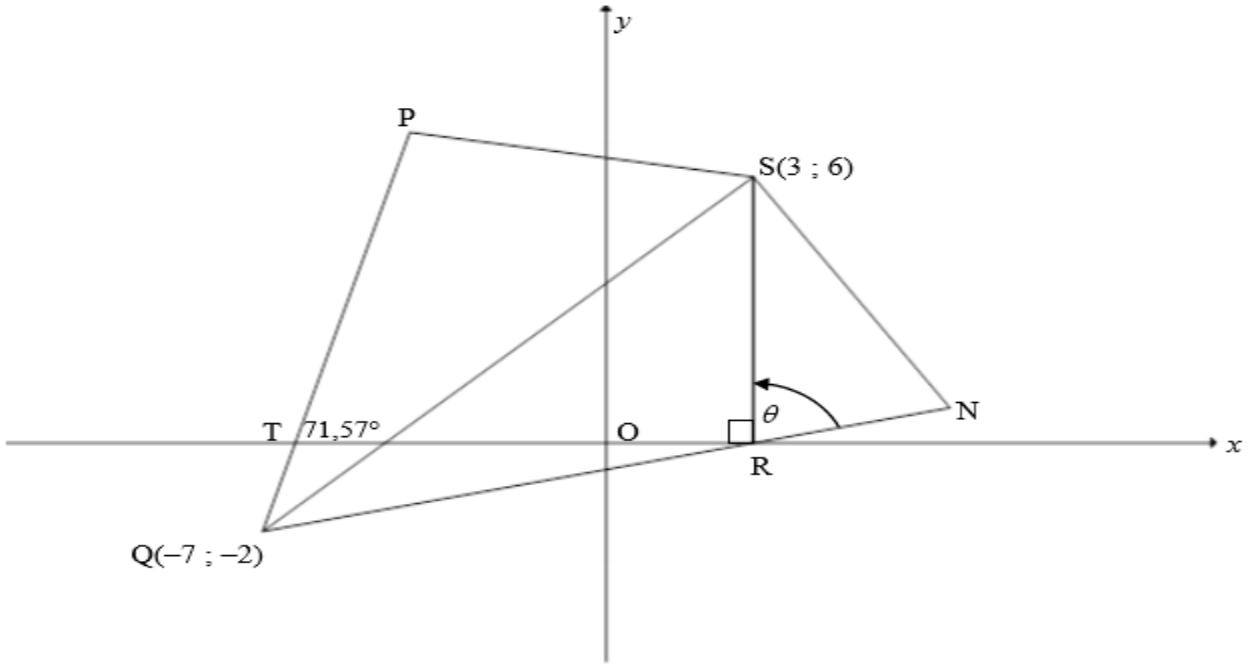


2.1	Outlier/*Uitskieter*: (100 ; 100)	accept: 100 as answer	✓ answer (1)
2.2	$a = 94{,}50273\ldots$ $b = 2{,}913729\ldots$ $\hat{y} = 94{,}50 + 2{,}91x$	Integral values: max 2/3 Swopped a and b: 2/3	✓ value of a ✓ value of b ✓ equation (3)
2.3	$\hat{y} = 2{,}91(240) + 94{,}50$ (CA from 2.1) $= 792{,}90$ Value = R793 000 **OR/*OF*** $\hat{y} = 793{,}7978142$ (calculator) Value = R794 000	Penalise 1 mark if answer not in thousands of Rands	✓ substitution ✓ answer in thousands of Rands (2) ✓✓ answer in thousands of Rands (2)
2.4	$b = 2{,}913729\ldots$ ∴ R2 914 **OR/*OF*** R2 910 (calculator)	Answer only: full marks	✓ value of b ✓ answer (2) **[8]**

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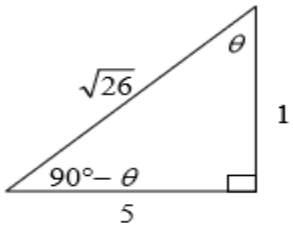
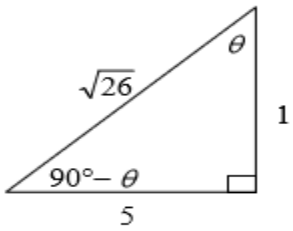
KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

QUESTION/*VRAAG* 3



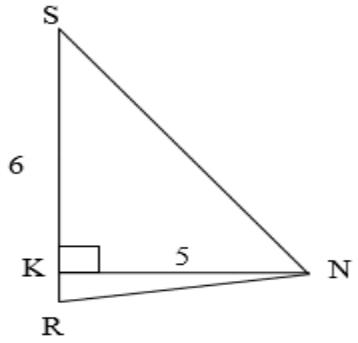
3.1	$x = 3$	✓ answer (1)
3.2	$m_{QP} = \tan 71{,}57°$ $= 3$ Answer only: full marks	✓ $m_{QP} = \tan 71{,}57°$ ✓ answer (2)
3.3	$y = mx + c$ or $y - y_1 = m(x - x_1)$ $-2 = 3(-7) + c$ or $y + 2 = 3(x + 7)$ $y = 3x + 19$	(m CA from 3.2 if > 0) ✓ substitution of m & Q ✓ equation (2)
3.4	R(3 ; 0) $QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-7 - 3)^2 + (-2 - 0)^2}$ $= \sqrt{104}$ or $2\sqrt{26}$	(wrong R: CA if $x > 0$) ✓ substitution ✓ answer (in surd form) (2)

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

3.5	$\tan(90°-\theta) = m_{QR}$ $= \frac{0-(-2)}{3-(-7)}$ $= \frac{1}{5}$ Answer only: full $\tan\theta = \frac{1}{5}$: 1/3	(wrong R: CA if $x > 0$) ✓ gradient of QR/RN/QN ✓ substitution of Q & R ✓ answer (3)
3.6	$RN = \frac{1}{2}.2\sqrt{26} = \sqrt{26}$ $SR = 6$  Area $\Delta RSN = \frac{1}{2}$ SR . RN. $\sin\theta$ $= \frac{1}{2} \times 6 \times \sqrt{26} \times \frac{5}{\sqrt{26}}$ = 15 square units **OR/*OF*** $RN = \frac{1}{2}.2\sqrt{26} = \sqrt{26}$ $SR = 6$  Area $\Delta RSN = \frac{1}{2}$ SR . RN. $\sin\theta$ $= \frac{1}{2}(6)(\frac{1}{2}QP).\sin\theta$ $= \frac{3}{2}(\sqrt{104}).\sin\theta$ $= \frac{3}{2}(\sqrt{104})\left(\frac{5}{\sqrt{26}}\right)$ = 15 square units	✓ RN ✓ SR ✓ diagram (5 & $\sqrt{26}$) ✓ use of correct area rule ✓ substitution of $\sin\theta$ ✓ answer (6) ✓ RN ✓ SR ✓ diagram ✓ use of correct area rule ✓ substitution of $\sin\theta$ ✓ answer (6)

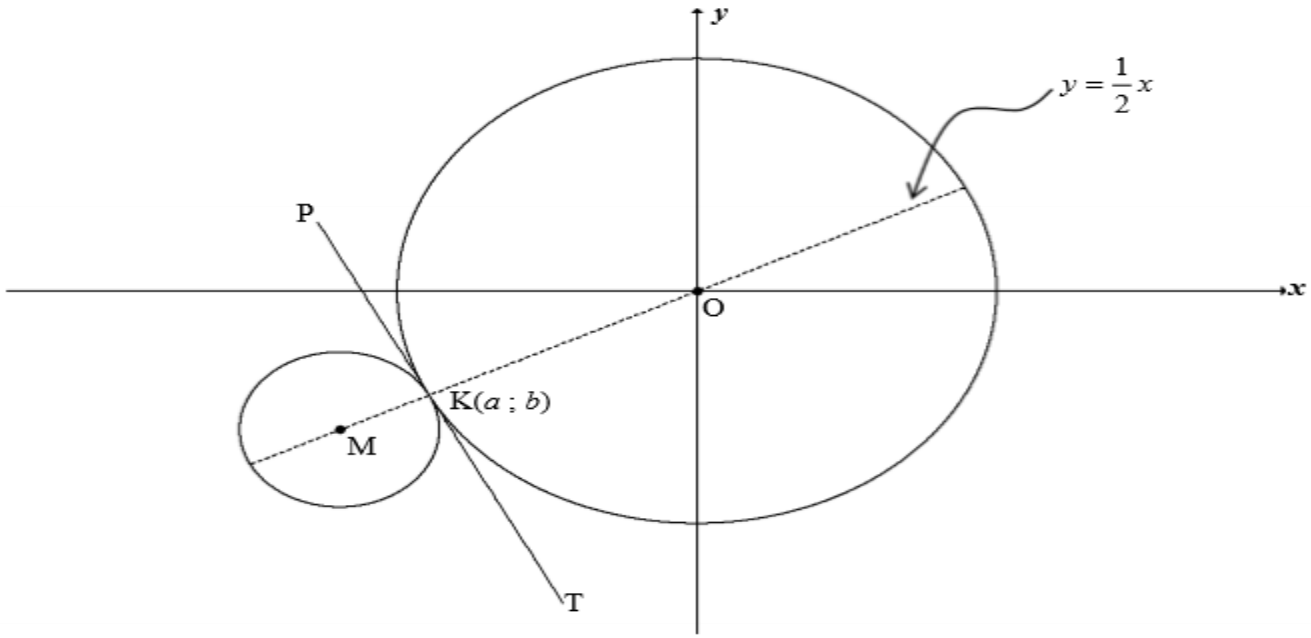
using calculator: max 4 marks

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	OR/*OF*	
	SR = 6 ⊥height = 5	✓ SR ✓✓ ⊥ height
		
	$A = \frac{1}{2}$ SR ×⊥h $= \frac{1}{2}(6)(5)$ = 30 square units	✓ use of correct area formula ✓ substitution of $\sin\theta$ ✓ answer (6)
	Using $A = \frac{1}{2}$ b×⊥h incorrectly: max 1/6	**[16]**

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QUESTION/*VRAAG* 4



4.1	OK $= \sqrt{180}$ or $6\sqrt{5}$	✓ answer (1)
4.2	$a^2 + b^2 = 180$ $b = \frac{1}{2}a$ $a^2 + \left(\frac{1}{2}a\right)^2 = 180$ $a^2 + \frac{1}{4}a^2 = 180$ $a^2 = 144 \quad \therefore a = -12$ $b = \frac{1}{2}(-12)$ K (–12 ; – 6) (given) **OR/*OF*** $a^2 + b^2 = 180$ $a = 2b$ $(2b)^2 + b^2 = 180$ $5b^2 = 180$ $b^2 = 36 \quad \therefore b = -6$ $a = 2(-6)$ K (–12 ; – 6) (given)	No penalty if x and y are not converted to a and b Error in simplification: max 2/4	✓ b in terms of a ✓ substitution ✓ $a^2 = 144$ ✓ substitution (4) ✓ a in terms of b ✓ substitution ✓ $b^2 = 36$ ✓ substitution (4)

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4.3.1

$m_{OK} = \frac{1}{2}$ $[y = \frac{1}{2}x]$

$m_{PT} = -2$ [radius ⊥ tangent/*raaklyn*]

$y = mx + c$ **OR/*OF*** $y - y_1 = m(x - x_1)$

$-6 = -2(-12) + c$ $y - (-6) = -2(x - (-12))$

$c = -30$ $c = -30$

$y = -2x - 30$

Using $m = \frac{1}{2}$: 0/3

Using $m = -\frac{1}{2}$ *or* 2 : 2/3

✓ $m_{PT} = -2$

✓ substitution of m & K(−12 ; − 6)

✓ equation

(3)

4.3.2

3MK = OK

$\Rightarrow$OM = $\frac{4}{3}$ OK

$M = \frac{4}{3}(-12 ; -6)$

$\therefore$M(−16 ; −8)

Answer only: 0/6

✓ 3MK = OK

✓ OM = $\frac{4}{3}$ OK

✓✓ $M = \frac{4}{3}(-12 ; -6)$

✓ x-coordinate

✓ y-coordinate

(6)

OR/*OF*

3MK = OK

$9MK^2 = OK^2 = 180$

$\therefore MK^2 = 20$

Let M(x ; y), then :

$(x+12)^2 + (y+6)^2 = 20$

$(x+12)^2 + \left(\frac{1}{2}x + 6\right)^2 = 20$

$x^2 + 24x + 144 + \frac{1}{4}x^2 + 6x + 36 = 20$

$\frac{5}{4}x^2 + 30x + 160 = 0$

$x^2 + 24x + 128 = 0$

$(x+16)(x+8) = 0$

$x = -16$ $x \neq -8$ [since M is outside the large circle]

$y = -8$

M(−16 ; − 8)

✓ 3MK = OK

✓ $MK^2 = 20$

✓ equation

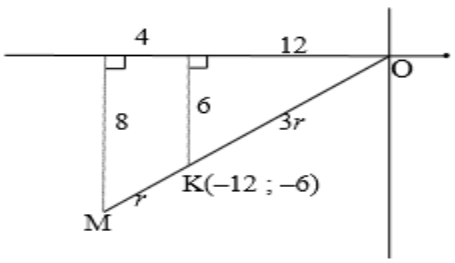
✓ substitution

✓ x-coordinate

✓ y-coordinate

(6)

OR/*OF*



∴M(−16 ; −8)

✓ 3MK = OK

✓ ✓✓ diagram with values **OR** valid explanation

✓ x-coordinate

✓ y-coordinate

(6)

OR/*OF*

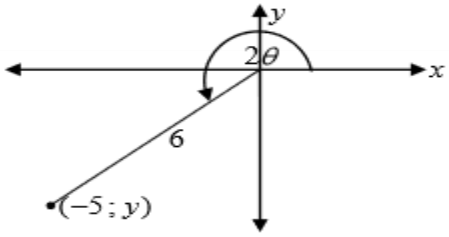
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	3MK = OK $9MK^2 = OK^2 = 180$ $\therefore MK^2 = 20$ Let M$(x\,;y)$, then $y = \frac{1}{2}x$: $(x+12)^2 + (y+6)^2 = 20$ $(x+12)^2 + \left(\frac{1}{2}x+6\right)^2 = 20$ $4(x+12)^2 + (x+12)^2 = 80$ $(x+12)^2 = 16$ $x+12 = \pm 4$ $x = -16 \quad x \neq -8$ [since M is outside the large circle] $y = -8$ M$(-16;-8)$	✓ 3MK = OK ✓ $MK^2 = 20$ ✓ equation ✓ substitution ✓ x-coordinate ✓ y-coordinate (6)
4.3.3	$(x-(-16))^2 + (y-(-8))^2 = \left(\frac{1}{3}\sqrt{180}\right)^2$ $(x+16)^2 + (y+8)^2 = 20$	✓ LHS (CA from 4.3.2) ✓ RHS (CA from 4.1) (2)
4.4	$OK < r < OK + 2KM$ $\sqrt{180} < r < \sqrt{180} + \frac{2}{3}\sqrt{180}$ Answer only: full marks (No need to simplify) $6\sqrt{5} < r < 10\sqrt{5}$	✓✓ values ✓ inequality (3)
4.5	$x^2 + 32x + (16)^2 + y^2 + 16y + (8)^2 = 256 + 64 - 240$ $(x+16)^2 + (y+8)^2 = 80$ New circle/*nuwe sirkel*: Centre/*middelpt* $(-16;-8)$ & $r = 4\sqrt{5}$ Original circle/*oorspronklike sirkel*: M$(-16;-8)$ & $r = 2\sqrt{5}$ This circle will never cut the circle with centre M as they have the **same centre (concentric circles) but unequal radii**/*Hierdie sirkel sal nooit die sirkel met middelpt M sny nie, want hulle is* ***konsentries, want het dieselfde middelpunt met verskillende radii.***	✓ equation in centre, radius form ✓ Centre: $(-16;-8)$ ✓ $r = 4\sqrt{5}$ (new) ✓ $r = 2\sqrt{5}$ (original) ✓ conclusion ("concentric" must be stated) (5) **[24]**

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QUESTION/*VRAAG* 5

5.1.1 **no calculator in 5.1**	$\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°; 270°]$  $y^2 = 6^2 - (-5)^2$ [Pythagoras] $y = \pm\sqrt{11}$ $(5 ; y)$ is in 3rd quadrant: $\therefore y = -\sqrt{11}$ $\sin 2\theta = -\frac{\sqrt{11}}{6}$ **OR/*OF*** Getting to $\sin 2\theta = \frac{\sqrt{11}}{6}$: 3/4 $\sin^2 2\theta = 1 - \cos^2 2\theta$ $= 1 - \left(-\frac{5}{6}\right)^2$ $= 1 - \frac{25}{36}$ $= \frac{11}{36}$ $\sin 2\theta = -\frac{\sqrt{11}}{6}$	✓ diagram (3rd quadrant only) ✓ using Pythagoras ✓ y – value ✓ answer (4) ✓ $\sin^2 2\theta = 1 - \cos^2 2\theta$ ✓ substitution ✓ value of $\sin^2 2\theta$ ✓ answer (4)
5.1.2	$\cos 2\theta = 1 - 2\sin^2\theta$ $2\sin^2\theta = 1 - \cos 2\theta$ $\sin^2\theta = \frac{1 - \left(-\frac{5}{6}\right)}{2}$ $= \frac{11}{6} \times \frac{1}{2}$ $= \frac{11}{12}$	✓ $\cos 2\theta = 1 - 2\sin^2\theta$ ✓ substitution ✓ answer (3)

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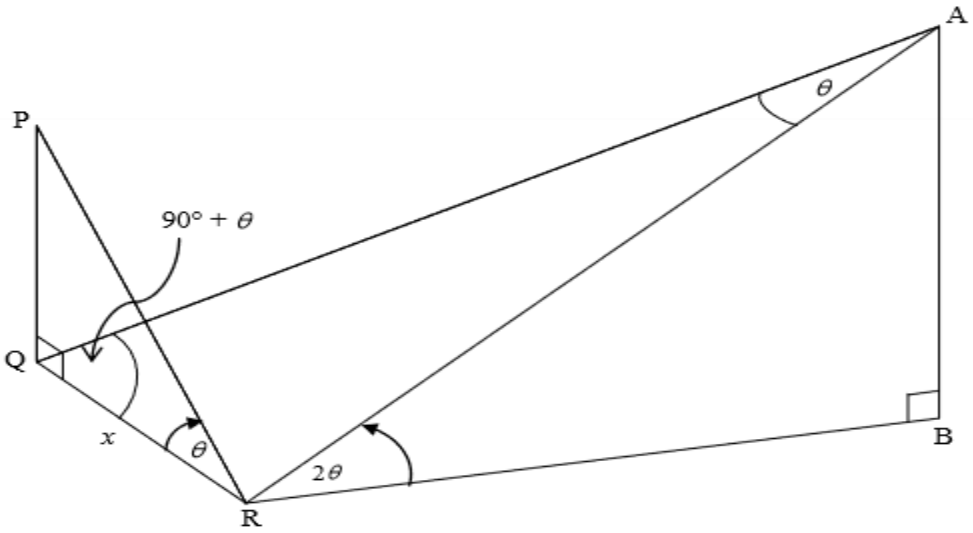
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5.2	$\sin(180°-x).\cos(-x)+\cos(90°+x).\cos(x-180°)$ $=\sin x.\cos x-\sin x(-c$ $=2\sin x.\cos x$ $=\sin 2x$ Second line written as $\sin x\cos x+\sin x\cos x$: max 5/6	✓ $\sin x$ ✓ $\cos x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ simplification ✓ answer (6)
5.3	$\sin 3x.\cos y+\cos 3x.\sin y$ $\sin(3x+y)$ $=\sin 270°$ $=-1$	✓ compound angle ✓ answer (2)
5.4.1	$2\cos x=3\tan x$ $2\cos x=\frac{3\sin x}{\cos x}$ $2\cos^2 x=3\sin x$ $2(1-\sin^2 x)=3\sin x$ $2-2\sin^2 x=3\sin x$ $2\sin^2 x+3\sin x-2=0$	✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ multiplying by $\cos\theta$ ✓ $\cos^2 x=1-\sin^2 x$ (3)
5.4.2	$2\sin^2 x+3\sin x-2=0$ $(2\sin x-1)(\sin x+2)=0$ $\sin x=\frac{1}{2}$ or $\sin x=-2$ (no solution) $x=30°+k.360°$ or $x=150°+k.360°$; $k\in Z$	✓ factors ✓ both values of $\sin x$ ✓ no solution ✓ $30°+k.360°$ ✓ $150°+k.360°$; $k\in Z$ (5)
5.4.3	$5y=30°+k.360°$ or $5y=150°+k.360°$ $y=6°+k.72°$ or $y=30°+k.72°$ $\therefore y=144°+6°$ or $y=144°+30°$ $y=150°$ or $y=174°$ **OR/*OF*** $144°\le y\le 216°$ $720°\le 5y\le 1080°$ $5y=750°$ or $5y=870°$ $y=150°$ or $y=174°$	✓ $y=6°+k.72°$ ✓ $y=30°+k.72°$ ✓ 150° ✓ 174° (4) ✓ $5y=750°$ ✓ $5y=870°$ ✓ 150° ✓ 174° (4)
5.5.1	$g(x)=-4\cos(x+30°)$ maximum value = 4	✓ answer (1)

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5.5.2	range of/*waardeversameling van* $g(x)$: $-4 \le y \le 4$ **OR/*OF*** $y \in [-4\ ;\ 4]$ $\therefore$ range of/*waardeversameling van* $g(x) + 1$: $-3 \le y \le 5$ **OR/*OF*** $y \in [-3\ ;\ 5]$ Answer only: full marks	✓ range of $g(x)$ ✓ answer (2)
5.5.3	$y = -4\cos(x + 30°)$ shifted to the left/*skuif na links*: $y = -4\cos(x + 30° + 60°)$ $= -4\cos(x + 90°)$ $= 4\sin x$ $\therefore h(x) = -4\sin x$ Answer only: full marks	✓ shift of 60° to the left ✓ reduction ✓ equation of h (3) **[33]**

QUESTION/*VRAAG* 6

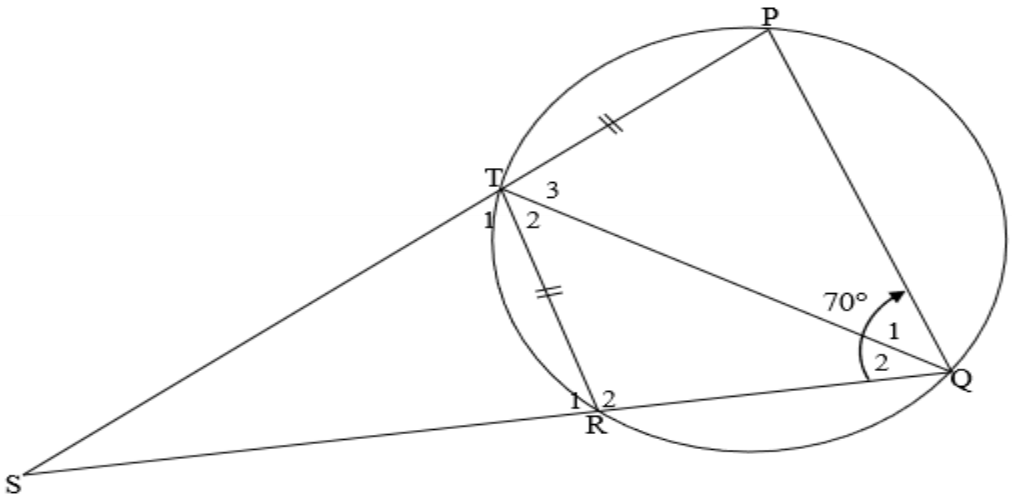


6.1.1	$\tan\theta = \frac{PQ}{QR} = \frac{PQ}{x}$ $\therefore PQ = x\tan\theta$ **OR/*OF*** $\frac{QR}{\sin P} = \frac{PQ}{\sin P\hat{R}Q}$ $\therefore PQ = \frac{x.\sin\theta}{\sin(90° - \theta)}$	Answer only: full marks	✓ trig ratio ✓ answer (2) ✓ trig ratio ✓ answer (2)
6.1.2	$\frac{AR}{\sin A\hat{Q}R} = \frac{QR}{\sin Q\hat{A}R}$ $AR = \frac{x\sin(90° + \theta)}{\sin\theta}$	Answer only: full marks	✓ use of sine rule ✓ substitution into sine rule correctly (2)

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6.2	$\sin 2\theta = \frac{AB}{AR}$ $AB = AR\sin 2\theta$ $= \frac{x\sin(90° + \theta).\sin 2\theta}{\sin\theta}$ $= \frac{x\cos\theta.\sin 2\theta}{\sin\theta}$ $= \frac{x\cos\theta.2\sin\theta\cos\theta}{\sin\theta}$ $= 2x\cos^2\theta$	✓ substitution into trig ratio and AB as subject ✓ substitution of AR ✓ co-ratio ✓ $\sin 2\theta = 2\sin\theta\cos\theta$ (4)
6.3	$\frac{AB}{QP} = \frac{2x\cos^2 12°}{x\tan 12°}$ $= 9$	✓ substitution CA from 6.1.1) ✓ answer (2) **[10]**

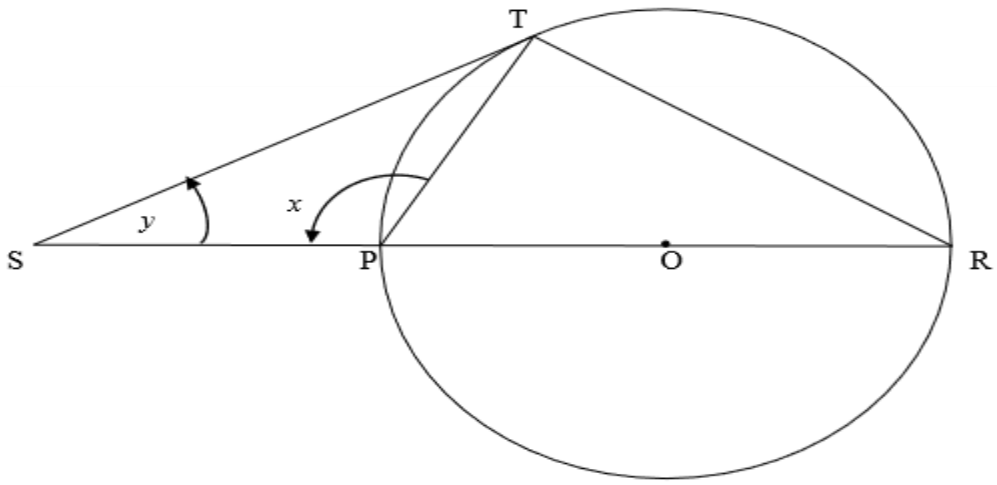
QUESTION/*VRAAG* 7



7.1.1	$\hat{T}_1 = 70°$	[ext ∠ of cyclic quad/*buite ∠ van koordevh*]	✓ S ✓ R (2)
7.1.2	$\hat{Q}_1 = \hat{Q}_2 = 35°$	[equal chords;equal∠s/*gelyke koorde; gelyke∠e*]	✓ S ✓ R (2)
7.2.1	$\hat{T}_2 = \hat{Q}_1 = 35°$	[alt ∠s/*verwiss ∠e*; PQ ∥ TR]	✓ S ✓ R (2)
7.2.2	$\frac{PT}{TS} = \frac{QR}{RS}$ $\therefore \frac{TR}{TS} = \frac{QR}{RS}$	[prop theorem/*eweredighst*; PQ ∥ TR] [PT = TR]	✓ S ✓ R (2) **[8]**

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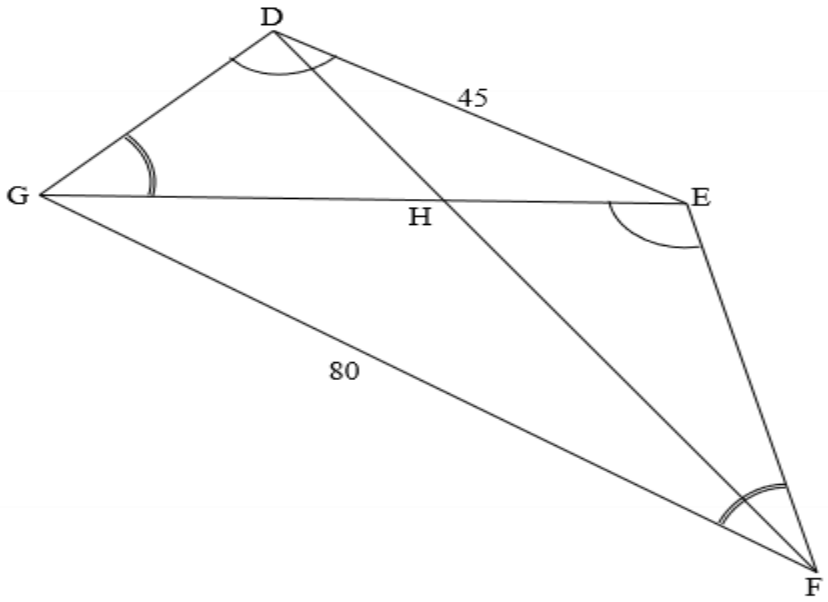
QUESTION/*VRAAG* 8



	$P\hat{T}R = 90°$	[$\angle$ in semi-circle/*halfsirkel*]	✓ S/R
	$x = 90° + \hat{R}$	[ext/*buite* $\angle$ of/*van* Δ]	✓ S/R
	$\therefore \hat{R} = x - 90°$		
	$S\hat{T}P = x - 90°$	[tan chord theorem/*raakl koordstelling*]	✓ S ✓R
	$x + x - 90° + y = 180°$	[sum of/*som van* $\angle$s/*e* in Δ]	✓ S
	$\therefore y = 270° - 2x$		✓ answer
			[6]

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QUESTION/*VRAAG* 9



9.1	equiangular Δs/*gelykhoekige Δe* **OR/*OF*** (∠∠∠)		✓ answer (1)
9.2	$\therefore \frac{GE}{GF} = \frac{DE}{GE}$ $GE^2 = 45 \times 80$ $GE = 60$	[||| Δs]	✓ proportion ✓ substitution ✓ answer (3)
9.3	In ΔDEH and ΔFGH: $D\hat{H}E = F\hat{H}G$ $D\hat{E}H = F\hat{G}H$ $E\hat{D}H = G\hat{F}H$ $\therefore$ ΔDEH ||| ΔFGH **OR/*OF*** In ΔDEH and ΔFGH: $D\hat{H}E = F\hat{H}G$ $D\hat{E}H = F\hat{G}H$ $\therefore$ ΔDEH ||| ΔFGH	 [vert opp ∠s =/*regoorst ∠e* =] [||| Δs] [sum of/*som van* ∠s/*e* in Δ] [vert opp ∠s =/*regoorst ∠e* =] [||| Δs] [∠∠∠]	 ✓ S/R ✓ S/R ✓ S (3) ✓ S/R ✓ S/R ✓ R (3)

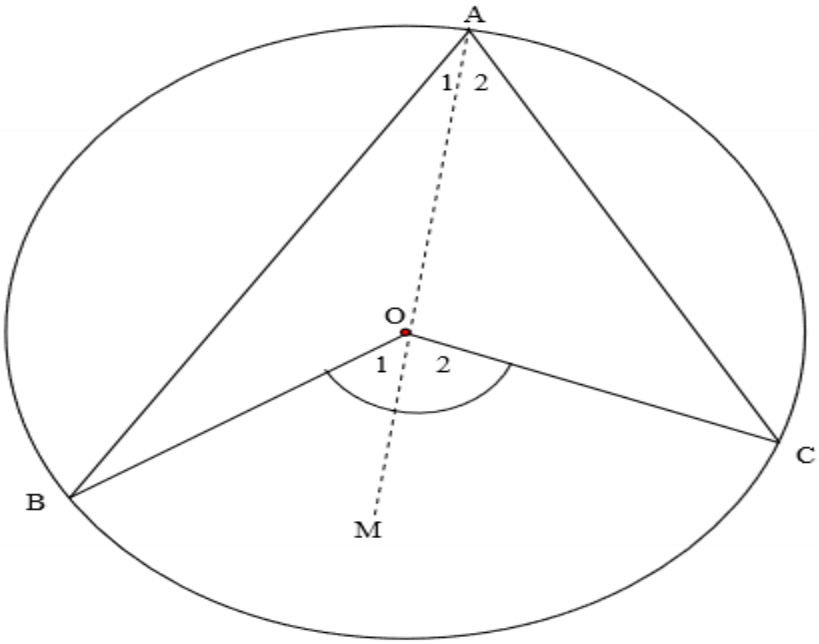
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9.4	$\frac{GH}{EH} = \frac{FG}{DE}$ $\frac{GH}{60 - GH} = \frac{80}{45}$ $45\ GH = 80(60 - GH)$ $45\ GH = 4800 - 80\ GH$ $125\ GH = 4800$ $GH = 38{,}4$	[||| Δs] [EH = 60 – GH]	✓S ✓ substitution ✓ answer (3) **[10]**

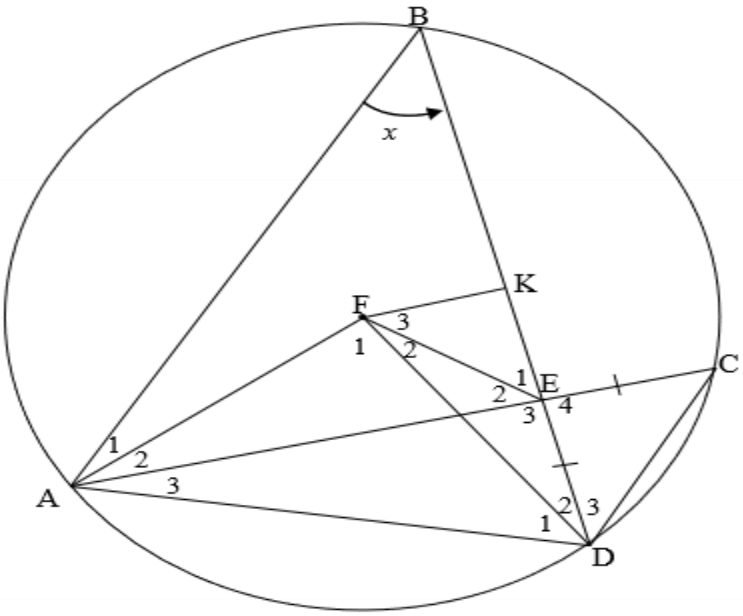
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QUESTION/*VRAAG* 10



10.1	Construction: AO is drawn and produced to M $\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext ∠ of Δ/*buite ∠ van Δ*] But $\hat{A}_1 = \hat{B}$ [∠s opp = radii/*∠e teenoor =radii*] $\therefore \hat{O}_1 = 2\hat{A}_1$ Similarly/*Netso*: $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$ $= 2(\hat{A}_1 + \hat{A}_2)$ $B\hat{O}C = 2B\hat{A}C$	✓ Constr ✓ S/R ✓ S/R ✓ S ✓ S (5)

10.2



10.2.1(a)	$\hat{F}_1 = 2x$	[∠ centre = 2∠ at circum/*midpts∠ =2omtreks∠*]	✓ S ✓ R (2)
10.2.1(b)	$\hat{C} = x$	[∠s in the same seg/*∠e in dieselfde segment*]	✓ S ✓ R (2)
	OR/*OF*		
	$\hat{C} = x$	[∠ centre = 2∠ at circum/*midpts∠ =2omtreks∠*]	✓ S ✓ R (2)
10.2.2	$\hat{D}_3 = x$	[∠s opp equal sides/*∠e teenoor = sye*]	✓ S/R
	$\hat{E}_3 = 2x$	[ext ∠ of Δ/*buite ∠ van Δ*]	✓ S/R
	$\therefore \hat{F}_1 = \hat{E}_3 = 2x$		✓ S
	∴ AFED is a cyclic quadrilateral	[converse ∠s in the same seg]/ *Is 'n koordevierhoek* [*omgekeerde ∠e in dieselfde segm*]	✓ R (4)

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10.2.3	$\hat{A}_2+\hat{A}_3+\hat{D}_1+\hat{F}_1=180°$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\hat{A}_2+\hat{A}_3=D_1$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore \hat{A}_2+\hat{A}_3=90°-x$ $\hat{E}_1=\hat{A}_2+\hat{A}_3$ [ext∠of cyclic quad/*buite∠ v koordevh*] $=90°-x$ $F\hat{K}E=90°$ [line from centre bisects chord]/ [*lyn van midpt halveer koord*] $\hat{F}_3=x$ [sum of ∠s in Δ/*som van ∠e in Δ*]	✓ S ✓ S ✓ R ✓ S ✓ S ✓ R (6)
10.2.4	$B\hat{A}C=\hat{D}_3$ [∠s in the same seg/*∠e in dieselfde segm*] $AE=BE$ [sides opp equal ∠s/*sye teenoor =∠e*] $\frac{\text{area }\Delta AEB}{\text{area }\Delta DEC}=\frac{\frac{1}{2}(BE)(AE).\sin A\hat{E}B}{\frac{1}{2}(EC)(ED).\sin D\hat{E}C}$ $6,25=\frac{AE^2}{ED^2}$ $\therefore \frac{AE}{ED}=2,5$	✓ S ✓ S ✓ substitution into area rule ✓ simplification of RHS ✓ answer (5) **[24]**

TOTAL/*TOTAAL*: 150

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Department:
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SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2016

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages, 1 information sheet and an answer book of 31 pages.

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QUESTION 1

On a certain day a tour operator sent 11 tour buses to 11 different destinations. The table below shows the number of passengers on each bus.

8	8	10	12	16	19	20	21	24	25	26

1.1 Calculate the mean number of passengers travelling in a tour bus. (2)

1.2 Write down the five-number summary of the data. (3)

1.3 Draw a box and whisker diagram for the data. Use the number line provided in the ANSWER BOOK. (2)

1.4 Refer to the box and whisker diagram and comment on the skewness of the data set. (1)

1.5 Calculate the standard deviation for this data set. (2)

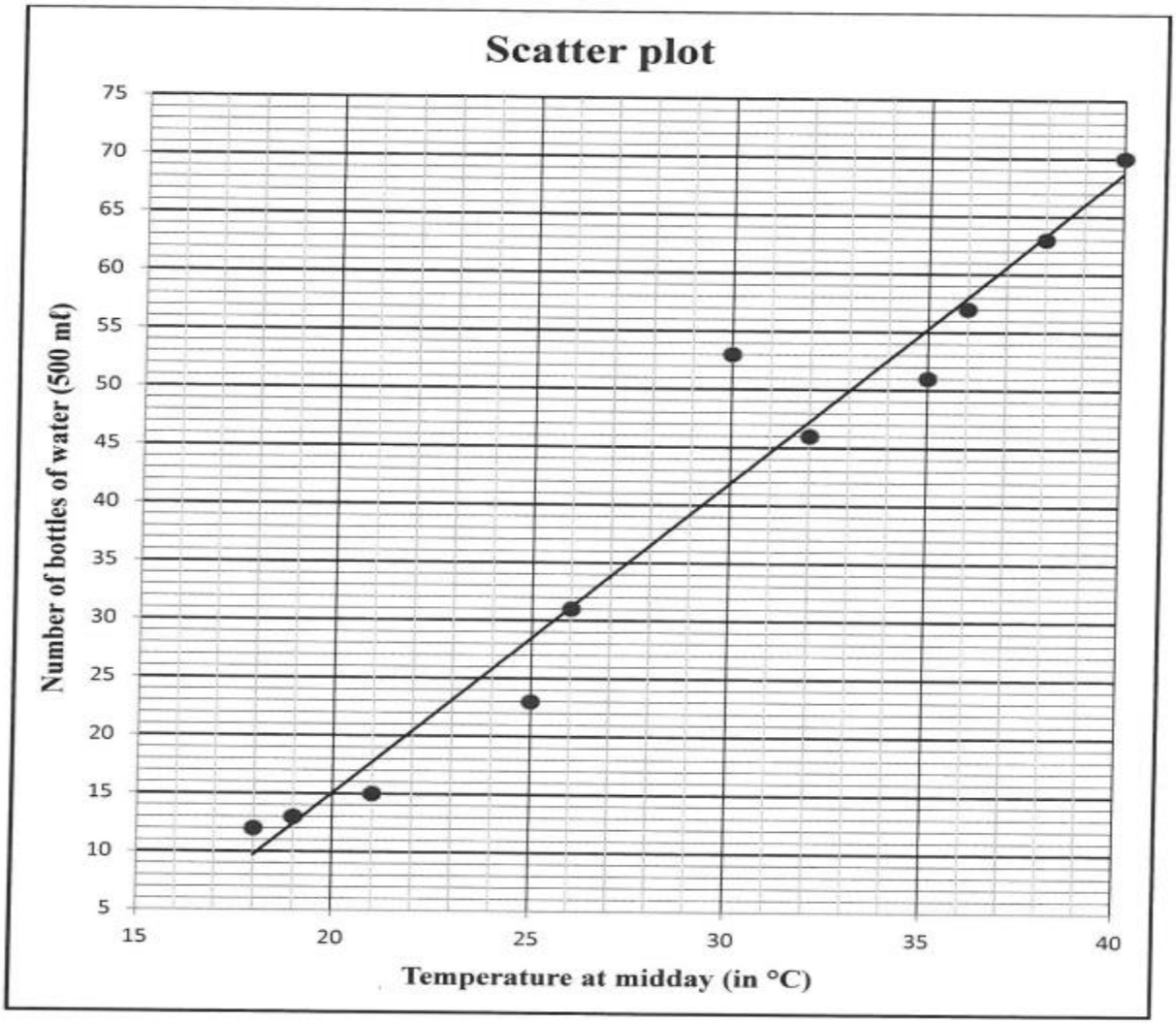
1.6 A tour is regarded as popular if the number of passengers on a tour bus is one standard deviation above the mean. How many destinations were popular on this particular day? (2)
[12]

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QUESTION 2

On the first school day of each month information is recorded about the temperature at midday (in °C) and the number of 500 mℓ bottles of water that were sold at the tuck shop of a certain school during the lunch break. The data is shown in the table below and represented on the scatter plot. The least squares regression line for this data is drawn on the scatter plot.

Temperature at midday (in °C)	18	21	19	26	32	35	36	40	38	30	25
Number of bottles of water (500 mℓ)	12	15	13	31	46	51	57	70	63	53	23



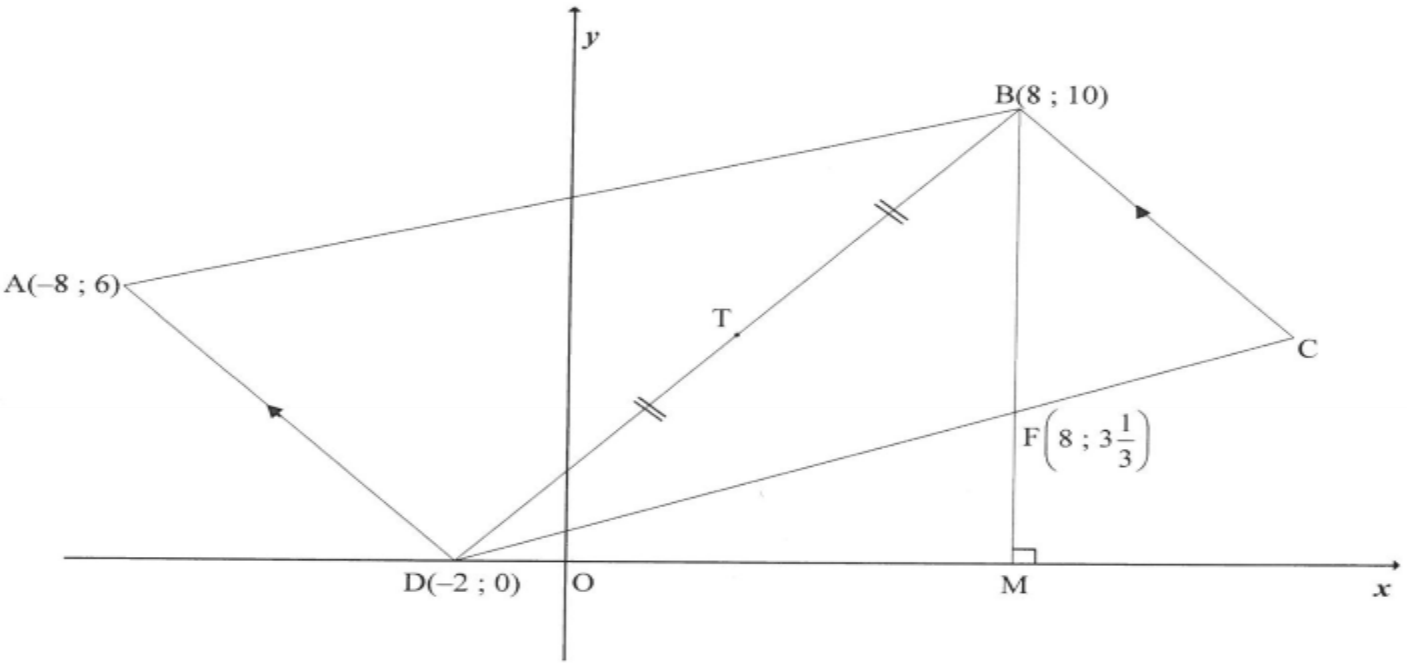
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2.1	Identify an outlier in the data.	(1)
2.2	Determine the equation of the least squares regression line.	(3)
2.3	Estimate the number of 500 mℓ bottles of water that will be sold if the temperature is 28 °C at midday.	(2)
2.4	Refer to the scatter plot. Would you say that the relation between the temperature at midday and the number of 500 mℓ bottles of water sold is weak or strong? Motivate your answer.	(2)
2.5	Give a reason why the observed trend for this data cannot continue indefinitely.	(1)
		[9]

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QUESTION 3

In the diagram below (not drawn to scale) A(–8 ; 6), B(8 ; 10), C and D(–2 ; 0) are the vertices of a trapezium having BC || AD. T is the midpoint of DB. From B, the straight line drawn parallel to the y-axis cuts DC in $F\left(8 ; 3\frac{1}{3}\right)$ and the x-axis in M.



3.1 Calculate the gradient of AD. (2)

3.2 Determine the equation of BC in the form $y = mx + c$. (3)

3.3 Prove that BD ⊥ AD. (3)

3.4 Calculate the size of $B\hat{D}M$. (2)

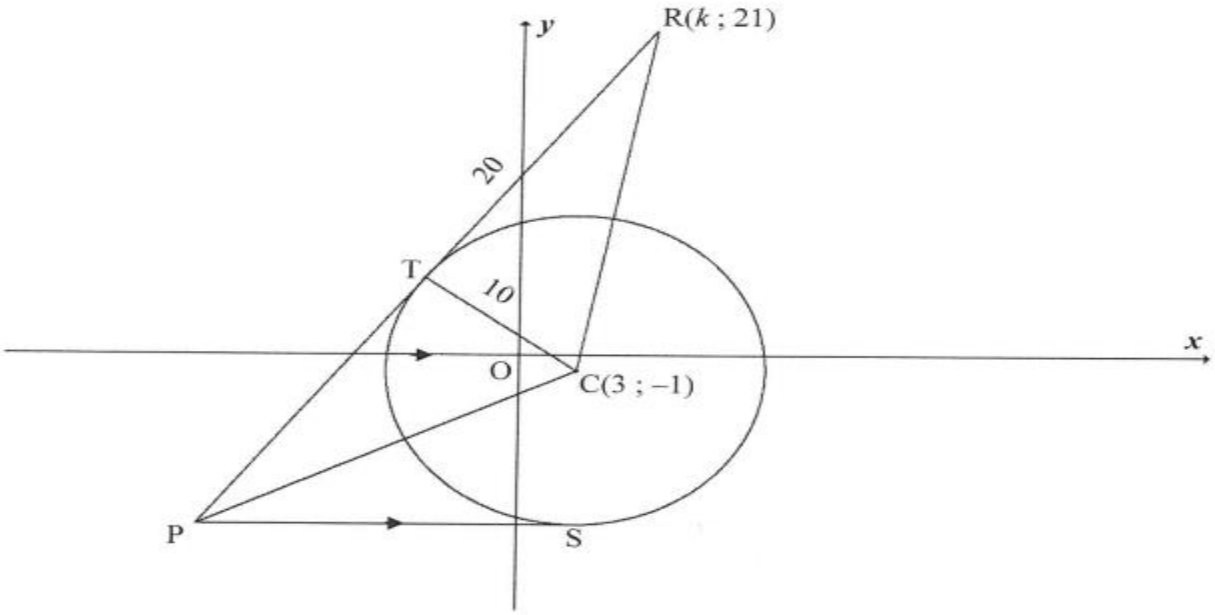
3.5 If it is given that TC || DM and points T and C are symmetrical about line BM, calculate the coordinates of C. (3)

3.6 Calculate the area of ΔBDF. (5)
[18]

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QUESTION 4

A circle having C(3 ; –1) as centre and a radius of 10 units is drawn. PTR is a tangent to this circle at T. R(k ; 21), C and P are the vertices of a triangle. TR = 20 units.



4.1 Give a reason why TC ⊥ TR. (1)

4.2 Calculate the length of RC. Leave your answer in surd form. (2)

4.3 Calculate the value of k if R lies in the first quadrant. (4)

4.4 Determine the equation of the circle having centre C and passing through T. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$ (2)

4.5 PS, a tangent to the circle at S, is parallel to the x-axis. Determine the equation of PS. (2)

4.6 The equation of PTR is $3y-4x=35$

4.6.1 Calculate the coordinates of P. (2)

4.6.2 Calculate, giving a reason, the length of PT. (3)

4.7 Consider another circle with equation $(x-3)^2+(y+16)^2=16$ and having centre M.

4.7.1 Write down the coordinates of centre M. (1)

4.7.2 Write down the length of the radius of this circle. (1)

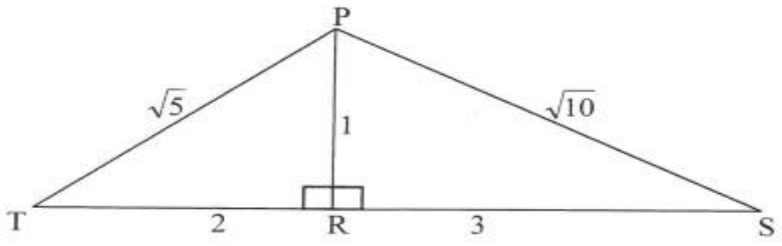
4.7.3 Prove that the circle with centre C and the circle with centre M do not intersect or touch. (3)

[21]

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QUESTION 5

5.1 In the diagram PR ⊥ TS in obtuse triangle PTS.
PT = $\sqrt{5}$; TR = 2; PR = 1; PS = $\sqrt{10}$ and RS = 3



5.1.1 Write down the value of:

(a) $\sin \hat{T}$ (1)

(b) $\cos \hat{S}$ (1)

5.1.2 Calculate, WITHOUT using a calculator, the value of $\cos(\hat{T} + \hat{S})$ (5)

5.2 Determine the value of:

$$\frac{1}{\cos(360° - \theta).\sin(90° - \theta)} - \tan^2(180° + \theta)$$ (6)

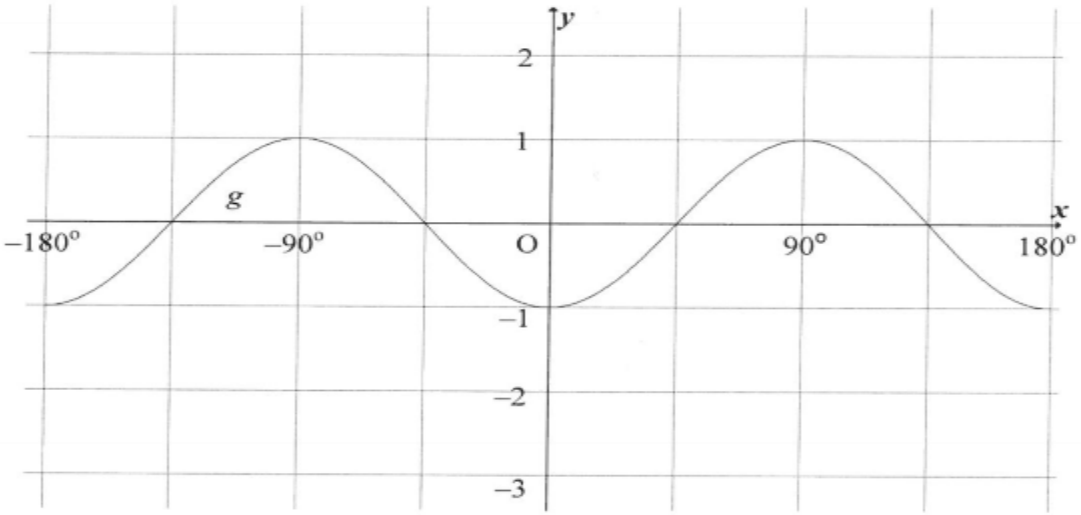
5.3 If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (5)
[18]

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QUESTION 6

6.1 Determine the general solution of $4\sin x + 2\cos 2x = 2$ (6)

6.2 The graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$ is drawn below.



6.2.1 Draw the graph of $f(x) = 2\sin x - 1$ for $x \in [-180° ; 180°]$ on the set of axes provided in the ANSWER BOOK. (3)

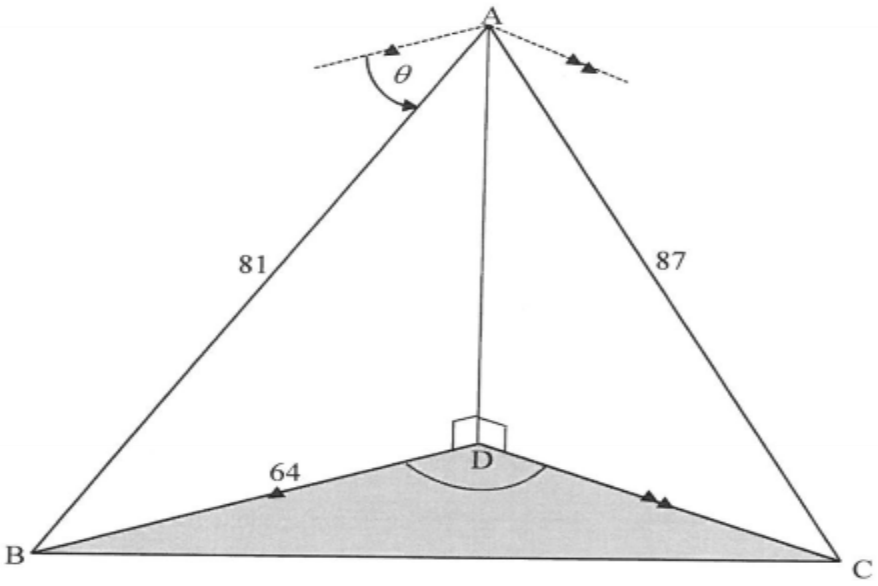
6.2.2 Write down the values of x for which g is strictly decreasing in the interval $x \in [-180° ; 0°]$ (2)

6.2.3 Write down the value(s) of x for which $f(x + 30°) - g(x + 30°) = 0$ for $x \in [-180° ; 180°]$ (2)
[13]

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QUESTION 7

From point A an observer spots two boats, B and C, at anchor. The angle of depression of boat B from A is θ. D is a point directly below A and is on the same horizontal plane as B and C. BD = 64 m, AB = 81 m and AC = 87 m.



7.1 Calculate the size of θ to the nearest degree. (3)

7.2 If it is given that $B\hat{A}C = 82{,}6°$, calculate BC, the distance between the boats. (3)

7.3 If $B\hat{D}C = 110°$, calculate the size of $D\hat{C}B$. (3)

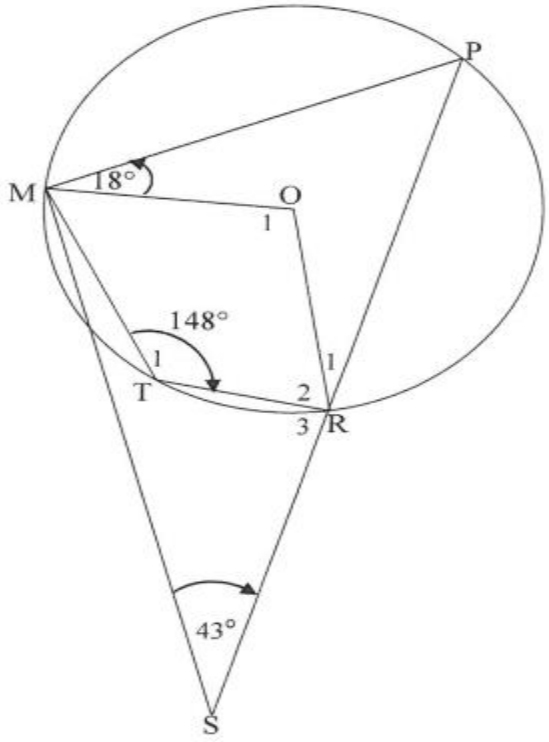
[9]

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Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 In the diagram below, P, M, T and R are points on a circle having centre O. PR produced meets MS at S. Radii OM and OR and the chords MT and TR are drawn. $\hat{T}_1 = 148°$, $P\hat{M}O = 18°$ and $\hat{S} = 43°$



Calculate, with reasons, the size of:

8.1.1 $\hat{P}$ (2)

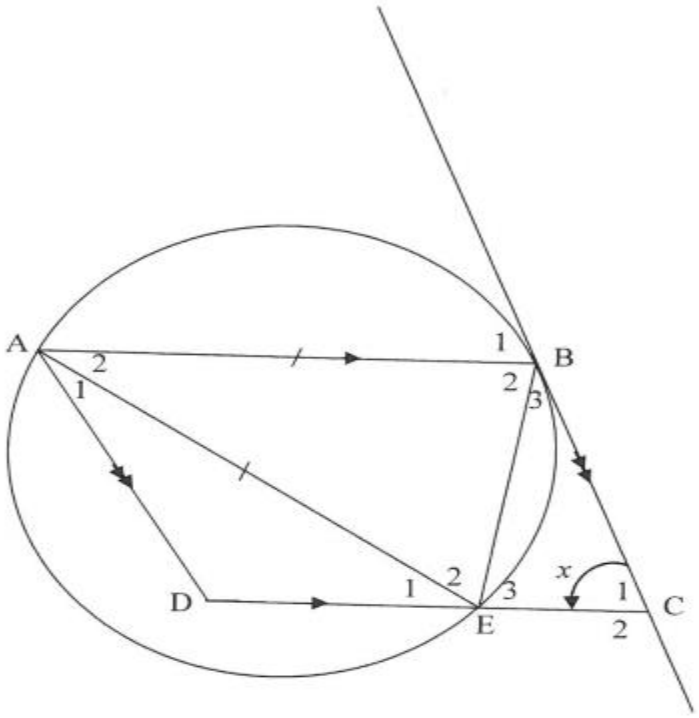
8.1.2 $\hat{O}_1$ (2)

8.1.3 $O\hat{M}S$ (2)

8.1.4 $\hat{R}_3$ if it is given that $T\hat{M}S = 6°$ (2)

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8.2 In the diagram below, the circle passes through A, B and E. ABCD is a parallelogram. BC is a tangent to the circle at B. AE = AB. Let $\hat{C}_1 = x$



8.2.1 Give a reason why $\hat{B}_1 = x$ (1)

8.2.2 Name, with reasons, THREE other angles equal in size to x. (6)

8.2.3 Prove that ABED is a cyclic quadrilateral. (3)
[18]

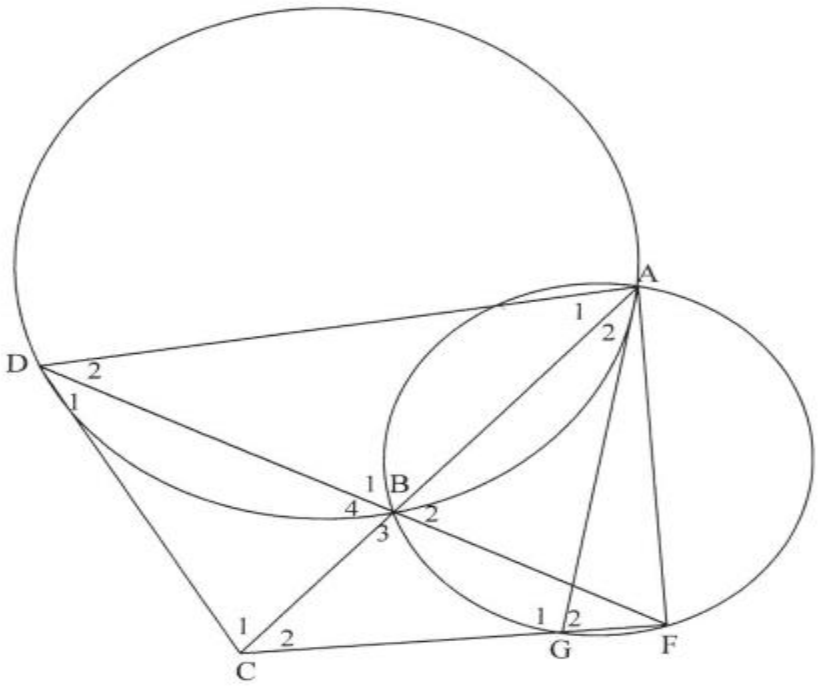
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QUESTION 9

9.1 Complete the statement so that it is TRUE:

The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle ... (1)

9.2 In the diagram below, two unequal circles intersect at A and B. AB is produced to C such that CD is a tangent to the circle ABD at D. F and G are points on the smaller circle such that CGF and DBF are straight lines. AD and AG are drawn.



Prove that:

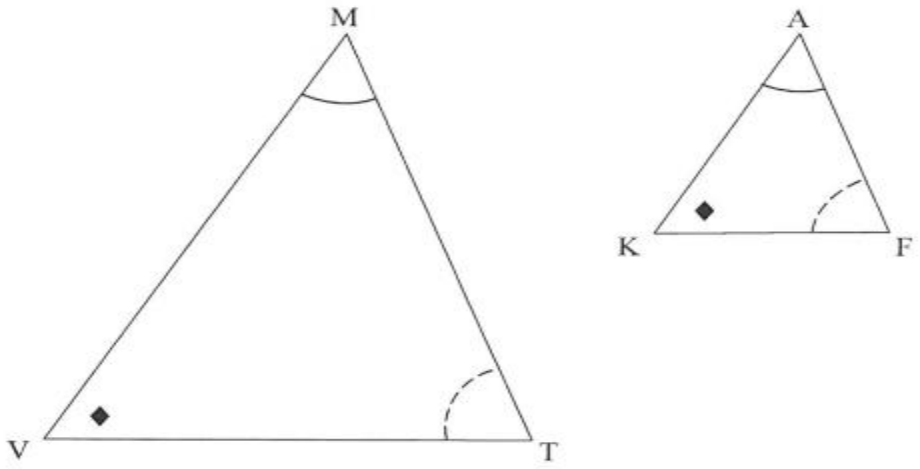
9.2.1 $\hat{B}_4 = \hat{D}_1 + \hat{D}_2$ (4)

9.2.2 AGCD is a cyclic quadrilateral (4)

9.2.3 DC = CF (4)
[13]

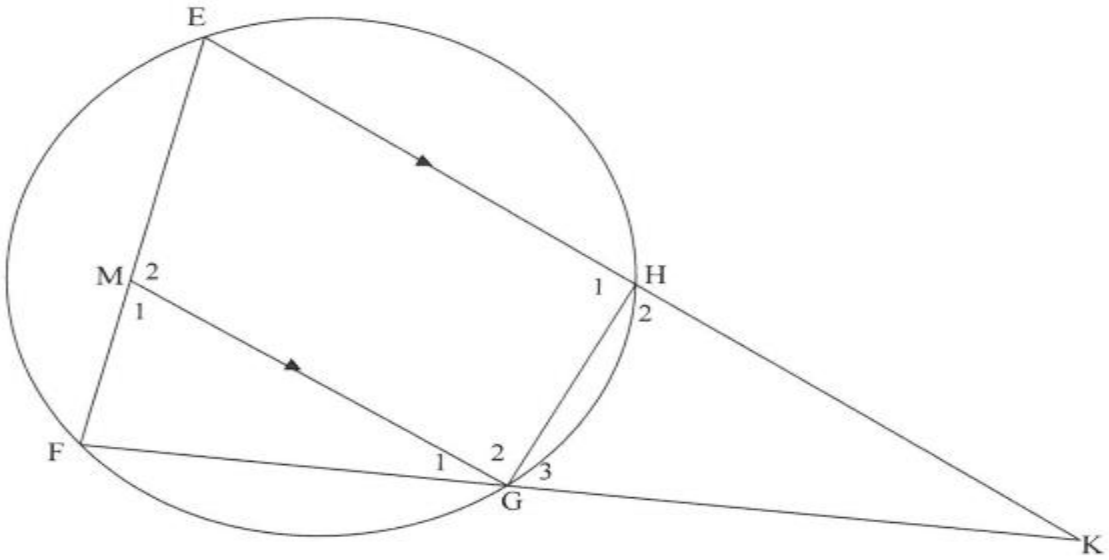
QUESTION 10

10.1 In the diagram below, ΔMVT and ΔAKF are drawn such that $\hat{M} = \hat{A}$, $\hat{V} = \hat{K}$ and $\hat{T} = \hat{F}$



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{MV}{AK} = \frac{MT}{AF}$ (7)

10.2 In the diagram below, cyclic quadrilateral EFGH is drawn. Chord EH produced and chord FG produced meet at K. M is a point on EF such that MG | | EK. Also KG = EF



10.2.1 Prove that:

(a) $\Delta KGH \,|||\, \Delta KEF$ (4)

(b) $EF^2 = KE \cdot GH$ (2)

(c) $KG^2 = EM \cdot KF$ (3)

10.2.2 If it is given that KE = 20 units, KF = 16 units and GH = 4 units, calculate the length of EM. (3)
[19]

TOTAL: 150

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SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

JUNE 2016

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 21 pages./
Hierdie memorandum bestaan uit 21 bladsye.

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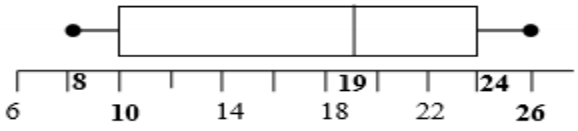
Please turn over/Blaai om asseblief

NOTE:
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

8	8	10	12	16	19	20	21	24	25	26

1.1	Mean/*Gemiddelde* $= \frac{189}{11}$ $= 17{,}18$ Answer only: Full marks *Slegs antwoord: Volpunte*	✓189 ✓ answer (2)
1.2	Min = 8, max = 26 Median/*Mediaan* = 19 $Q_1 = 10$, $Q_3 = 24$ ∴ (8 ; 10 ; 19 ; 24 ; 26)	✓ min, max ✓ median ✓ Q_1 & Q_3 (3)
1.3		✓ box/*boks/mond* ✓ whiskers/*snor* (2)
1.4	The data is skewed to the left/*Die data is skeef na links.* **OR/*OF*** Negatively skewed/*Negatief skeef*	✓ answer (1) ✓ answer (1)
1.5	SD/*SA* = 6,46	✓✓ answer (2)
1.6	17,18 + 6,46 = 23,64 ∴ 3 destinations/*bestemmings*	✓ interval ✓ answer (2) **[12]**

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QUESTION/*VRAAG* 2

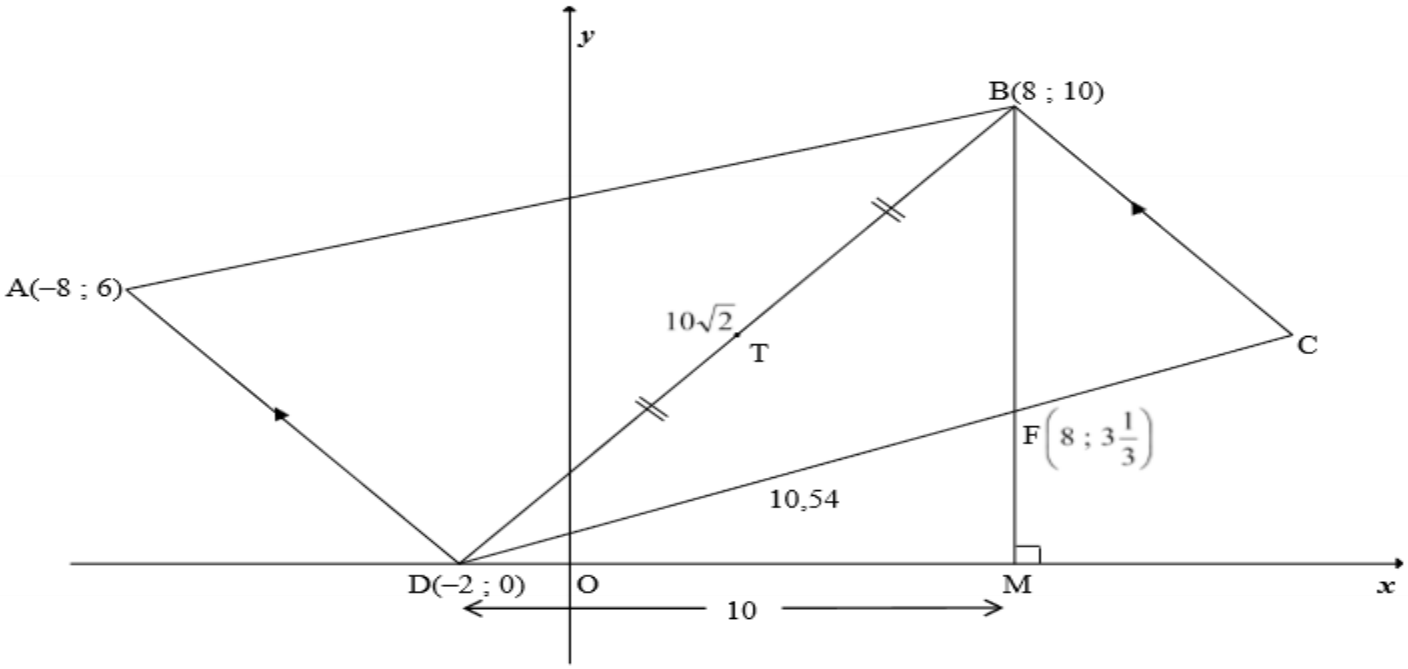
Temperature at midday (in °C) ***Middaguur-temperatuur (in °C)***	18	21	19	26	32	35	36	40	38	30	25
Number of bottles of water (500 mℓ) ***Getal bottels water (500 mℓ)***	12	15	13	31	46	51	57	70	63	53	23

2.1	(30 ; 53)	✓answer (1)
2.2	$a = -38{,}51$ $b = 2{,}68$ $\therefore \hat{y} = 2{,}68x - 38{,}51$	✓ value *a* ✓ value *b* ✓ equation (3)
2.3	$\therefore \hat{y} \approx 36{,}53$ bottles **OR/*OF*** $\hat{y} \approx 2{,}68(28) - 38{,}51$ $\approx 36{,}53$ bottles	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	Strong/*Sterk* The majority of the points lie **close to** the regression line./*Die meerderheid punte lê naby die regressielyn.* **OR/*OF*** Strong/*Sterk* $r = 0{,}98$	✓ strong/*sterk* ✓ reason/*rede* (2) ✓ strong/*sterk* ✓ reason/*rede* (2)
2.5	Temperature cannot rise beyond a certain point as this would be life threatening **OR** there is only so much water one can consume before it becomes a risk to your health (hyponatremia)./*Temperatuur kan nie hoër as 'n sekere punt styg nie, anders raak dit lewensgevaarlik.* ***OF*** *'n persoon kan net 'n sekere hoeveelheid water inneem, anders raak dit 'n gesondheidsrisiko*	✓ reason/*rede* (1) **[9]**

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QUESTION/*VRAAG* 3



3.1	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0-6}{-2+8}$ $= \frac{-6}{6} = -1$	✓substitution ✓–1 (2)
3.2	$m_{BC} = -1$ [BC||AD] $y = -x + c$ $10 = -8 + c$ $c = 18$ $y = -x + 18$ **OR/*OF*** $m_{BC} = -1$ [BC||AD] $y - y_1 = m(x - x_1)$ $y - 10 = -(x - 8)$ $y = -x + 18$	✓ gradient ✓substitute m and (8 ; 10) ✓ equation (3) ✓ gradient ✓substitute m and (8 ; 10) ✓ equation (3)

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3.3	$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{10-0}{8+2} = 1$ $m_{BD} \times m_{AD} = 1 \times -1 = -1$ $\therefore DB \perp AD$ **OR** $AD^2 = 72$ *or* $AD = \sqrt{72}$ *or* $6\sqrt{2}$ $AB^2 = 272$ *or* $AB = \sqrt{272}$ *or* $4\sqrt{17}$ $BD^2 = 200$ *or* $BD = \sqrt{200}$ *or* $10\sqrt{2}$ $\therefore AB^2 = AD^2 + BD^2$ $\therefore A\hat{D}B = 90°$ [converse Pyth th/ *omgekeerde Pyth st*]	✓substitution ✓ answer ✓ $m_{BD} \times m_{AD} = -1$ (3) ✓✓ calculating all 3 sides ✓ $AB^2 = AD^2 + BD^2$ (3)
3.4	$\tan B\hat{D}M = m_{BD} = 1$ $\therefore B\hat{D}M = 45°$ **OR** $\sin B\hat{D}M = \frac{BM}{BD} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\therefore B\hat{D}M = 45°$	✓ $\tan B\hat{D}M = m_{BD}$ ✓ answer (2) ✓ $\sin B\hat{D}M = \frac{1}{\sqrt{2}}$ ✓ answer (2)
3.5	$T(x\,;y) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $= \left(\frac{-2+8}{2}; \frac{0+10}{2}\right)$ $= (3\,;\,5)$ T symmetrical about BM/*T is simmetries om BM* $\therefore$ distance of T to BM = 5 units = distance from BM to C $\therefore$ C(13 ; 5) **OR/*OF***	✓T(3 ; 5) ✓value of x ✓ value of y (3)

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	$m_{DF} = \frac{3\frac{1}{3} - 0}{8 - (-2)} = \frac{1}{3}$ Equation of DF: $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{3}(x + 2)$ $y = \frac{1}{3}x + \frac{2}{3}$ Equation of BC: $y = -x + 18$ $\frac{1}{3}x + \frac{2}{3} = -x + 18$ $4x = 52$ $x = 13$ $\therefore y = -13 + 18 = 5$ $\therefore$ C(13 ; 5)	✓eq of DF ✓value of x ✓ value of y (3)
3.6	area/*opp* ΔBDF = area/*opp* ΔBDM – area/*opp* ΔDFM $= \frac{1}{2}(10)(10) - \frac{1}{2}(10)\left(\frac{10}{3}\right)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,3 square units/*vk eenh* **OR/*OF*** area/*opp* ΔBDF $= \frac{1}{2}$.BF.DM $= \frac{1}{2}\left(\frac{20}{3}\right)(10)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,3 square units/*vk eenh* **OR/*OF***	✓ formula/method ✓ 10 (DM) ✓ 10 (BM) ✓ $\frac{10}{3}$ or $3\frac{1}{3}$ ($\perp$h) ✓ answer (5) ✓ formula/method ✓ BF ✓✓ DM ✓ answer (5)

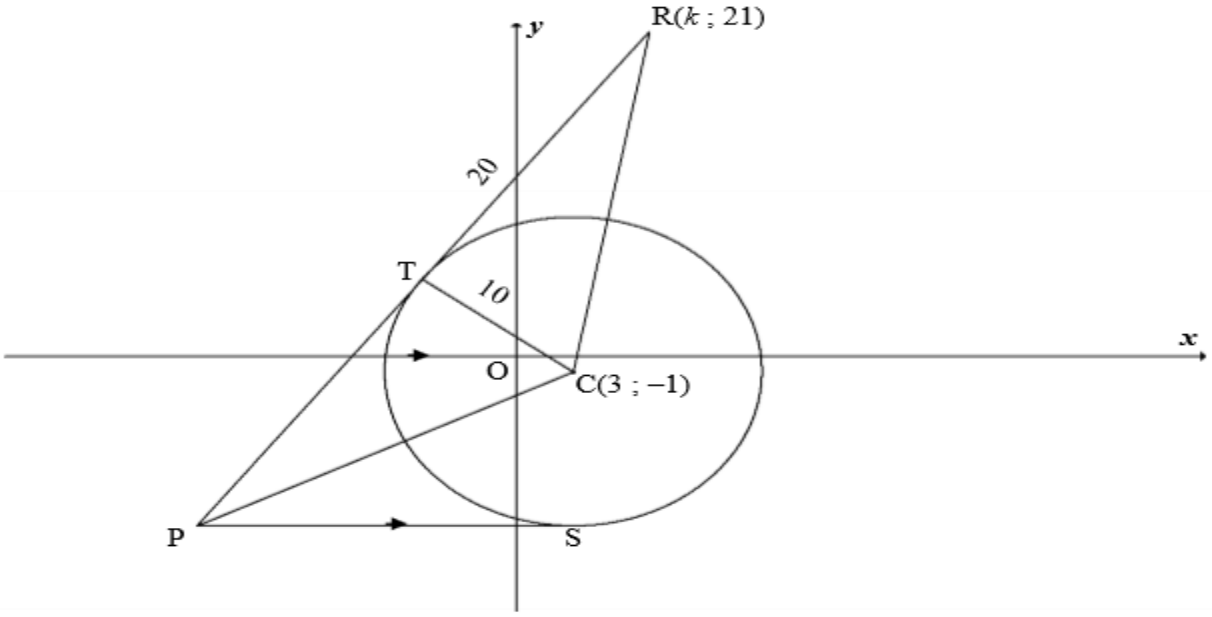
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	$\tan \hat{FDM} = m_{DC} = \frac{5-0}{13+2} = \frac{1}{3}$ or $\tan \hat{FDM} = \frac{FM}{DM} = \frac{\frac{10}{3}}{10} = \frac{1}{3}$	✓ gradient/ratio
	$\hat{FDM} = 18{,}43°$ $\therefore B\hat{F}D = 108{,}43°$ [ext $\angle$ Δ]	✓ $B\hat{F}D$
	$BF = \frac{20}{3}$ or $6\frac{2}{3}$	
	$DF^2 = (10)^2 + \left(3\frac{1}{3}\right)^2$ [Pyth ΔDFM]	✓ DF
	$DF = 10{,}54$ or $\frac{\sqrt{1000}}{3}$ or $\frac{10\sqrt{10}}{3}$	$BD = \sqrt{(10-0)^2 + (8+2)^2}$ $= \sqrt{200}$ *or* $10\sqrt{2}$
	$\therefore$ area/*opp* $\Delta BDF = \frac{1}{2}.BF.FD.\sin B\hat{F}D$ $= \frac{1}{2}\left(\frac{20}{3}\right)\left(\frac{10\sqrt{10}}{3}\right)(\sin 108{,}43)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓correct substitution into area rule ✓ answer (5)
	OR/*OF*	
	$BF = \frac{20}{3}$ or $6\frac{2}{3}$	✓ BF
	$BD = \sqrt{(10-0)^2 + (8+2)^2}$ $= \sqrt{200}$ *or* $10\sqrt{2}$	✓ BD
	area/*opp* $\Delta BDF = \frac{1}{2}.BF.BD.\sin D\hat{B}F$ $= \frac{1}{2}\left(\frac{20}{3}\right)\left(\sqrt{200}\right)(\sin 45°)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓ formula/method ✓correct substitution into area rule ✓ answer (5)
	OR/*OF* area/*opp* ΔBDF = area/*opp* ΔBCD – area/*opp* ΔBCF $= \frac{1}{2}\left(10\sqrt{2}\right)\left(5\sqrt{2}\right) - \frac{1}{2}\left(\frac{20}{3}\right)(5)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓ formula/method ✓ BD = $10\sqrt{2}$ ✓ BC = $5\sqrt{2}$ ✓ BF = $\frac{20}{3}$ ✓ answer (5)
	OR/*OF*	

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	$\tan \hat{FDM} = m_{DC} = \frac{5-0}{13+2} = \frac{1}{3}$ or $\tan \hat{FDM} = \frac{FM}{DM} = \frac{\frac{10}{3}}{10} = \frac{1}{3}$ $\hat{FDM} = 18{,}43°$ $\hat{BDF} = 26{,}56°$ area / *opp* ΔBDF $= \frac{1}{2}.BD.DF.\sin \hat{BDF}$ $= \frac{1}{2}.\left(10\sqrt{2}\right)\left(\frac{10\sqrt{10}}{3}\right).\sin 26{,}56°$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓ gradient/ratio ✓ $\hat{BDF}$ ✓ DF **OR/*OF*** BD ✓correct substitution into area rule ✓ answer (5) **[18]**

QUESTION/*VRAAG* 4



4.1	radius ⊥ tangent /*raaklyn*	✓ R (1)
4.2	$CR^2 = TR^2 + CT^2$ (Pyth) $CR^2 = 20^2 + 10^2 = 500$ $CR = \sqrt{500}$ or $10\sqrt{5}$	✓ substitution ✓ answer (2)
4.3	$CR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $500 = (k-3)^2 + (21+1)^2$ $k^2 - 6k + 9 + 484 = 500$ $k^2 - 6k - 7 = 0$ $(k-7)(k+1) = 0$ $k = 7$ or $k \neq -1$ **OR/*OF*** $CR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $500 = (k-3)^2 + (21+1)^2$ $(k-3)^2 = 16$ $k - 3 = 4$ or $k - 3 = -4$ $k = 7$ or $k \neq -1$	✓ substitution ✓ standard form ✓ factors ✓ $k = 7$ (4) ✓ substitution ✓ square form ✓ square root ✓ $k = 7$ (4)

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4.4	$(x-3)^2+(y+1)^2=100$	✓✓ answer (2)
4.5	CS = 10 and CS ⊥ PS ∴ S(3 ; −11) ∴ $y=-11$	✓ S(3 ; −11) ✓ answer (2)
4.6.1	S(3;−11) ∴ $3(-11)-4x=35$ $x=-17$ ∴ P(−17 ; −11) **OR/*OF*** $\frac{4}{3}x+\frac{35}{3}=-11$ $\frac{4}{3}x=\frac{-68}{3}$ $x=-17$ P(−17 ; −11)	✓ substituting ✓ answer (2) ✓ equating ✓ answer (2)
4.6.2	PT = PS [tangents from common point/*rklyne vanaf dies pt*] = 17 + 3 = 20 units **OR** $PC=\sqrt{(-17-3)^2+(-11+1)^2}$ $=\sqrt{500}$ *or* $10\sqrt{5}$ $PT^2=PC^2-TC^2$ [Pyth th] $=500-100$ $=400$ ∴ PT = 20 **OR** $PC=\sqrt{(-17-3)^2+(-11+1)^2}$ $=\sqrt{500}$ *or* $10\sqrt{5}$ Δ PTC ≡ ΔRTC [90°HS] ∴ PT = TR ∴ PT = 20	✓ S ✓R ✓ answer (3) ✓ value of PC ✓ using Pyth ✓ answer (3) ✓ value of PC ✓ S/R or proved ✓ answer (3)
4.7.1	M(3 ; −16)	✓answer (1)

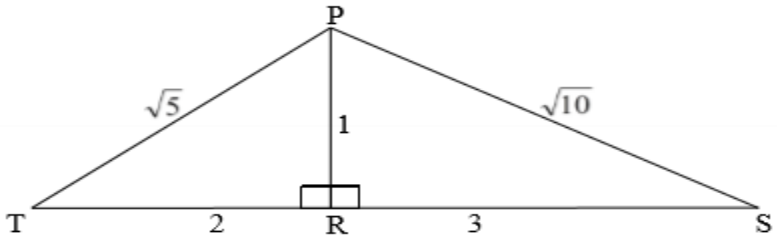
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4.7.2	Radius = 4	✓ answer (1)
4.7.3	$r_1 + r_2 = 10 + 4 = 14$ distance CM $= \sqrt{(3-3)^2 + (-1+16)^2}$ $= \sqrt{225}$ $= 15$ CM > $r_1 + r_2$ Therefore the two circles do not intersect or touch./*Daarom sny of raak die twee sirkels nie.*	✓ $r_1 + r_2$ ✓ 15 ✓explanation (3) **[21]**

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QUESTION/*VRAAG* 5

5.1



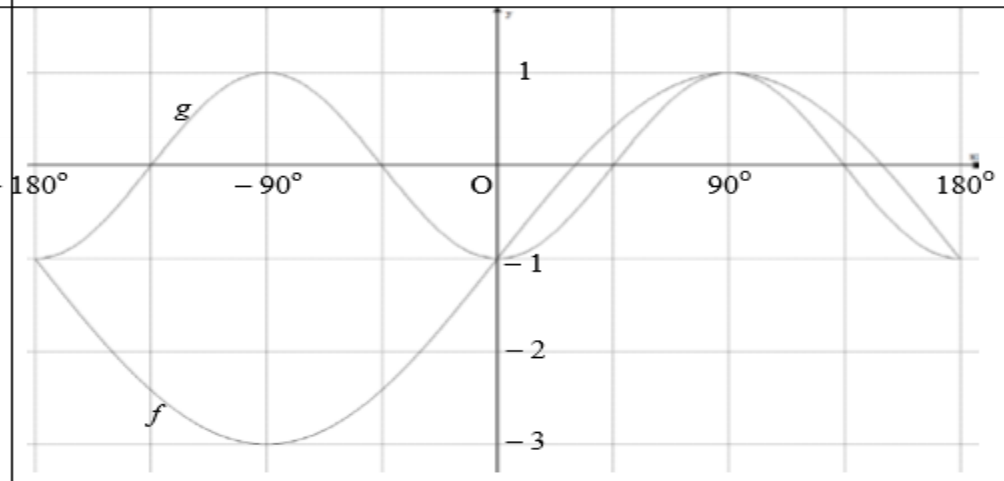
5.1.1(a)	$\sin T = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = 0{,}45$	✓value (1)
5.1.1(b)	$\cos S = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} = 0{,}95$	✓value (1)
5.1.2	$\cos(T+S) = \cos T \cos S - \sin T \sin S$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right)$ $= \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$ $= \frac{5}{\sqrt{50}}$ *or* $\frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓expansion ✓ $\frac{2}{\sqrt{5}}$ ✓ $\frac{1}{\sqrt{10}}$ ✓ simplification ✓ answer (5)
5.2	$\frac{1}{\cos(360° - \theta)\sin(90° - \theta)} - \tan^2(180° + \theta)$ $= \frac{1}{(\cos\theta)(\cos\theta)} - \tan^2\theta$ $= \frac{1}{\cos^2\theta} - \left(\frac{\sin^2\theta}{\cos^2\theta}\right)$ $= \frac{1-\sin^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta}$ *OR* $\frac{1-\sin^2\theta}{1-\sin^2\theta}$ $= 1$	✓ $\cos\theta$ ✓ $\cos\theta$ ✓ $\tan^2\theta$ ✓ $\frac{\sin^2\theta}{\cos^2\theta}$ ✓identity ✓ answer (6)

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5.3	$(\sin x-\cos x)^2=\left(\frac{3}{4}\right)^2$ $\sin^2 x-2\sin x\cos x+\cos^2 x=\frac{9}{16}$ $1-2\sin x\cos x=\frac{9}{16}$ $2\sin x\cos x=\frac{7}{16}$ $\therefore \sin 2x=\frac{7}{16}$	✓ squaring both sides ✓expanding LHS ✓ using identity ✓ simplifying ✓answer (5) **[18]**

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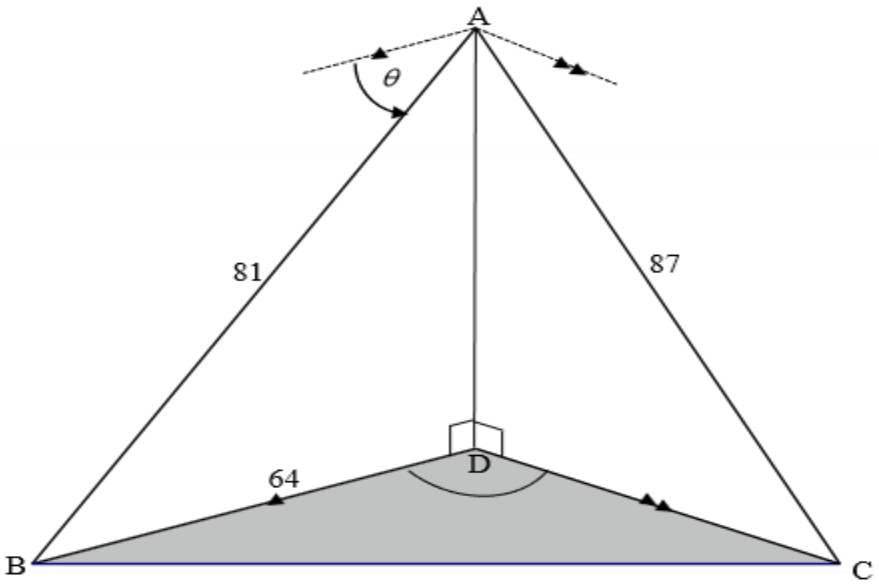
QUESTION/*VRAAG* 6

6.1	$4\sin x + 2\cos 2x = 2$ $2\sin x + \cos 2x - 1 = 0$ $2\sin x + (1 - 2\sin^2 x) - 1 = 0$ $2\sin^2 x - 2\sin x = 0$ $2\sin x(\sin x - 1) = 0$ $2\sin x = 0$ or $\sin x - 1 = 0$ $\sin x = 0$ $\sin x = 1$ $x = k.180°$ or $x = 90° + k.360$, $k \in Z$	✓using identity ✓ standard form ✓ factors ✓ $\sin x = 0$ or $\sin x = 1$ ✓ $k.180°$ ✓ $90° + k.360, k \in Z$ (6)
6.2.1		✓ turning point $(-90° ; -3)$ ✓ turning point $(90° ; 1)$ ✓ $(-180° ; -1)$ & $(0° ; -1)$ (3)
6.2.2	$(-90° ; 0°)$ **OR/*OF*** $-90° < x < 0°$	✓ ✓ answer (2) ✓ ✓ answer (2)
6.2.3	$f(x) = g(x)$ $\therefore -180° ; 0° ; 90° ; 180°$ $f(x + 30°) = g(x + 30°)$ $\therefore x = -30° ; 60° ; 150°$	✓ any ONE correct ✓ other 2 correct (2) **[13]**

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QUESTION/*VRAAG* 7



7.1	$A\hat{B}D = \theta$ [alternate $\angle$ s; $\|\|$ lines] $\cos\theta = \frac{BD}{AB} = \frac{64}{81}$ $\theta = 38°$ OR/*OF* $\sin B\hat{A}D = \frac{64}{81}$ $B\hat{A}D = 52{,}18°$ $\theta = 38°$	✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3) ✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3)
7.2	$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$ $= 81^2 + 87^2 - 2(81)(87)\cos 82{,}6°$ $= 12314{,}754...$ $BC = 110{,}97\text{ m}$	✓use cosine rule ✓correct substitution into cosine rule ✓answer (3)

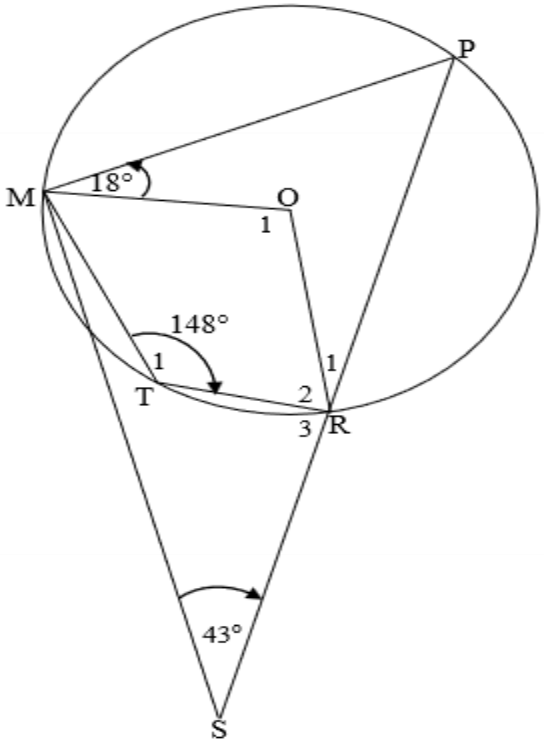
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7.3	$\frac{\sin D\hat{C}B}{BD} = \frac{\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{BD.\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{64.\sin 110°}{110,97}$ $\therefore D\hat{C}B = 32,82°$	✓ use sine rule ✓ substitution ✓ answer (3) **[9]**

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QUESTION/*VRAAG* 8

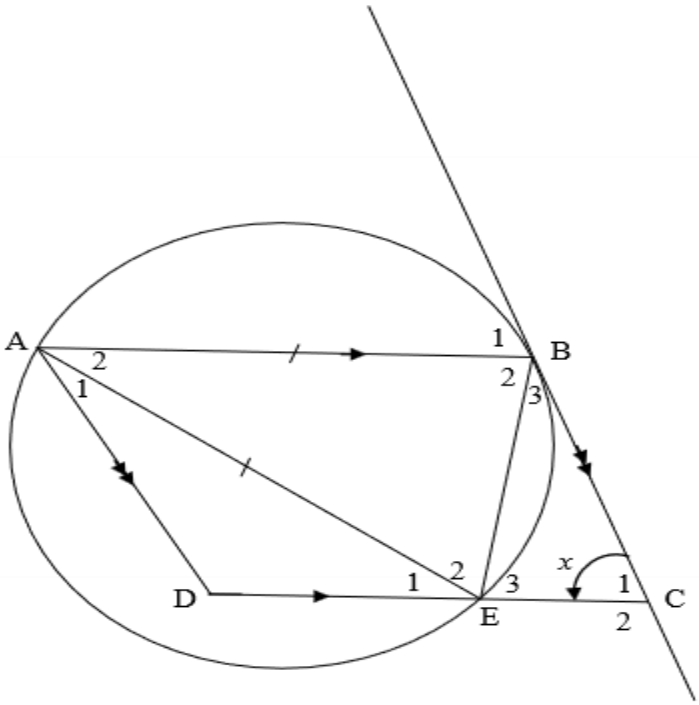
8.1



8.1.1	$\hat{P} = 32°$ [opp ∠s of cyclic quad/*teenoorst∠e v koordevh*]	✓ S ✓ R (2)
8.1.2	$\hat{O}_1 = 2(32°) = 64°$ [∠ centre = 2 ∠ at circum/*midpts∠ = 2 omtreks∠*] **OR/*OF*** reflex $\hat{O} = 296°$ [∠ centre = 2 ∠ at circum/*midpts∠ = 2 omtreks∠*] $\hat{O}_1 = 64°$ [∠s around a point/*∠e om 'n punt*]	✓ S ✓R (2) ✓ S and R ✓ S (2)
8.1.3	$O\hat{M}S = 180° - (32° + 18° + 43°)$ [sum ∠s Δ / *som ∠e Δ*] $= 87°$	✓ S ✓ S (2)
8.1.4	$\hat{R}_3 = T\hat{M}P$ [ext ∠ cyclic quad/*buite ∠ koordevh*] $= 87° + 18° - 6°$ $= 99°$	✓ R ✓ S (2)

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8.2



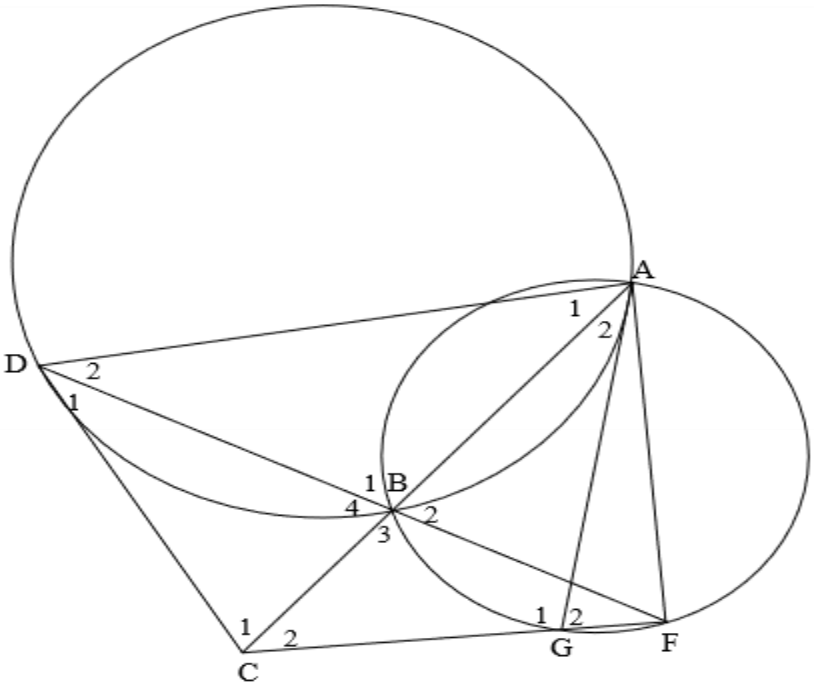
8.2.1	corres ∠s/*ooreenk ∠e*; AB||DC	✓ R (1)
8.2.2	$\hat{E}_2 = x$ [tan - chord theorem/*raakl – koordst*] $\hat{B}_2 = x$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{E}_3 = x$ [alt ∠s/*verwiss ∠e*; AB||DC] $D\hat{A}B = x$ [opp ∠s $\|^m$ /*teenoor ∠e* $\|^m$ **OR/*OF*** alternate/*verwiss* ∠s/*e*; BC ||AD] Any 3 ∠s correct	✓S ✓R ✓ S ✓ R ✓ S ✓ R (6)
8.2.3	$\hat{D} = 180° - x$ [co - int ∠s suppl/*ko – binne∠e* suppl; AD||BC] $\therefore \hat{B}_2 + \hat{D} = 180°$ ∴ ABED a cyc quad/*kdvh* [converse opp ∠s of cyclic quad/ *omgek teenoorst ∠e koordevh*] **OR/*OF*** $D\hat{A}B = x$ [opp ∠s/*teenoor ∠e* ||^m] OR/*OF* [alt∠s/*verwiss ∠e*; BC||AD] $\hat{E}_3 = D\hat{A}B = x$ ∴ ABED a cyc quad/*kdvh* [converse ext ∠ of cyc quad/*omgek buite∠ v koordevh*]	✓S ✓ R ✓ R (3) ✓S ✓ R ✓ R (3) **[18]**

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QUESTION/*VRAAG* 9

9.1	… in the alternate segment/*…in die(teen)oorstaande segment*	✓ answer (1)

9.2

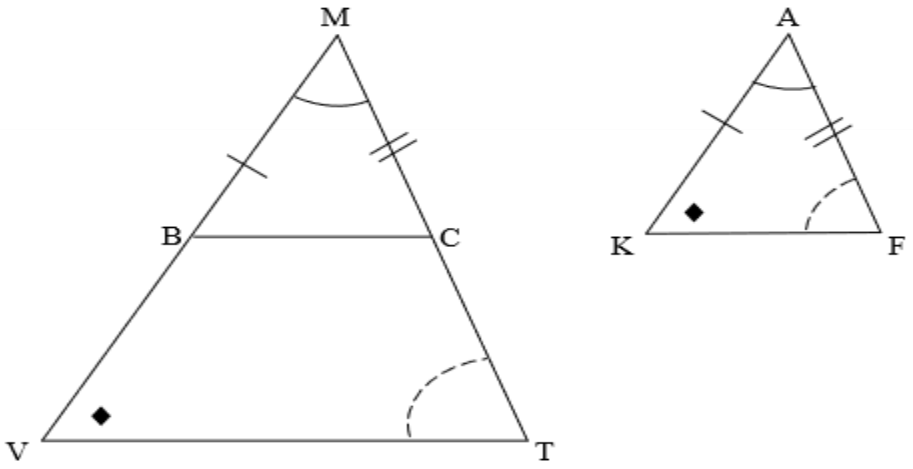


9.2.1	$\hat{A}_1 = \hat{D}_1$ [tan chord theorem/*raakl – koordst*] $\hat{B}_4 = \hat{A}_1 + \hat{D}_2$ [ext ∠ Δ/*buite*∠ Δ] $= \hat{D}_1 + \hat{D}_2$	✓S ✓R ✓ S ✓ R (4)
9.2.2	$\hat{B}_4 = \hat{B}_2$ [vert opp ∠s/*regoorst ∠e*] $\hat{D}_1 + \hat{D}_2 = \hat{B}_2$ [proven/*bewys*] $= \hat{G}_2$ [∠s in same segment/∠e *in dies segment*] ∴ AGCD is cyc quad/*kvh* [converse ext∠ cyc quad/*omgek buite∠ kvh*]	✓ S ✓ S ✓ R ✓ R (4)
9.2.3	$\hat{D}_1 = \hat{A}_2$ [∠s in same segment/∠e *in dies segment*] $\hat{A}_2 = \hat{F}$ [∠s in same segment/∠e *in dies segment*] ∴ $\hat{D}_1 = \hat{F}$ ∴ DC = CF [sides opp = ∠s/*sye teenoor = ∠e*]	✓ S ✓ R ✓ S ✓ R (4) **[13]**

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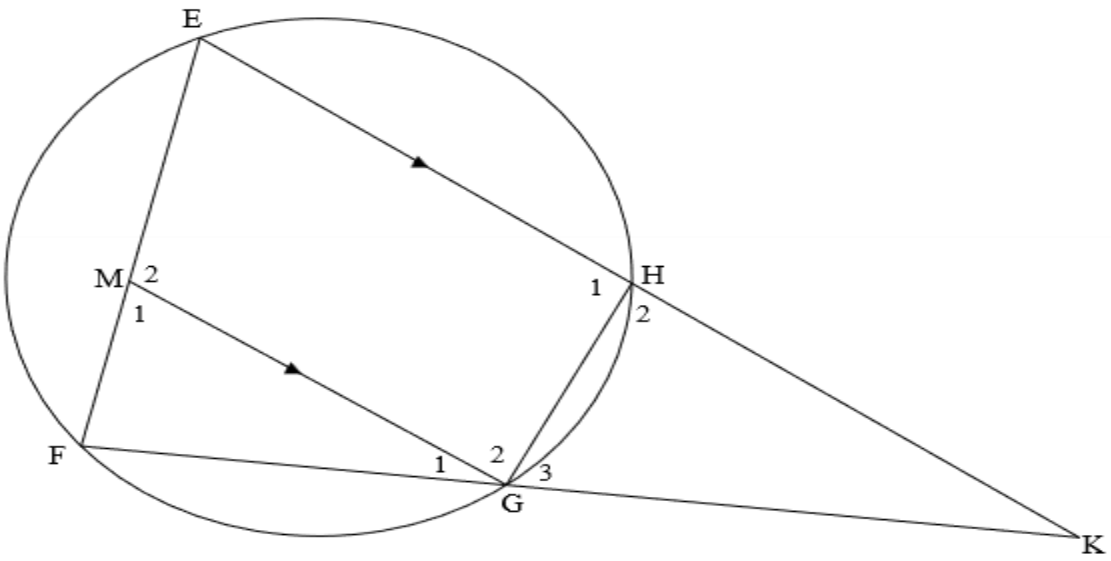
QUESTION/*VRAAG* 10



10.1	Constr/*Konstr* : Draw line BC such that MB = AK and MC = AF *Trek lyn BC sodat MB = AK en MC = AF* Proof/*Bewys* : In ΔBMC and/*en* ΔKAF MB = AK [constr/*konstr*] $\hat{M} = \hat{A}$ [given/*gegee*] MC = AF [constr/*konstr*] ΔBMC ≡ ΔKAF [s ∠ s] $\therefore M\hat{B}C = A\hat{K}F$ or $M\hat{C}B = A\hat{F}K$ [≡ Δ] but /*maar* $\hat{V} = \hat{K}$ or $\hat{T} = \hat{F}$ [given/*gegee*] $\therefore M\hat{B}C = \hat{V}$ or $M\hat{C}B = \hat{T}$ But these are corresponding ∠s/*maar hulle is ooreenk ∠e* ∴ BC || VT [corr ∠s = /*ooreenk∠e* =] $\therefore \frac{MV}{MB} = \frac{MT}{MC}$ [prop theorem/*eweredighst*; BC||VT] but /*maar* MB = AK and MC = AF [constr/*konstr*] $\therefore \frac{MV}{AK} = \frac{MT}{AF}$	✓ constr/*konstr* ✓ S / R ✓ S ✓ S ✓ S / R ✓S ✓R (7)

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10.2



10.2.1(a)	In ΔKGH and ΔKEF $\hat{K}$ is common/*gemeen* $\hat{H}_2 = \hat{F}$ [ext $\angle$ cyclic quad/*buite $\angle$ koordevh*] $\hat{G}_3 = \hat{E}$ [sum$\angle$s Δ OR ext $\angle$ cyclic quad/*som$\angle$eΔ OR buite$\angle$koordevh*] $\therefore \Delta KGH \,\|\|\| \,\Delta KEF$ [$\angle\angle\angle$]	✓ S ✓S ✓R ✓ naming third angle OR $\angle\angle\angle$ (4)
10.2.1(b)	$\frac{EF}{GH} = \frac{KE}{KG}$ [$\|\|\|$ Δs] $\therefore \frac{EF}{GH} = \frac{KE}{EF}$ [KG = EF] $\therefore EF^2 = KE.GH$	✓ S ✓ S (2)
10.2.1(c)	$\frac{KG}{KF} = \frac{EM}{EF}$ [prop theorem/*eweredighst*; MG $\|\|$ EK] but EF = KG [given/*gegee*] $\frac{KG}{KF} = \frac{EM}{KG}$ $KG^2 = EM.KF$	✓ S ✓ R ✓ S (3)
10.2.2	KE.GH = EM.KF $EM = \frac{20 \times 4}{16}$ = 5 units	✓ KE.GH = EM.KF ✓ substitution ✓ answer (3) **[19]**

TOTAL/*TOTAAL*: 150

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2017

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 27 pages.

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QUESTION 1

An IT company writes programs for apps. The time taken (in hours) to write the programs and the cost (in thousands of rands) are shown in the table below.

TIME TAKEN (IN HOURS)	5	7	5	8	10	13	15	20	18	25	23
COST (IN THOUSANDS OF RANDS)	10	10	15	12	20	25	28	32	28	40	30

1.1 Determine the equation of the least squares regression line. (3)

1.2 Use the equation of the least squares regression line to predict the cost, in rands, of an app that will take 16 hours to write. (2)

1.3 Calculate the correlation coefficient of the data. (1)

1.4 For each app that the company writes, there is a cost that is independent of the number of hours spent on writing the app. Calculate this cost (in rands). (2)
[8]

QUESTION 2

The commission earned, in thousands of rands, by the salesmen of a particular company in a certain month is shown in the table below.

COMMISSION EARNED (IN THOUSANDS OF RANDS)	FREQUENCY
$20 < x \leq 40$	7
$40 < x \leq 60$	6
$60 < x \leq 80$	8
$80 < x \leq 100$	10
$100 < x \leq 120$	4

2.1 Write down the modal class of the data. (1)

2.2 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.3 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

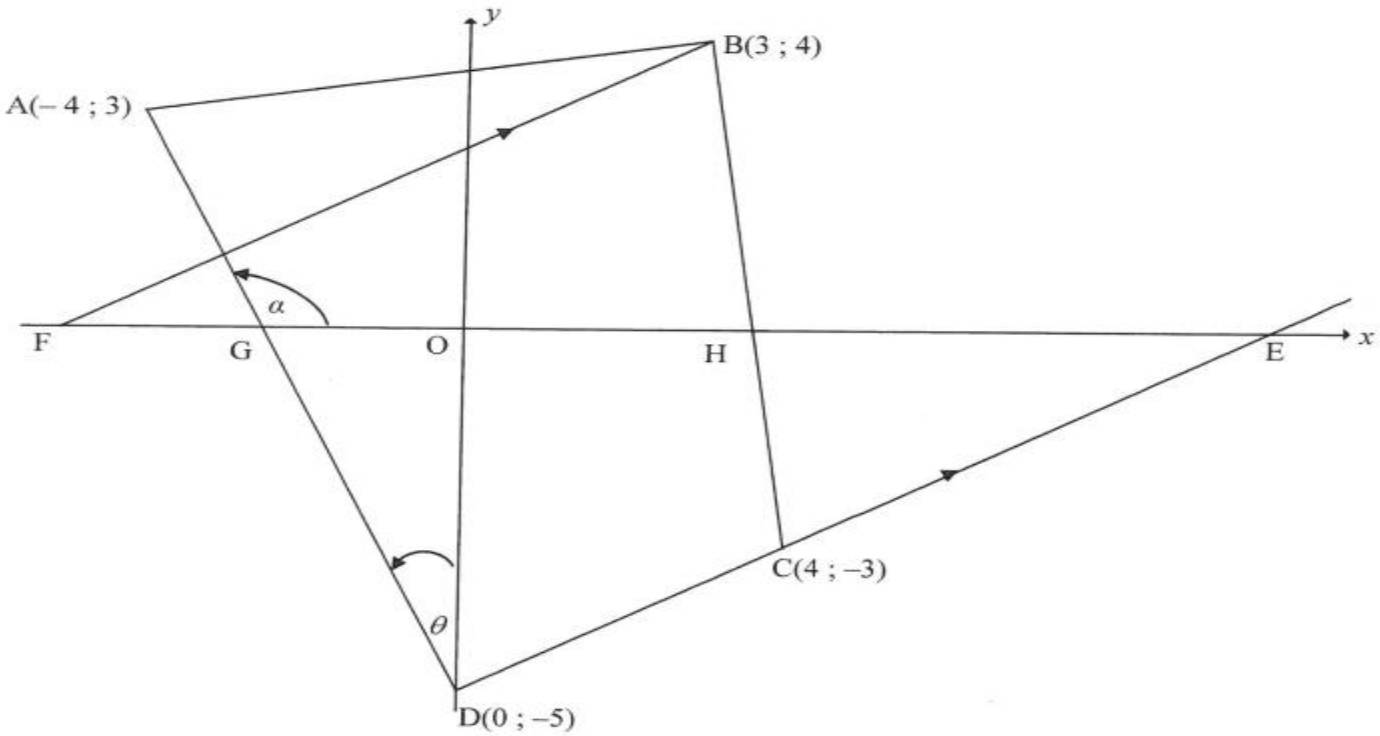
2.4 A salesman receives a bonus if his commission is more than R90 000 for the month. Calculate how many of the salesmen received bonuses for this month. (2)

2.5 Determine the approximate mean commission earned by the salesmen in this month correct to the nearest thousand rand. (3)
[12]

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QUESTION 3

In the diagram, ABCD is a quadrilateral having vertices A(– 4 ; 3), B(3 ; 4), C(4 ; –3) and D(0 ; –5). DC produced cuts the x-axis at E, BC cuts the x-axis at H and AD cuts the x-axis at G. F is a point on the x-axis such that BF ∥ DE. $A\hat{G}O = \alpha$ and $A\hat{D}O = \theta$.



3.1 Calculate the gradient of DC. (2)

3.2 Prove that AD ⊥ DC. (3)

3.3 Show by calculation that ΔABC is an isosceles. (4)

3.4 Determine the equation of BF in the form $y = mx + c$. (3)

3.5 Calculate the size of θ. (3)

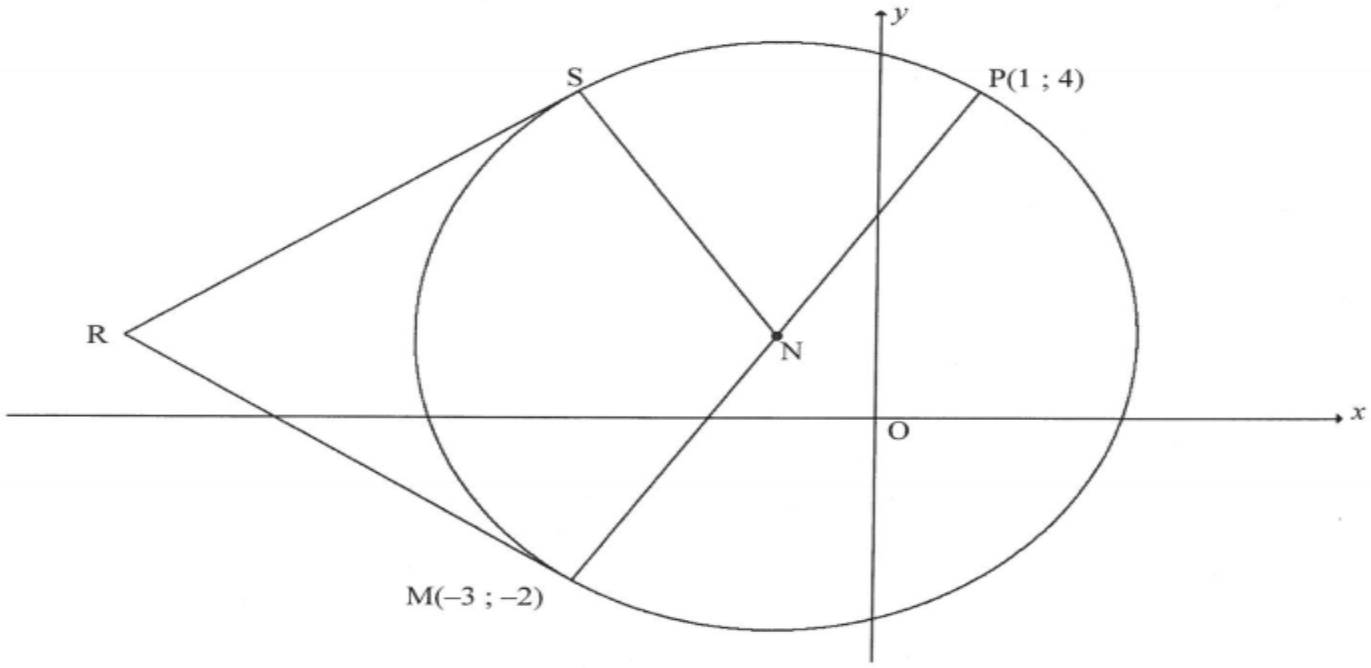
3.6 Determine the equation of the circle, with the centre as the origin and passing through point C, in the form $x^2 + y^2 = r^2$. (2)

[17]

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QUESTION 4

In the diagram, N is the centre of the circle. M(–3 ; –2) and P(1 ; 4) are points on the circle. MNP is the diameter of the circle. Tangents drawn to circle N from point R, outside the circle, meet the circle at S and M respectively.



4.1 Determine the coordinates of N. (3)

4.2 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (4)

4.3 Determine the equation of the tangent RM in the form $y=mx+c$. (5)

4.4 If it is given that the line joining S to M is perpendicular to the x-axis, determine the coordinates of S. (2)

4.5 Determine the coordinates of R, the common external point from which both tangents to the circle are drawn. (4)

4.6 Calculate the area of RSNM. (4)
[22]

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QUESTION 5

5.1 Given: $\sin A = 2p$ and $\cos A = p$

5.1.1 Determine the value of $\tan A$. (2)

5.1.2 **Without using a calculator,** determine the value of p, if $A \in [180° ; 270°]$. (3)

5.2 Determine the general solution of $2\sin^2 x - 5\sin x + 2 = 0$ (6)

5.3 5.3.1 Expand $\sin(x + 300°)$ using an appropriate compound angle formula. (1)

5.3.2 **Without using a calculator,** determine the value of $\sin(x + 300°) - \cos(x - 150°)$. (5)

5.4 Prove the identity: $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x.$ (5)

5.5 Consider: $\sin x + \cos x = \sqrt{1 + k}$

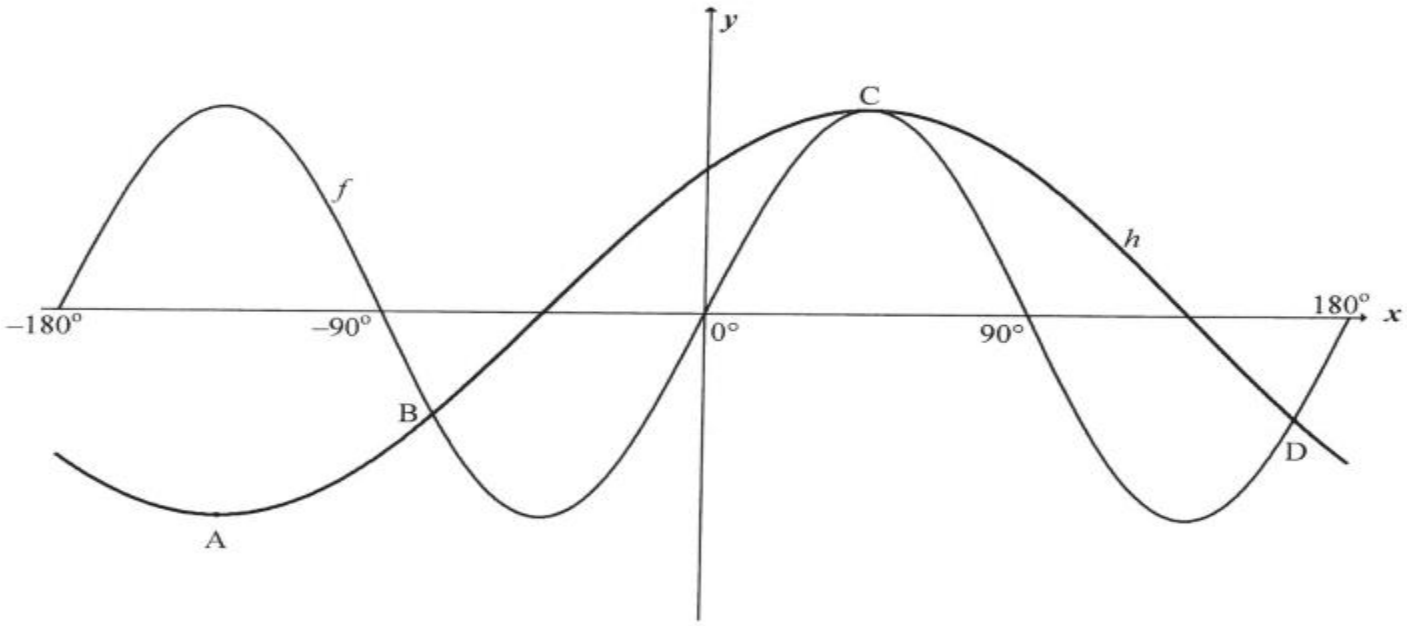
5.5.1 Determine k as a single trigonometric ratio. (3)

5.5.2 Hence, determine the maximum value of $\sin x + \cos x$. (2)
[27]

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QUESTION 6

In the diagram are the graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45°)$ for the interval $x \in [-180° ; 180°]$. A$(-135° ; -1)$ is a minimum point on graph h and C$(45° ; 1)$ is a maximum point on both graphs. The two graphs intersect at B, C and D$\left(165°; -\frac{1}{2}\right)$.



6.1 Write down the period of f. (1)

6.2 Determine the x-coordinate of B. (1)

6.3 Use the graphs to solve $2\sin x.\cos x \le \frac{1}{\sqrt{2}}(\cos x + \sin x)$ for the interval $x \in [-180° ; 180°]$. Show ALL working. (4)
[6]

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QUESTION 7

A rectangular box with lid ABCD is given in FIGURE (i) below. The lid is opened through 60° to position HKCD, as shown in the FIGURE (ii) below. EF = 12 cm, FG = 6 cm and BG = 8 cm.

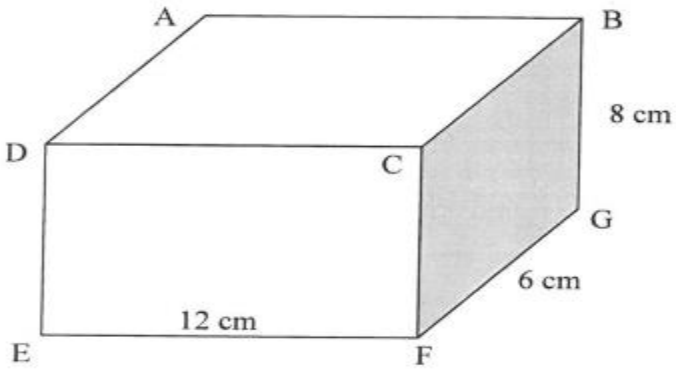


FIGURE (I)

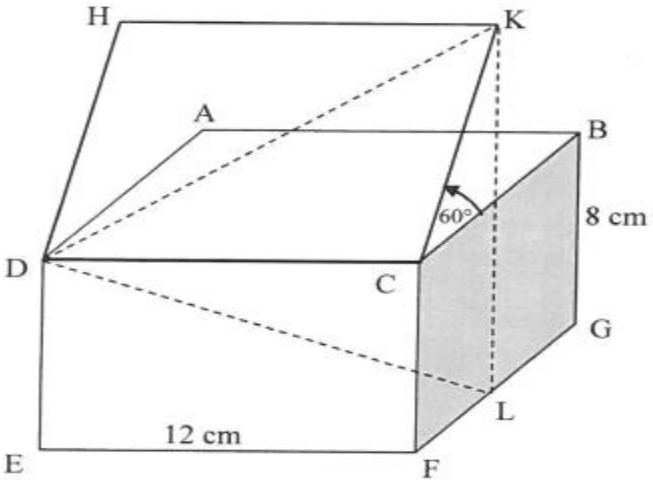


FIGURE (II)

7.1 Write down the length of KC. (1)

7.2 Determine KL, the perpendicular height of K, above the base of the box. (3)

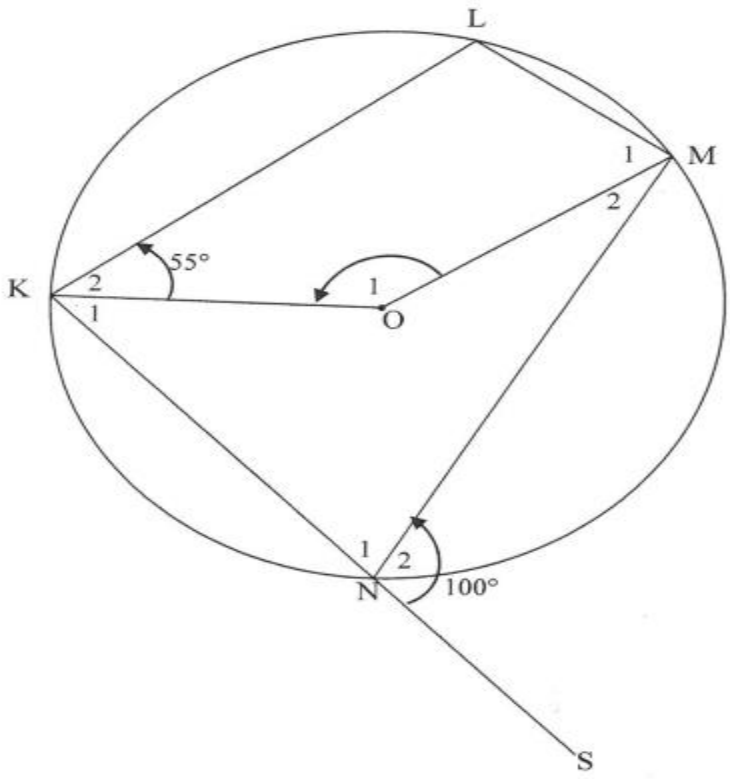
7.3 Hence, determine the value of $\frac{\sin K\hat{D}L}{\sin D\hat{L}K}$. (4)

[8]

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QUESTION 8

In the diagram, O is the centre of circle KLMN and KO and OM are joined. Chord KN is produced to S. $\hat{K}_2 = 55°$ and $\hat{N}_2 = 100°$.



Determine, with reasons, the size of the following:

8.1 $\hat{L}$ (2)

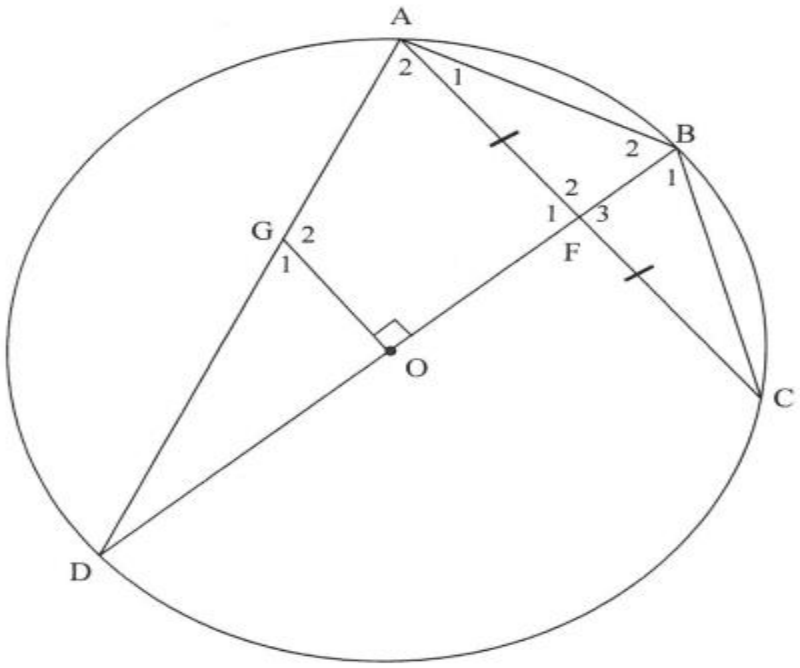
8.2 $\hat{O}_1$ (3)

8.3 $\hat{M}_1$ (2)
[7]

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QUESTION 9

In the diagram, O is the centre of circle ABCD and BOD is a diameter. F, the midpoint of chord AC, lies on BOD. G is a point on AD such that GO ⊥ DB.



9.1 Give a reason why:

9.1.1 $D\hat{A}B = 90°$ (1)

9.1.2 AGOB is a cyclic quadrilateral (1)

9.2 Prove that:

9.2.1 AC ∥ GO (3)

9.2.2 $\hat{G}_1 = \hat{B}_1$ (4)

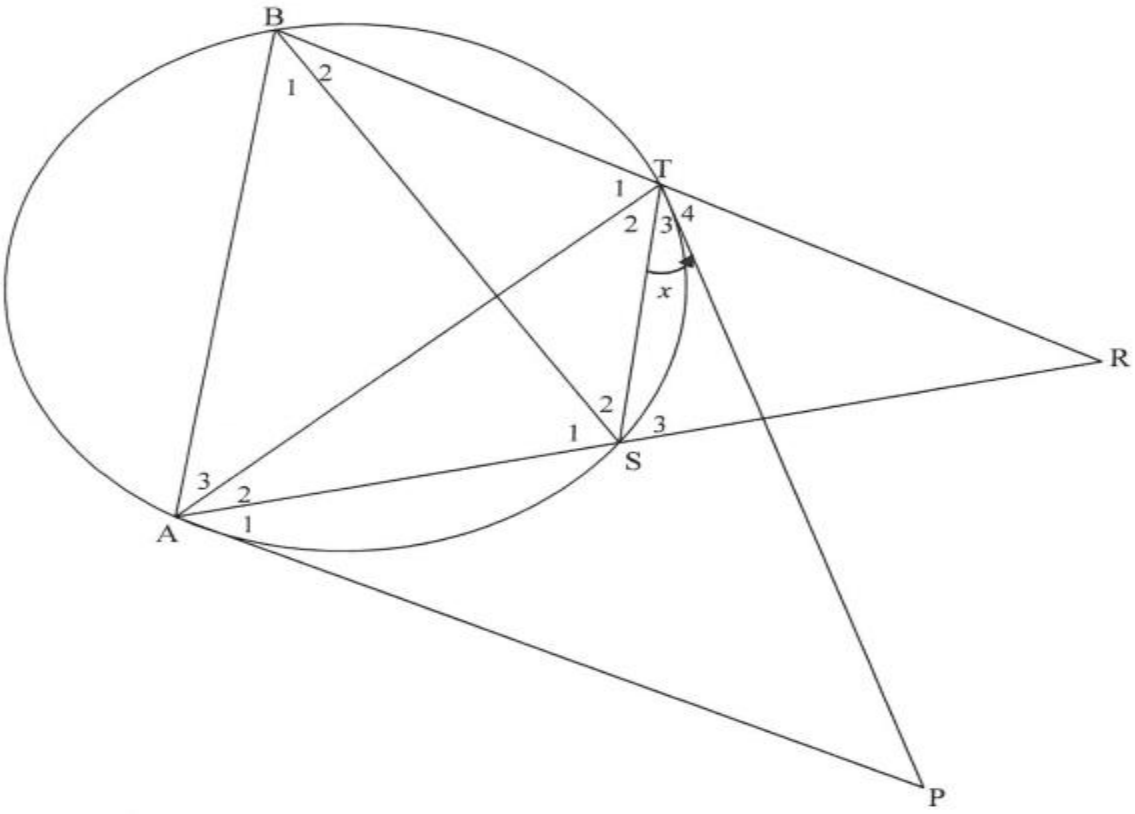
9.3 If it is given that $FB = \frac{2}{5}r$, where r is the radius of the circle, determine, with reasons, the ratio of $\frac{DG}{DA}$. (3)

[12]

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QUESTION 10

In the diagram, PA and PT are tangents to a circle at A and T respectively. B and S are points on the circle such that BT produced and AS produced meet at R and BR = AR. BS, AT and TS are drawn. $\hat{T}_3 = x$.



10.1 Give a reason why $\hat{T}_3 = \hat{A}_2 = x$. (1)

10.2 Prove that:

10.2.1 AB || ST (5)

10.2.2 $\hat{T}_4 = \hat{A}_1$ (5)

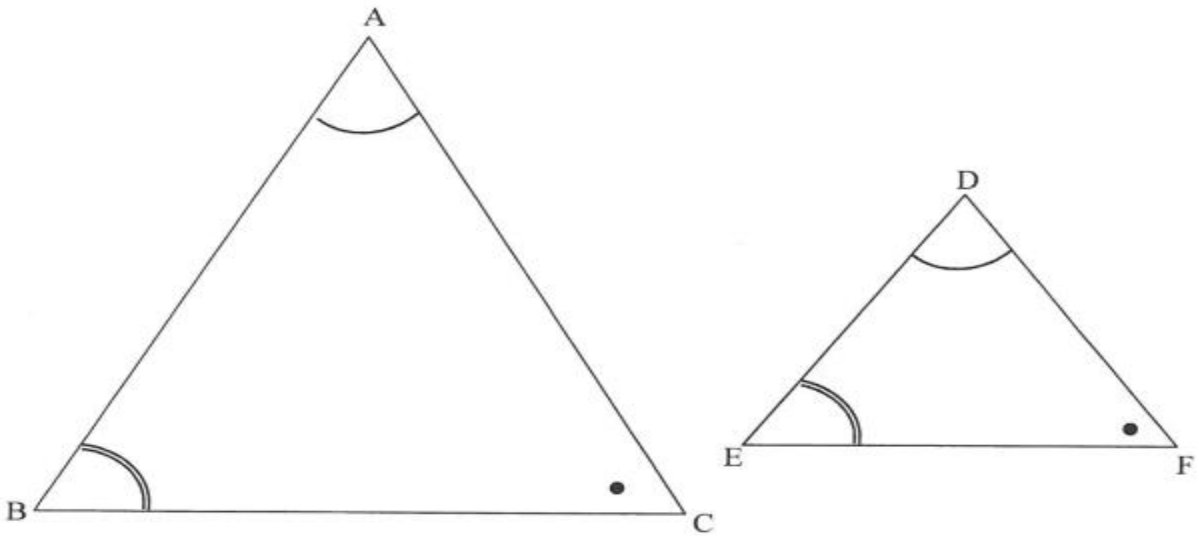
10.2.3 RTAP is a cyclic quadrilateral (2)

[13]

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QUESTION 11

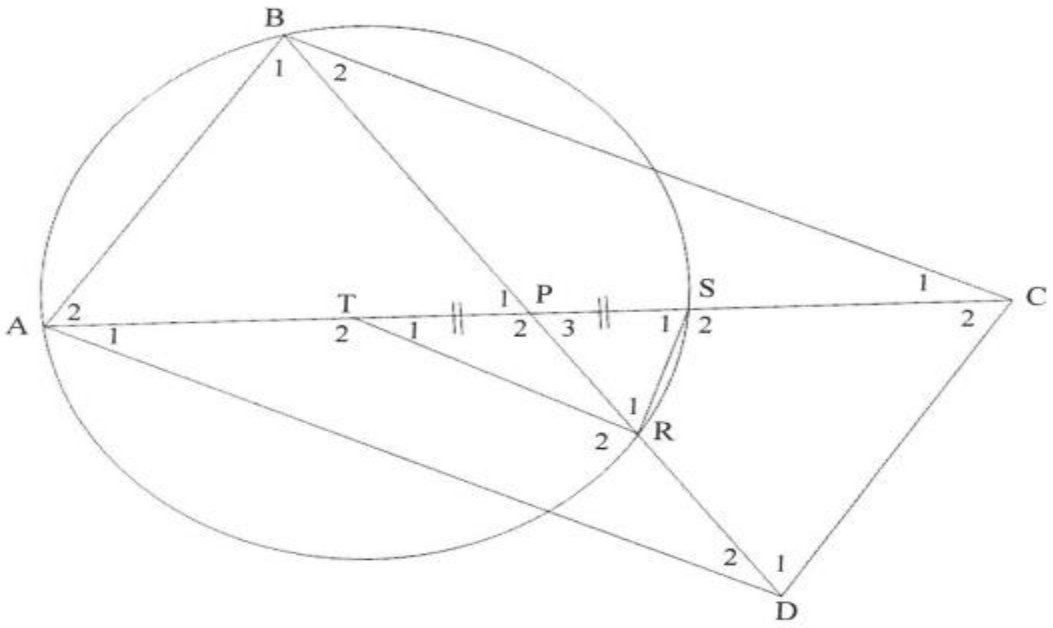
11.1 In the diagram, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem which states that if two triangles, ΔABC and ΔDEF, are equiangular, then $\frac{DE}{AB} = \frac{DF}{AC}$. (6)

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11.2 In the diagram, ABCD is a parallelogram with A and B on the circle. The diagonals BD and AC intersect in P. PC and PD intersect the circle at S and R respectively. T is a point on AP such that TP = PS. TR is drawn.



11.2.1 Prove that:

(a) AT = SC (2)

(b) ΔPSR ||| ΔPBA (5)

11.2.2 If it is further given that $\frac{PR}{PA} = \frac{TR}{AD}$, prove that:

(a) ΔRPT ||| ΔAPD (3)

(b) ATRD is a cyclic quadrilateral (2)

[18]

TOTAL: 150

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SENIOR CERTIFICATE EXAMNATIONS
SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS: 150
PUNTE: 150

These marking guidelines consist of 22 pages.
Hierdie nasienriglyne bestaan uit 22 bladsye..

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***QUESTION/VRAAG/VRAAG* 1**

TIME TAKEN (IN HOURS)	5	7	5	8	10	13	15	20	18	25	23
COST (IN THOUSANDS OF RANDS)	10	10	15	12	20	25	28	32	28	40	30

1.1	$a = 4{,}806\ldots = 4{,}81$ $b = 1{,}323\ldots = 1{,}32$ $y = 4{,}81 + 1{,}32x$	✓ $a = 4{,}81$ ✓ $b = 1{,}32$ ✓ equation (3)
1.2	Cost = 25,974… = 25,97 thousand rand (calculator) = R25 970 **OR/*OF*** $y = 4{,}81 + 1{,}32(16)$ $y = 25{,}93$ Cost = R25 930	✓ 25,97 ✓ answer (in Rands) (2) ✓ substitution ✓ answer (in Rands) (2)
1.3	$r = 0{,}949\ldots = 0{,}95$	✓ answer (1)
1.4	$x = 0$ $y = 4{,}81$ **OR** (4,80647) $\therefore$ R4 810 **OR** R4806,47	✓ $x = 0$ ✓ answer (2) **[8]**

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QUESTION/VRAAG 2

2.1 modal class: $80 < x \leq 100$ — ✓ correct class (1)

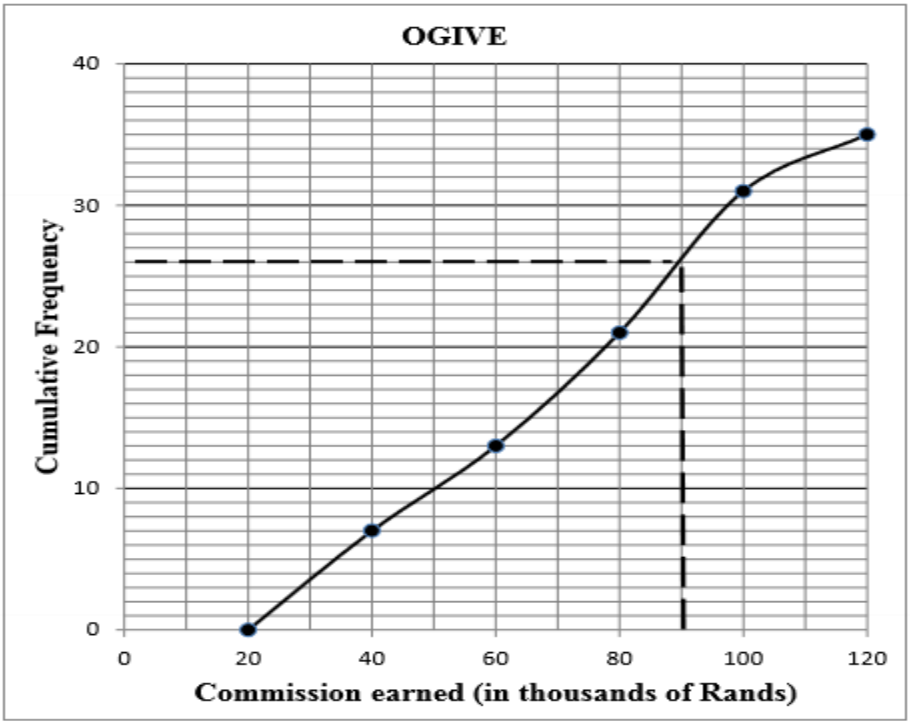
2.2

Commission earned (in thousands of Rands)	Frequency	Cumulative Frequency
$20 < x \leq 40$	7	7
$40 < x \leq 60$	6	13
$60 < x \leq 80$	8	21
$80 < x \leq 100$	10	31
$100 < x \leq 120$	4	35

✓ 13 ; 21

✓ 31 ; 35 (2)

2.3



✓ grounded/*geanker*
✓ upper limits/ *boonste limiet*
✓ cum frequency / Kum frekwensie
✓ shape/*vorm*

(4)

2.4 No. of salesmen awarded bonuses: $35 - 26$
$= 9$ salesmen

✓ accept $(25 - 27)$
✓ accept $(8 - 10)$
(2)

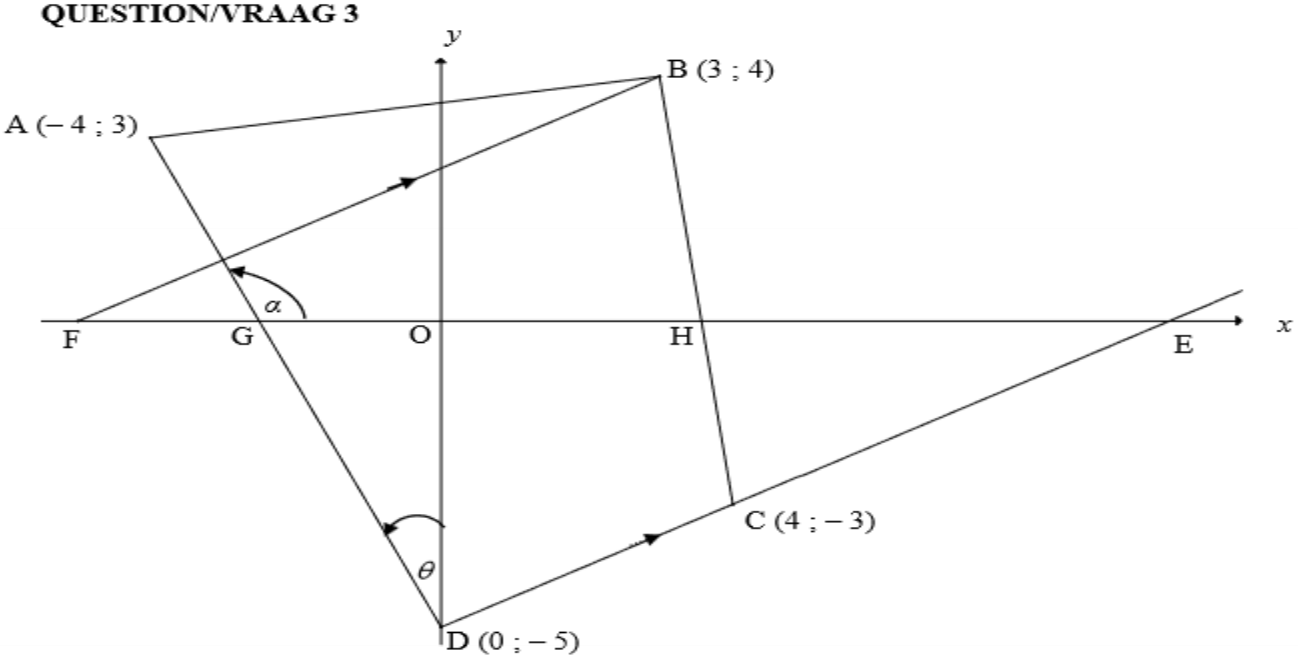
2.5

$$\text{Estimated mean} = \frac{(30 \times 7) + (50 \times 6) + (70 \times 8) + (90 \times 10) + (110 \times 4)}{35}$$
$$= \frac{2410}{35}$$
$$= 68{,}86 \text{ thousand rand or R68 857,14}$$
$$= \text{R69 000 or 69 thousand rand}$$

✓ top line using midpts & freq
✓ 2410
✓ answer (nearest)
(3)
[12]

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

QUESTION/VRAAG 3

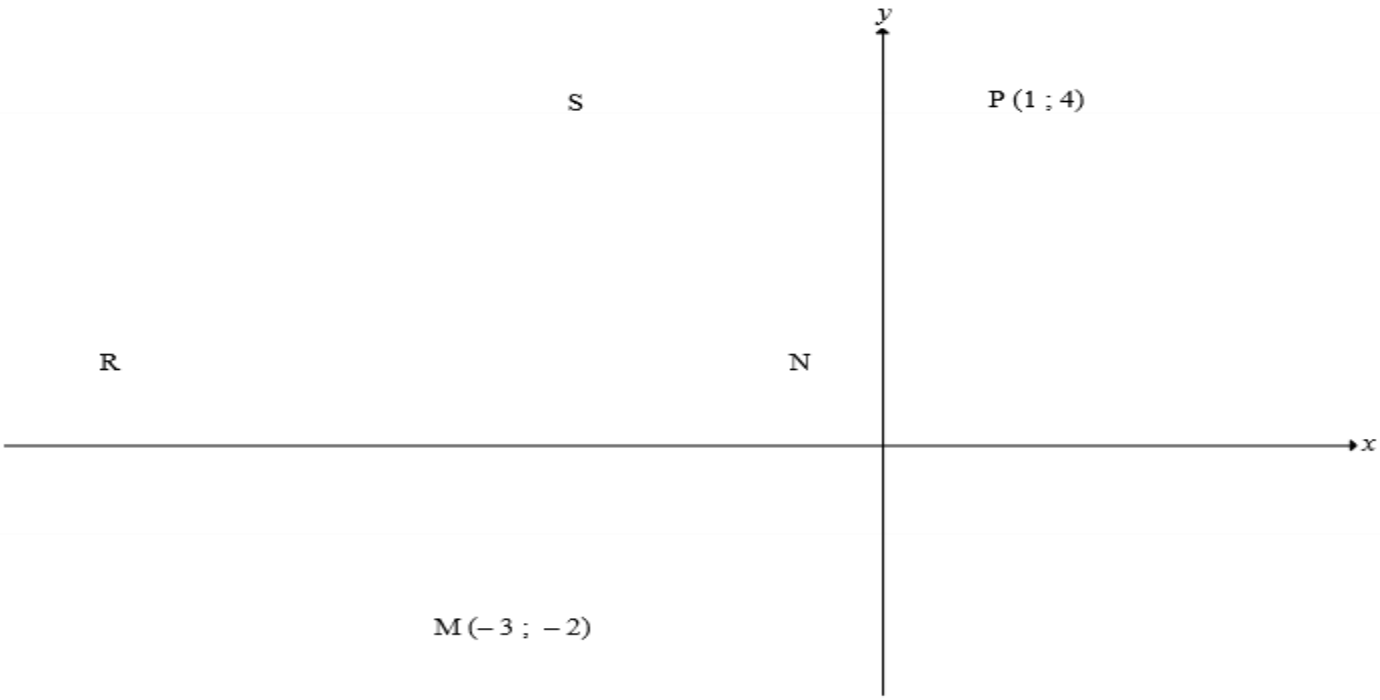


3.1	$m_{CD} = \frac{-3-(-5)}{4-0}$ $= \frac{-3+5}{4-0}$ $= \frac{1}{2}$	✓ substitution of C & D ✓answer (2)
3.2	$m_{AD} = \frac{-5-3}{0-(-4)}$ $= -2$ $m_{CD} \times m_{AD} = \frac{1}{2} \times -2$ $= -1$ $\therefore$ AD ⊥ DC	✓ substitution of A & D ✓ $m_{AD} = -2$ ✓ product = –1 (3)
3.3	$AB = \sqrt{(3+4)^2 + (4-3)^2} = \sqrt{50} = 5\sqrt{2}$ $BC = \sqrt{(4-3)^2 + (-3-4)^2} = 5\sqrt{2}$ AB = BC $\therefore \Delta ABC$ is an iscosceles triangle/*'n gelykbenige driehoek*	✓ correct substitution ✓ length of AB ✓ correct substitution ✓ length of BC (4)

3 .4	$m_{CD} = m_{BF} = \frac{1}{2}$ [BF || DC] $4 = \frac{1}{2}(3) + c$ $c = \frac{5}{2}$ $y = \frac{1}{2}x + \frac{5}{2}$ **OR/*OF*** $y - 4 = \frac{1}{2}(x - 3)$ $y - 4 = \frac{1}{2}x - 1\frac{1}{2}$ $y = \frac{1}{2}x + 2\frac{1}{2}$	✓ $m_{BF} = \frac{1}{2}$ ✓ substitution of B(3 ; 4) ✓equation (3)
3.5	$\tan\alpha = -2$ $\therefore \alpha = 116.57°$ $\alpha = 90° + \theta$ [ext ∠Δ] $\therefore \theta = 26{,}57°$ **OR/*OF*** $\tan\alpha = -2$ OR $m_{AD} = -2$ $\therefore \tan\theta = \frac{1}{2}$ $\therefore \theta = 26{,}57°$ **OR/*OF*** Inclination of DE is β: $\tan\beta = \frac{1}{2}$ $\therefore \beta = 26{,}57°$ $\therefore$ OD̂E = 63,43° $\therefore \theta = 90° - 63{,}43°$ $= 26{,}57°$	✓ $\tan\alpha = -2$ ✓ $\alpha = 116.57°$ ✓ $\theta = 26{,}57°$ (3) ✓ $\tan\alpha = -2$ ✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26{,}57°$ (3) ✓ $\beta = 26{,}57°$ ✓ OD̂E = 63,43° ✓ $\theta = 26{,}57°$ (3)
3.6	$x^2 + y^2 = r^2$ $(4)^2 + (-3)^2 = 25$ $x^2 + y^2 = 25$	✓ $r^2 = 25$ ✓ equation (2) **[17]**

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

QUESTION/VRAAG 4



4.1	$N\left(\frac{1+(-3)}{2};\frac{4+(-2)}{2}\right)$ $N(-1;1)$ is the centre of the circle	✓ substitution M & P ✓ x-value of N ✓ y-value of N (3)
4.2	$r=\sqrt{(1-(-1))^2+(4-1)^2}$ $r=\sqrt{13}$ = radius $(x+1)^2+(y-1)^2=13$ **OR/*OR*** $r=\sqrt{(-3-(-1))^2+(-2-1)^2}$ $r=\sqrt{13}$ = radius $(x+1)^2+(y-1)^2=13$	✓ substitution N & P ✓ $r=\sqrt{13}$ ✓ LHS of eq ✓ RHS of eq (4) ✓ substitution N & M ✓ $r=\sqrt{13}$ ✓ LHS of eq ✓ RHS of eq (4)

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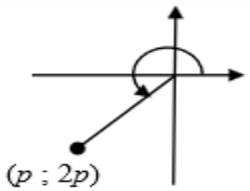
4.3	$m_{NM} \times m_{MR} = -1$ [radius ⊥ tangent/*raakklyn*] $m_{NM} = \frac{1-(-2)}{-1-(-3)}$ *OR* $m_{PM} = \frac{4-(-2)}{1-(-3)}$ $= \frac{3}{2}$ $= \frac{3}{2}$ $m_{MR} = -\frac{2}{3}$ $y - y_1 = -\frac{2}{3}(x - x_1)$ **OR/*OF*** $y = -\frac{2}{3}x + c$ $y + 2 = -\frac{2}{3}(x + 3)$ **OR/*OF*** $-2 = -\frac{2}{3}(-3) + c$ $y = -\frac{2}{3}x - 4$	✓ correct substitution ✓ m_{NM} ✓ m_{MR} ✓ substitution of m_{MR} &(–3 ;–2) ✓ equation (5)
4.4	Symmetry of a kite: S(–3 ; 4) **OR/*OF*** $P\hat{S}M = 90°$ [∠ in semi circle] PS ⊥ SM ∴ S(–3 ; 4) **OR/*OF*** $(NS)^2 = (\text{radius})^2$ $(-3+1)^2 + (y-1)^2 = 13$ $(y-1)^2 = 9$ $y - 1 = \pm 3$ $y = 4$ *OR* $y \neq -2$ ∴ S(–3 ; 4)	✓ x-value of S ✓ y-value of S (2) ✓ x-value of S ✓ y-value of S (2) ✓ x-value of S ✓ y-value of S (2)
4.5	$(SR)^2 = (RM)^2$ ...Tangents from common pt/*rklyne v dies punt* $(x+3)^2 + (y-4)^2 = (x+3)^2 + (y+2)^2$ $y^2 - 8y + 16 = y^2 + 4y + 4$ $-12y = -12$ $y = 1$ $\frac{2}{3}x = -4 - 1$ or $1 = -\frac{2}{3}x - 4$ $x = -\frac{15}{2}$ $x = -7\frac{1}{2}$ ∴ $R\left(-7\frac{1}{2}; 1\right)$ **OR/*OF***	✓ equating lengths ✓ simplification ✓ y-value of R ✓ x-value of R (4)

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	R$(x;1)$ [RN is a horizontal line] $\therefore 1=-\frac{2}{3}x-4$ $5=-\frac{2}{3}x$ $x=-\frac{15}{2}$ $\therefore \text{R}\left(-\frac{15}{2};1\right)$ **OR/*OF*** $m_{\text{NS}}=\frac{1-4}{-1+3}=-\frac{3}{2}$ $\therefore\ m_{\text{RS}}=\frac{2}{3}$ $y-4=\frac{2}{3}(x+3)$ $y=\frac{2}{3}x+6$ $-\frac{2}{3}x-4=\frac{2}{3}x+6$ $x=-7\frac{1}{2}$ $y=\frac{2}{3}(-\frac{15}{2})+6=1$ $\therefore \text{R}\left(-\frac{15}{2};1\right)$	✓ $y_{\text{R}}=1$ ✓ horizontal line OR R lies on $y=1$ ✓ equating ✓ x-value of R $(x<-4,6)$ (4) ✓ $y=\frac{2}{3}x+6$ ✓ equating ✓ x-value of R $(x<-4,6)$ ✓ y-value of R (4)
4.6	RS $=\sqrt{(-3+7,5)^2+(4-1)^2}$ OR/OF RM $=\sqrt{(-3+7,5)^2+(-2-1)^2}$ RS $=\frac{3\sqrt{13}}{2}=5,41$ area of RSNM = 2area of ΔRSN $=2\left(\frac{1}{2}\right)(\sqrt{13})\left(\frac{3\sqrt{13}}{2}\right)$ $=\frac{39}{2}$ **OR/*OF*** 19,5 square units **OR/*OF***	✓ RS **OR** RM ✓ method ✓ $\sqrt{13}$ and $\left(\frac{3\sqrt{13}}{2}\right)$ ✓ answer (4) ✓ method ✓ MS = 6 ✓ RN = 6,5 ✓ answer

	area RSNM $= \frac{1}{2}$(MS × RN) (area of a kite/*opp v vlieër*) $= \frac{1}{2}(6)(6{,}5)$ $= \frac{39}{2}$ OR 19,5 square units **OR/*OF*** RS $= \sqrt{(-3+7{,}5)^2+(4-1)^2}$ OR/OF RM $= \sqrt{(-3+7{,}5)^2+(-2-1)^2}$ RS $= \frac{3\sqrt{13}}{2}$ *or* 5,41 area of ΔRSN $= \left(\frac{1}{2}\right)(\sqrt{13})\left(\frac{3\sqrt{13}}{2}\right)$ $= \frac{39}{4}$ **OR/*OF*** 9,75 square units area of RSNM = 2area of ΔRSN $= \frac{39}{2}$ **OR/*OF*** 19,5 square units **OR/*OF*** SM = 6 area of RSNM = Area of ΔSMN + Area of ΔRSM $= \frac{1}{2}(6)(1) + \frac{1}{2}(6)(5\frac{1}{2})$ $= 3 + 16\frac{1}{2}$ $= 19\frac{1}{2}$	(4) ✓ RS **OR** RM ✓ $\left(\frac{1}{2}\right)\sqrt{13}\left(\frac{3\sqrt{13}}{2}\right)$ ✓ method ✓ answer (4) ✓ method ✓ MS = 6 ✓ $h = 1$ & $5\frac{1}{2}$ ✓ answer (4)
		[22]

QUESTION/VRAAG 5

5.1.1	$\tan A = \frac{\sin A}{\cos A}$ $= \frac{2p}{p}$ $= 2$ **OR/*OF*** $\tan A = \frac{2p}{p}$ $= 2$  	✓identity ✓value of tan A (2) ✓ $\frac{y}{x}$ ✓value of tan A (2)
5.1.2	$\sin^2 A + \cos^2 A = 1$ $(2p)^2 + p^2 = 1$ $4p^2 + p^2 = 1$ $5p^2 = 1$ $p^2 = \frac{1}{5}$ $\therefore p = -\frac{1}{\sqrt{5}}$	✓ $(2p)^2 + p^2 = 1$ ✓ simplification of LHS ✓answer (3)
5.2	$2\sin^2 x - 5\sin x + 2 = 0$ $(2\sin x - 1)(\sin x - 2) = 0$ $\sin x = \frac{1}{2}$ or $\sin x = 2$(no solution) ref $\angle = 30°$ $\therefore x = 30° + k.360°$ or $x = 150° + k.360°$; $k \in Z$	✓ factors or formula ✓ both equations ✓ no solution/*geen opl* ✓ $30° + k.360°$ ✓ $150° + k.360°$; ✓ $k \in Z$ (6)
5.3.1	$\sin(x + 300°) = \sin x \cos 300° + \cos x \sin 300°$	✓ expansion/*uitbreiding* (1)
5.3.2	$\sin(x + 300°) - \cos(x - 150°)$ $= \sin x \cos 300° + \cos x \sin 300° - (\cos x \cos 150° + \sin x \sin 150°)$ $= \sin x \cos 60° - \cos x \sin 60° - (-\cos x \cos 30° + \sin x \sin 30°)$ $= \sin x \cos 60° - \cos x \sin 60° + \cos x \cos 30° - \sin x \sin 30°$ $= \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ $= 0$ **OR/*OF***	✓2nd expansion/ *2de uitbreiding* ✓✓ reduction/*reduksie* ✓ special angle values/ *spesiale hoekwaardes* ✓ answer (5)

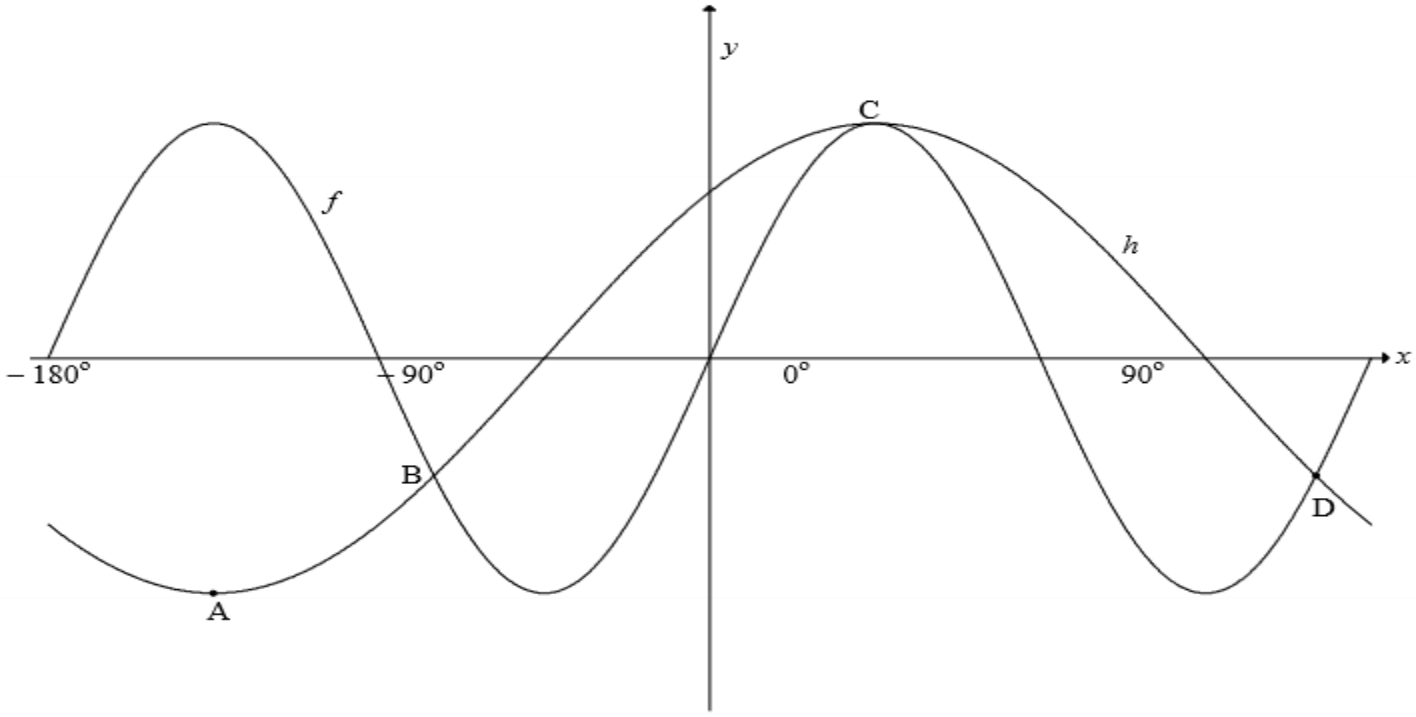
KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

	$\sin(x+300°)-\cos(x-150°)$ $=\sin x\cos 300°+\cos x\sin 300°-(\cos x\cos 150°+\sin x\sin 150°)$ $=\sin x\cos 60°-\cos x\sin 60°-(-\cos x\cos 30°+\sin x\sin 30°)$ $=\sin x\cos 60°-\cos x\sin 60°+\cos x\cos 30°-\sin x\sin 30°$ $=\sin x\sin 30°-\cos x\sin 60°+\cos x\sin 60°-\sin x\sin 30°$ $=0$	✓2nd expansion/ *2de uitbreiding* ✓✓ reduction/*reduksie* ✓ co-ratios / *ko-verh* ✓ answer (5)
5.4	Consider: $\frac{\tan x+1}{\sin x\tan x+\cos x}=\sin x+\cos x$ $\text{LHS}=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\left(\sin x.\frac{\sin x}{\cos x}+\cos x\right)}=\frac{\left(\frac{\sin x+\cos x}{\cos x}\right)}{\left(\frac{\sin^2 x+\cos^2 x}{\cos x}\right)}$ $=\frac{\frac{\sin x+\cos x}{\cos x}}{\frac{1}{\cos x}}$ $=\frac{\sin x+\cos x}{\cos x}\times\frac{\cos x}{1}$ $=\sin x+\cos x$ $=\text{RHS}$ **OR/*OF*** $\text{LHS}=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\left(\sin x.\frac{\sin x}{\cos x}+\cos x\right)}=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\left(\frac{\sin^2 x+\cos^2 x}{\cos x}\right)}$ $=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\frac{1}{\cos x}}$ $=\left(\frac{\sin x}{\cos x}+1\right)\times\frac{\cos x}{1}$ $=\sin x+\cos x$ $=\text{RHS}$	✓ identity of tan x ✓ $\frac{\sin x+\cos x}{\cos x}$ ✓ $\frac{\sin^2 x+\cos^2 x}{\cos x}$ ✓ $\sin^2 x+\cos^2 x=1$ ✓ simplify (5) ✓ identity of tan x ✓ $\frac{\sin^2 x+\cos^2 x}{\cos x}$ ✓ $\sin^2 x+\cos^2 x=1$ ✓ simplify ✓ multiplication (5)
5.5.1	$\left(\sqrt{1+k}\right)^2=(\sin x+\cos x)^2$ $1+k=\sin^2 x+2\sin x\cos x+\cos^2 x$ $1+k=1+\sin 2x$ $k=\sin 2x$	✓ square both sides ✓ $\sin^2 x+\cos^2 x=1$ ✓ $\sin 2x$ (3)

5.5.2	From 5.5.1 $\sin x + \cos x = \sqrt{1+\sin 2x}$ $\therefore$ max value : $\sin x + \cos x = \sqrt{1+1}$ $= \sqrt{2}$ **OR/*OF*** Maximum value of $1 + \sin 2x = 1 + 1$ $= 2$ $\therefore$ maximum value of $\sin x + \cos x = \sqrt{2}$ **OR/*OF*** $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$ $= 1 + \sin 2x$ $\therefore$ max value $(\sin x + \cos x)^2 = 1 + 1 = 2$ $\therefore$ max value $\sin x + \cos x = \sqrt{2}$	✓ max of $\sin 2x = 1$ ✓ answer (2) ✓ max of $\sin 2x = 1$ ✓ answer (2) ✓ max of $\sin 2x = 1$ ✓ answer (2)
		[27]

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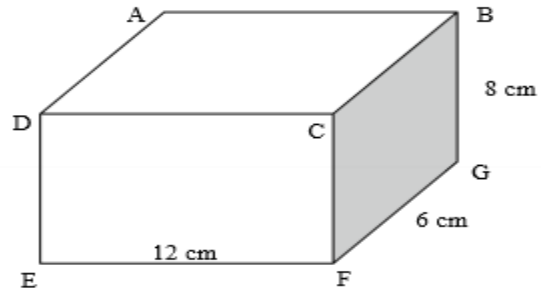
QUESTION/VRAAG 6



6.1	Period = 180°	✓ answer (1)
6.2	$-75°$	✓ answer (1)
6.3	$\sin 2x \le \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$ $\sin 2x \le \cos 45°.\cos x + \sin 45°.\sin x$ $\sin 2x \le \cos(x - 45°)$ $x \in [-75° ; 165°]$	✓ $\cos 45°.\cos x + \sin 45°.\sin x$ ✓ $\cos(x - 45°)$ ✓ ✓ answer (4)
		[6]

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QUESTION/VRAAG 7



Figure/*Figuur* (i)

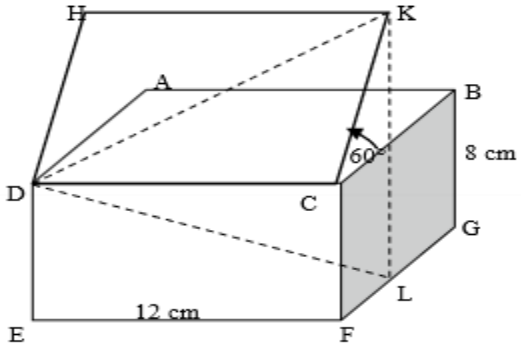
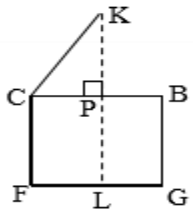
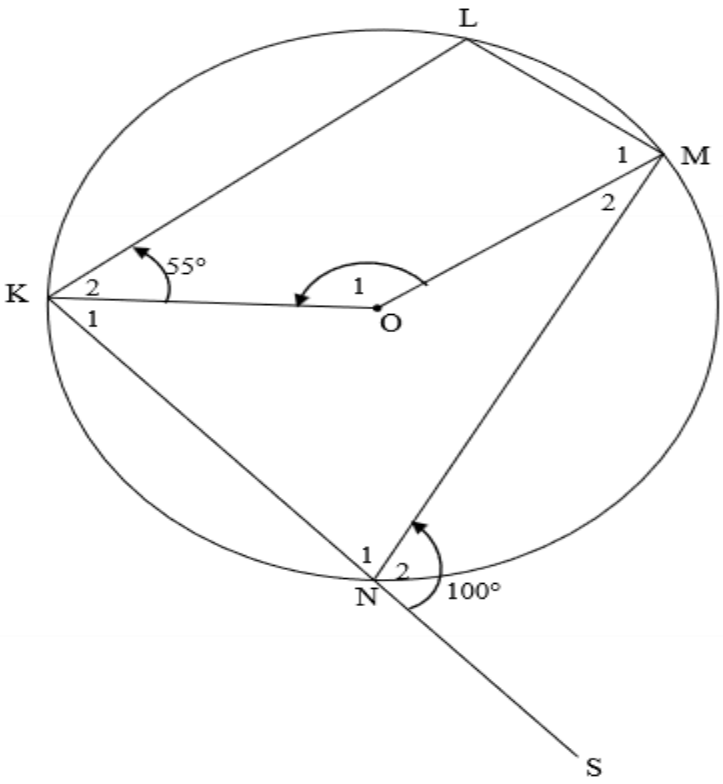


Figure / *Figuur* (ii)

7.1	KC = 6 cm	✓ answer (1)
7.2	Let P be the point of intersection of KL and CB $\frac{KP}{KC} = \sin 60°$ $KP = 6\sin 60°$ $KP = 3\sqrt{3}$ or 5,20 $\therefore KL = 8 + 3\sqrt{3}$ or 13,20 cm 	✓ trig ratio ✓ length of KP ✓ answer (3)
7.3	$DK^2 = 6^2 + 12^2$ $DK = \sqrt{180}$ or $6\sqrt{5}$ or 13,42 cm $\frac{\sin K\hat{D}L}{KL} = \frac{\sin D\hat{L}K}{DK}$ $\frac{\sin K\hat{D}L}{\sin D\hat{L}K} = \frac{KL}{DK}$ $= \frac{8+3\sqrt{3}}{6\sqrt{5}}$ or $\frac{13,20}{13,42}$ or 0,98	✓ $DK = 6\sqrt{5}$ ✓ use of sine rule ✓ $\frac{\sin K\hat{D}L}{\sin D\hat{L}K} = \frac{KL}{DK}$ ✓ answer (4) **[8]**

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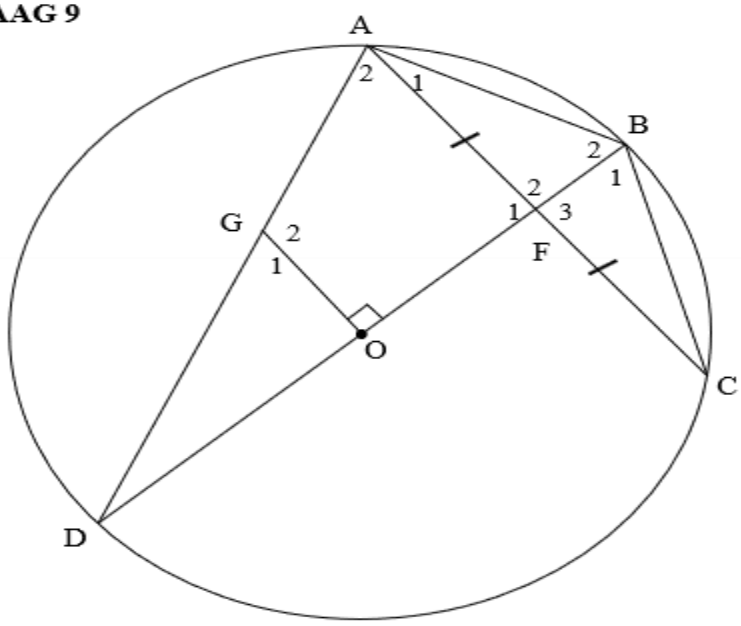
QUESTION/VRAAG 8



8.1	$\hat{L} = 100°$ [ext ∠ cyclic quad = int opp ∠ / *buite ∠ kdvh = tos ∠*] **OR/*OF*** $\hat{N}_1 = 80°$ [∠s on straight line] $\hat{L} = 100°$ [opp ∠s of cyclic quad]	✓S ✓R (2) ✓S ✓R (2)
8.2	$\hat{N}_1 = 80°$ [∠s on straight line/∠e *op reguitlyn*] $\therefore \hat{O}_1 = 160°$ [∠ at centre = 2 × ∠ at circumference/midpts ∠ = 2 omtreks ∠] **OR/*OF*** reflex $K\hat{O}M = 200°$ [∠ at centre=2 × ∠ at circumference/*midpts∠ = 2×omtreks∠*] $\therefore \hat{O}_1 = 160°$ [∠s around a pt/ ∠e om 'n pt]	✓S ✓S ✓R (3) ✓S ✓R ✓S (3)
8.3	$\hat{M}_1 = 360° - (100° + 55° + 160°)$ [sum ∠s of quad/*som ∠e v vierhoek*] $\therefore \hat{M}_1 = 45°$	✓S ✓S (2) **[7]**

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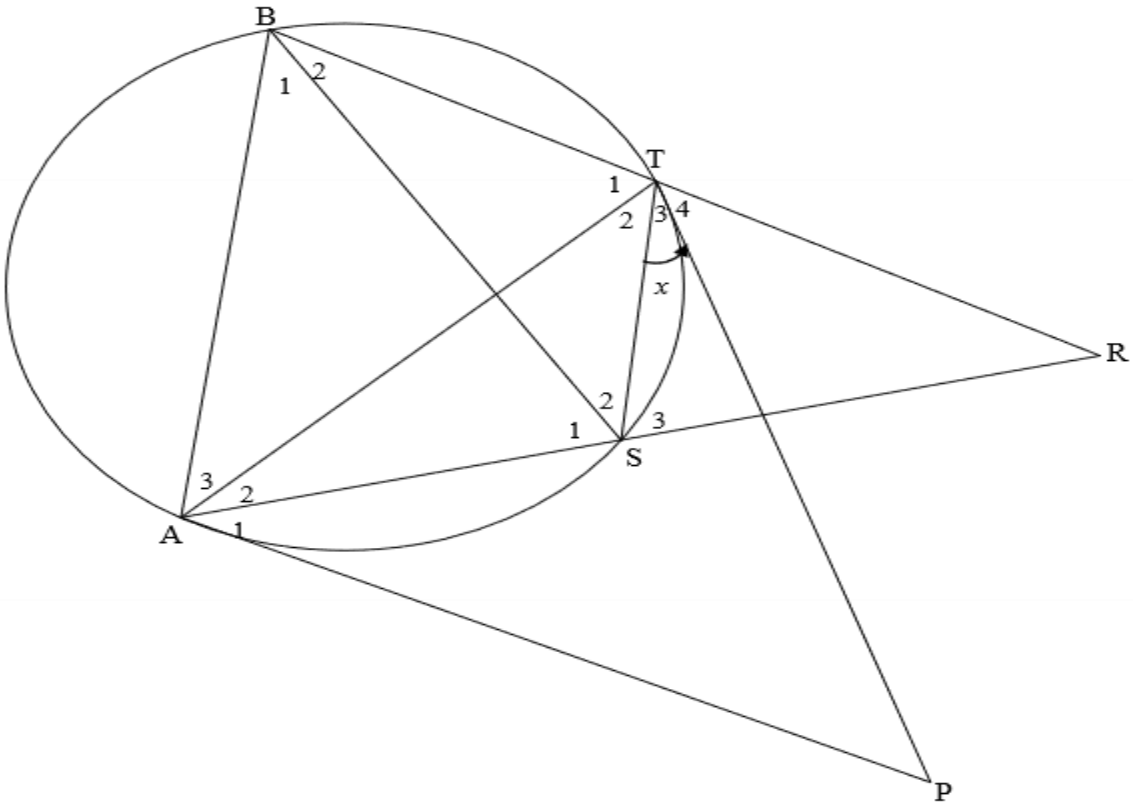
QUESTION/VRAAG 9



9.1.1	∠ in semi-circle/∠ *in halfsirkel*	✓answer (1)
9.1.2	**Opp ∠s** of quad = **180°**/*Teenoorst ∠e v vierhoek=180°*	✓answer (1)
9.2.1	OF ⊥ AC [line from centre bisects chord/*lyn v midpt halv kd*] ∴ AC ‖ GO [co-interior/*ko-binne* ∠s = 180° **OR/*OF*** corresp/*ooreenkomstige* ∠s =]	✓ S ✓ R ✓ R (3)
9.2.2	$\hat{G}_1 = \hat{A}_2$ [corresp/*ooreenk* ∠s; AC ‖ GO] $\hat{A}_2 = \hat{B}_1$ [∠s in same segment/∠e *in dies segment*] $\therefore \hat{G}_1 = \hat{B}_1$ **OR/*OF*** $\hat{G}_1 = \hat{B}_2$ [ext ∠ cyclic quad/*buite ∠ koordevh*] but ΔABF ≡ ΔCBF [s, ∠, s] $\therefore \hat{B}_2 = \hat{B}_1$ $\therefore \hat{G}_1 = \hat{B}_1$	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ R ✓ S (4)
9.3	OF : FB = 3 : 2 DB = 2*r* ∴ DO = 5*k* and DF = 8*k* **OR/*OF*** $DF = 2r - \frac{2}{5}r = \frac{8}{5}r$ $\therefore \frac{DG}{DA} = \frac{DO}{DF} = \frac{r}{\frac{8}{5}r}$ [line ‖ side of Δ/*lyn ‖ sy v* Δ] $\therefore \frac{DG}{DA} = \frac{5}{8}$	✓ S ✓ R ✓ S (3)
		[12]

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QUESTION/VRAAG 10



10.1	Tangent-chord theorem		✓ R (1)
10.2.1	$\hat{A}_2 + \hat{A}_3 = \hat{B}_1 + \hat{B}_2$	[$\angle^s$ opp = sides/$\angle$e teenoor = sye]	✓ S ✓ R
	$\hat{S}_3 = \hat{B}_1 + \hat{B}_2$	[ext $\angle$ cyclic quad/*buite* $\angle$ *koordevh*]	✓ S ✓ R
	$\therefore \hat{S}_3 = \hat{A}_2 + \hat{A}_3$		
	$\therefore AB \parallel ST$	[corresp/*ooreenk* $\angle^s$ =]	✓ R (5)
	OR/*OF*		
	$R\hat{T}S = B\hat{A}S$	[ext $\angle$ cyclic quad/*buite* $\angle$ *koordevh*]	✓ S ✓ R
	$B\hat{A}S = A\hat{B}T$	[$\angle^s$ opp = sides/$\angle$e teenoor = sye]	✓ S ✓ R
	$\therefore R\hat{T}S = A\hat{B}T$		
	$\therefore AB \parallel ST$	[corresp/*ooreenk* $\angle^s$ =]	✓ R (5)

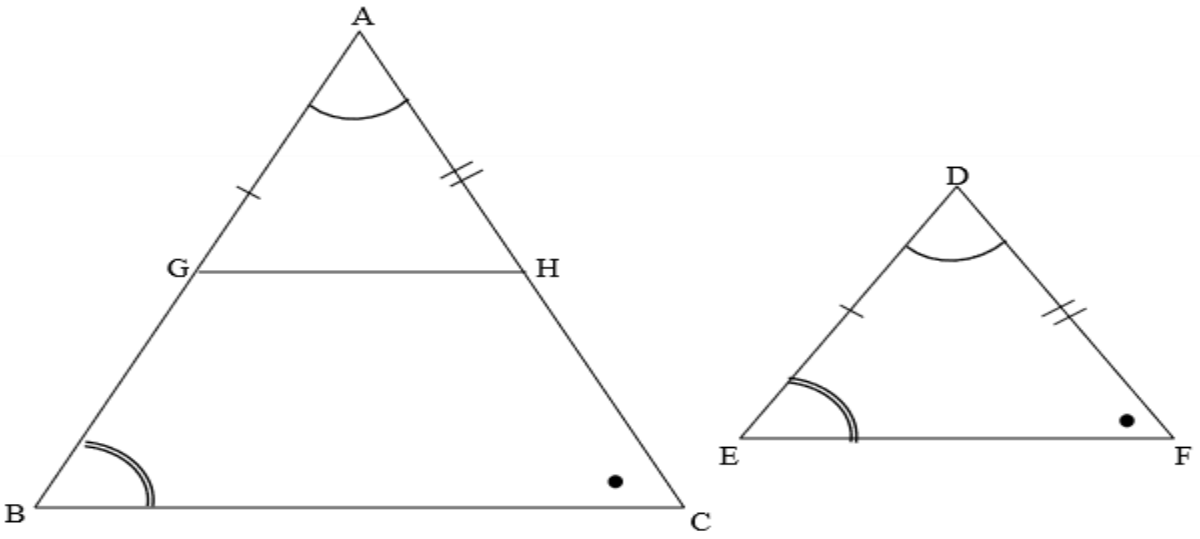
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10.2.2	$\hat{B}_2 = x$ [tan chord theorem/*raakl – koordst*] $x + \hat{T}_4 = \hat{B}_1 + \hat{B}_2$ [corresp/*ooreenk* $\angle^s$; AB // ST] $\therefore \hat{T}_4 = \hat{B}_1$ $\hat{B}_1 = \hat{A}_1$ [tan chord theorem/*raakl – koordst*] $\therefore \hat{T}_4 = \hat{A}_1$	✓ S ✓ R ✓ S ✓ R ✓ R (5)
10.2.3	$\hat{T}_4 = \hat{A}_1$ [proven/*bewys in* 10.2.2] $\therefore$ RTAP is a cyclic quadrilateral [line subtends $= \angle^s$] *Is 'n koordevierhoek* [*lyn onderspan* $= \angle e$]	✓ S ✓ R (2)
		[13]

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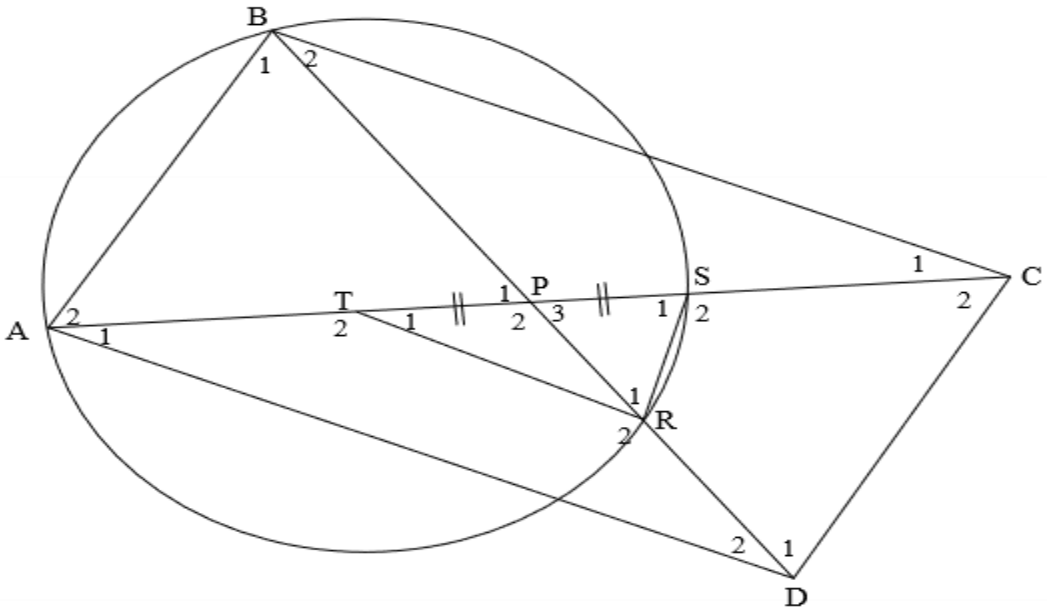
QUESTION/VRAAG 11

11.1



11.1	Constr: On sides AB and AC of ΔABC, mark points G and H respectively such that AG = DE and AH = DF. Draw GH/*Merk punt G en H op sy AB en AC van ΔABC onderskeidelik af sodanig dat AG = DE en AH = DF. Trek GH.*	✓ construction/ *konstruksie*
	Proof/*Bewys*:	
	$\Delta AGH \equiv \Delta DEF$ [s, ∠, s]	✓ S/R
	$\therefore A\hat{G}H = \hat{E}$	
	$= \hat{B}$ [$\hat{B} = \hat{E}$, given/*gegee*]	✓ S
	$\therefore GH \parallel BC$ [corresp/*ooreenk* ∠^{s} =]	✓ S /R
	$\therefore \frac{AG}{AB} = \frac{AH}{AC}$ [line ∥ side of Δ / *lyn* ∥ *sye v* Δ]	✓ S ✓ R
	$\therefore \frac{DE}{AB} = \frac{DF}{AC}$ [constr/*konstruksie*]	(6)

11.2



11.2.1(a)	AP = PC [diag $\|\|^m$ bisect each other/*hoekl* $\|\|^m$ *halveer mekaar*] But TP = PS [given/*gegee*] AP – TP = PC – PS ∴ AT = SC	✓S ✓S OR S 2)
11.2.1(b)	In ΔPSR and ΔPBA : $\hat{P}_1 = \hat{P}_3$ [vertically opp $\angle^s$ / *regoorst* $\angle e$] $\hat{B}_1 = \hat{S}_1$ [$\angle^s$ in same segment/$\angle e$ *in dies segment*] ∴ ΔPSR ||| ΔPBA [∠, ∠, ∠] **OR/*OF*** In ΔPSR and ΔPBA : $\hat{P}_1 = \hat{P}_3$ [vertically opp $\angle^s$ / *regoorst* $\angle e$] $\hat{B}_1 = \hat{S}_1$ [$\angle^s$ in same segment/$\angle e$ *in dies segment*] $\hat{A}_2 = \hat{R}_1$ [sum $\angle^s$ Δ / *som* $\angle e$ Δ] ∴ ΔPSR ||| ΔPBA [∠, ∠, ∠]	✓S ✓R ✓S ✓R ✓R (5) ✓S ✓R ✓S ✓R ✓S (5)

11.2.2(a)	$\frac{PR}{PA}=\frac{PS}{PB}$ [$\|\|\|\Delta s$] $\therefore \frac{PR}{PA}=\frac{TR}{AD}=\frac{PS}{PB}$ [given $\frac{PR}{PA}=\frac{TR}{AD}$] $\therefore \frac{PR}{PA}=\frac{TR}{AD}=\frac{TP}{PD}$ [PS = TP; PB = PD] $\therefore \Delta RPT \|\|\| \Delta APD$ [sides of Δ in prop/*sye v* Δ *in dies verhouding*]	✓ S (all 3 ratios) ✓ S ✓ R (3)
11.2.2(b)	$\hat{T}_1 = \hat{D}_2$ [$\|\|\|$ Δs] ∴ ATRD is a cyclic quad [converse: ext ∠ of cyclic quad/ *Omgekeerde buite* ∠ v koordevh]	✓ S ✓ R (2)
		[18]

TOTAL/*TOTAAL*: 150

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2018

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 27 pages.

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QUESTION 1

The monthly profit (in thousands of rands) made by a company in a year is given in the table below.

110	112	156	164	167	169
171	176	192	228	278	360

1.1 Calculate the:

1.1.1 Mean profit for the year (3)

1.1.2 Median profit for the year (1)

1.2 On the number line provided in the ANSWER BOOK, draw a box and whisker diagram to represent the data. (2)

1.3 Hence, determine the interquartile range of the data. (1)

1.4 Comment on the skewness in the distribution of the data. (1)

1.5 For the given data:

1.5.1 Calculate the standard deviation (1)

1.5.2 Determine the number of months in which the profit was less than one standard deviation below the mean (2)
[11]

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QUESTION 2

It is said that the number of times that a cricket chirps in a minute gives a very good indication of the air temperature (in °C). The table below shows the information recorded during an observation study.

CHIRPS PER MINUTE	AIR TEMPERATURE IN °C
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

2.1 Represent the data above on the grid provided in the ANSWER BOOK. (3)

2.2 Explain why the claim, 'gives a very good indication', is TRUE. (1)

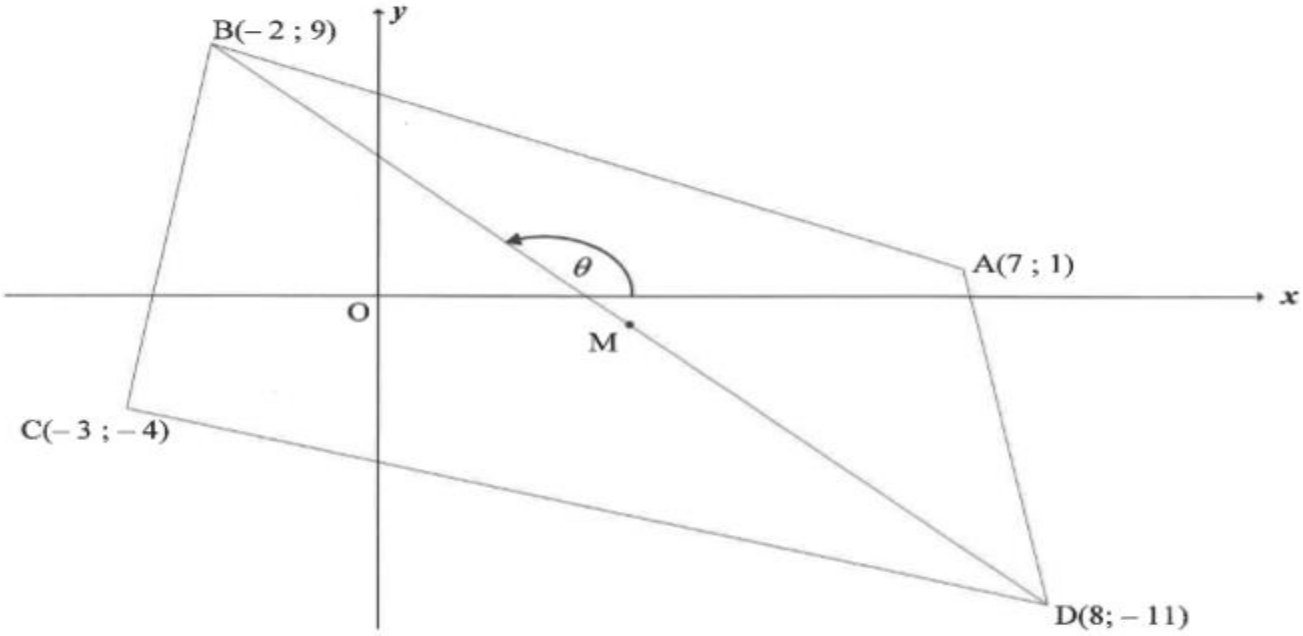
2.3 Determine the equation of the least squares regression line of the data. (3)

2.4 Predict the air temperature (in °C) if a cricket chirps 80 times a minute. (2)
[9]

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QUESTION 3

In the diagram, ABCD is a quadrilateral having vertices A(7 ; 1), B(– 2 ; 9), C(– 3 ; – 4) and D(8 ; – 11). M is the midpoint of BD.



3.1 Calculate the gradient of AC. (2)

3.2 Determine:

3.2.1 The equation of AC in the form $y = mx + c$ (2)

3.2.2 Whether M lies on AC (4)

3.3 Prove that BD ⊥ AC. (3)

3.4 Calculate:

3.4.1 θ, the inclination of BD (2)

3.4.2 The size of $C\hat{B}D$ (3)

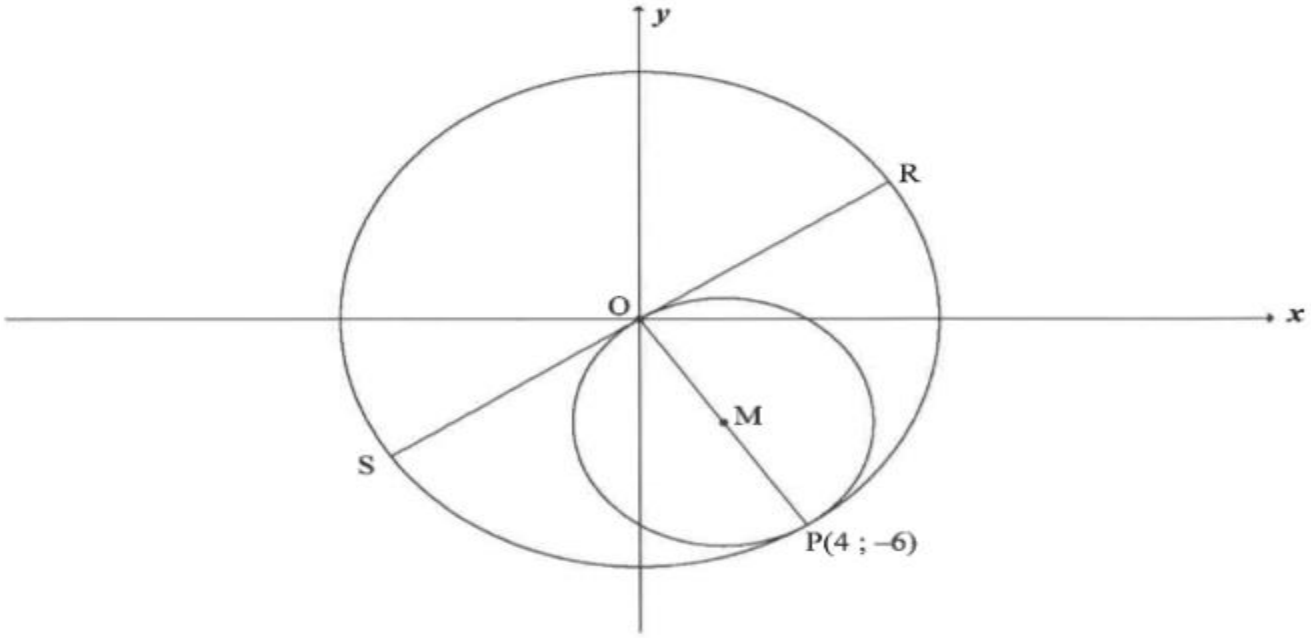
3.4.3 The length of AC (2)

3.4.4 The area of ABCD (5)
[23]

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QUESTION 4

In the diagram, a circle having centre at the origin passes through P(4 ; – 6). PO is the diameter of a smaller circle having centre at M. The diameter RS of the larger circle is a tangent to the smaller circle at O.



4.1 Calculate the coordinates of M. (2)

4.2 Determine the equation of:

4.2.1 The large circle (2)

4.2.2 The small circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ (3)

4.2.3 The equation of RS in the form $y = mx + c$ (3)

4.3 Determine the length of chord NR, where N is the reflection of R in the y-axis. (4)

4.4 The circle with centre at M is reflected about the x-axis to form another circle centred at K. Calculate the length of the common chord of these two circles. (3)
[17]

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QUESTION 5

5.1 In ΔMNP, $\hat{N} = 90°$ and $\sin M = \frac{15}{17}$.
Determine, **without using a calculator**:

5.1.1 tan M (3)

5.1.2 The length of NP if MP = 51 (2)

5.2 Simplify to a single term: $\cos(x-360°).\sin(90°+x)+\cos^2(-x)-1$ (4)

5.3 Consider: $\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)$

5.3.1 Write as a single trigonometric term in its simplest form. (2)

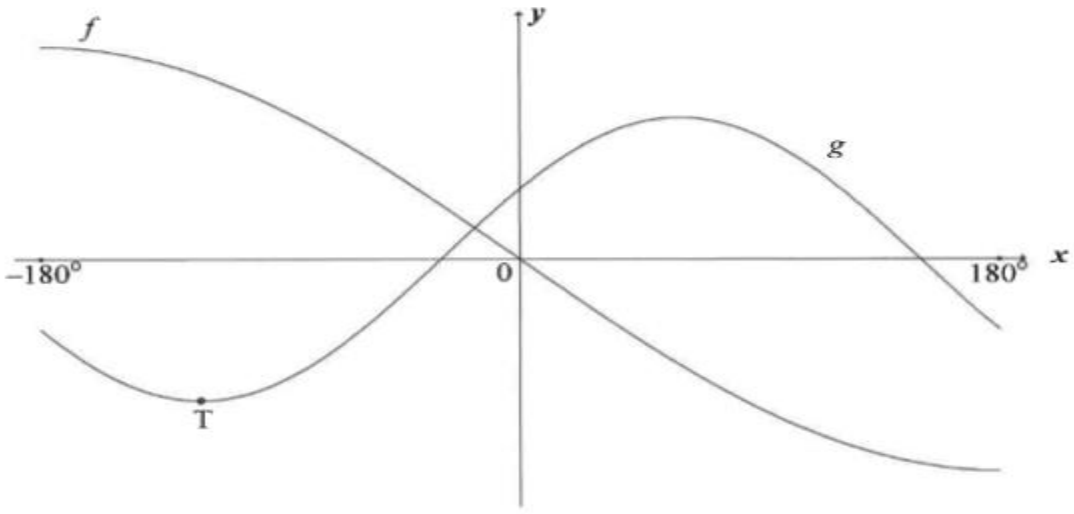
5.3.2 Determine the general solution of the following equation:

$\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)=\cos(2x-20°)$ (7)
[18]

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QUESTION 6

In the diagram, the graphs of $f(x) = -3\sin\frac{x}{2}$ and $g(x) = 2\cos(x - 60°)$ are drawn in the interval $x \in [-180°;180°]$. T$(p\,;q)$ is a turning point of g with $p < 0$.



6.1 Write down the period of f. (1)

6.2 Write down the range of g. (2)

6.3 Calculate $f(p) - g(p)$. (3)

6.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180°;180°]$ for which:

6.4.1 $g(x) > 0$ (3)

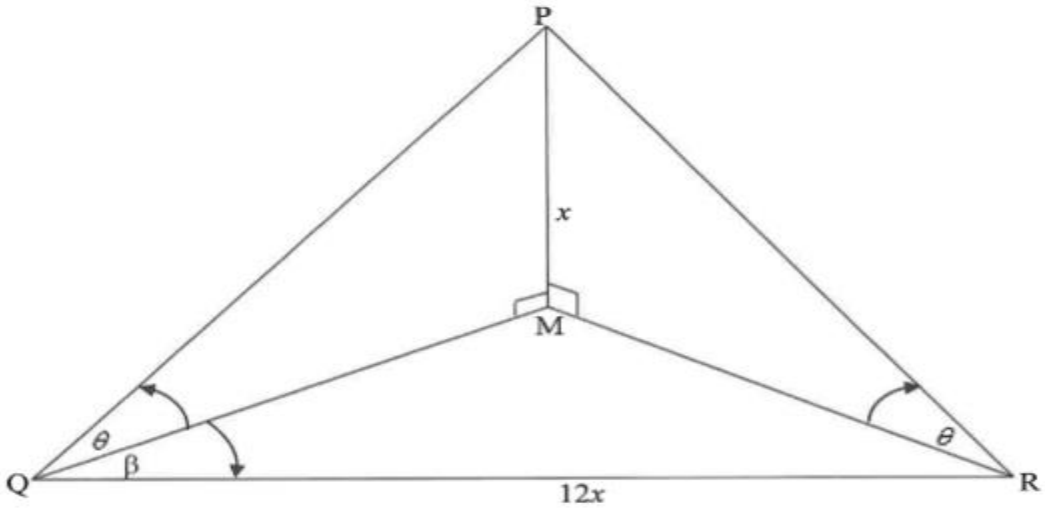
6.4.2 $g(x).g'(x) > 0$ (4)

[13]

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QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ. Q, M and R lie in the same horizontal plane.



7.1 Write QM in terms of x and θ. (2)

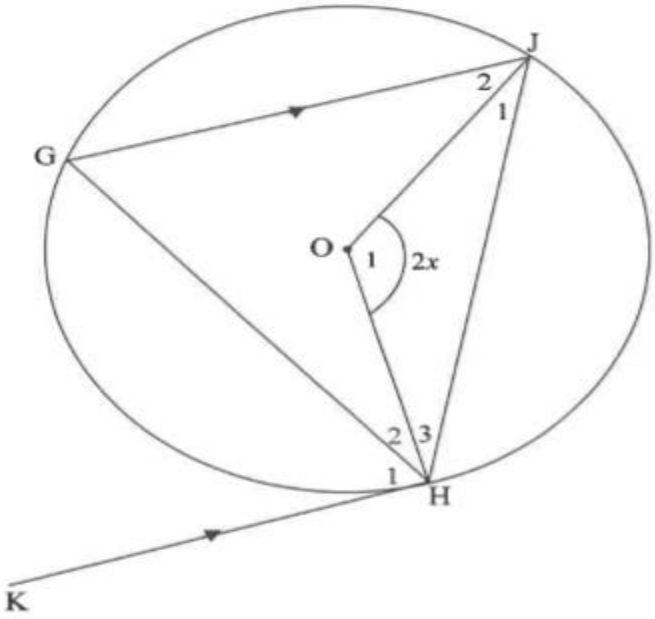
7.2 Prove that $\tan\theta = \frac{\cos\beta}{6}$. (4)

7.3 If $\beta = 40°$ and QM = 60 metres, calculate the height of the lighthouse **to the nearest metre**. (3)
[9]

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QUESTION 8

8.1 In the diagram, O is the centre of the circle. Radii OH and OJ are drawn. A tangent is drawn from K to touch the circle at H. ΔHGJ is drawn such that GJ ∥ KH. $\hat{O}_1 = 2x$.

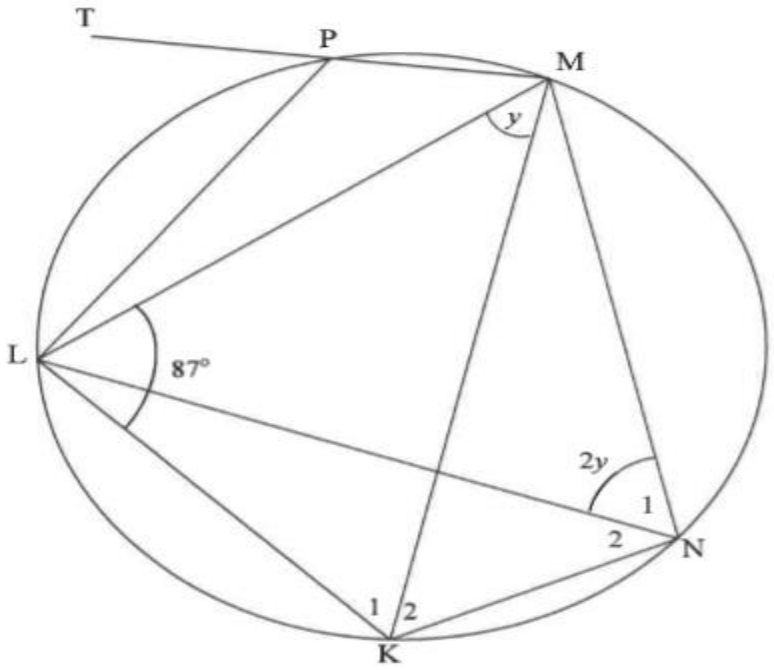


8.1.1 Name, giving reasons, THREE angles, each equal to x. (5)

8.1.2 Prove that $\hat{H}_2 = \hat{H}_3$. (3)

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8.2 In the diagram, KLMN is a cyclic quadrilateral with $K\hat{L}M = 87°$. Diagonals LN and MK are drawn. P is a point on the circle and MP is produced to T, a point outside the circle. Chord LP is drawn. $L\hat{M}K = y$ and $\hat{N}_1 = 2y$.



8.2.1 Name, giving a reason, another angle equal to y. (2)

8.2.2 Calculate, giving reasons, the size of:

(a) y (3)

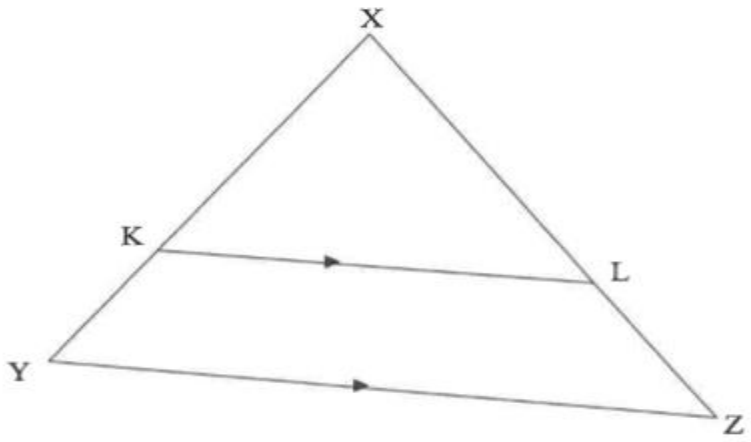
(b) $T\hat{P}L$ (2)

[15]

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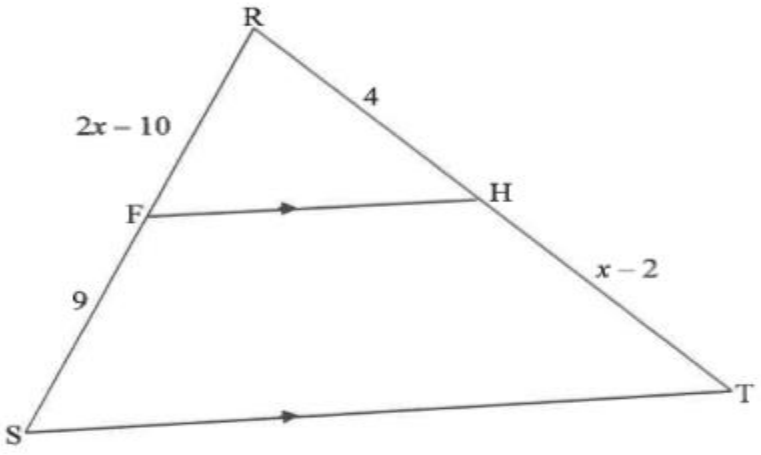
QUESTION 9

9.1 Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY} = \frac{XL}{LZ}$.



(5)

9.2 In ΔRST, F is a point on RS and H is a point on RT such that FH $\parallel$ ST. $RF = 2x - 10$, $FS = 9$, $RH = 4$ and $HT = x - 2$.



9.2.1 Determine, giving a reason, the value of x. (5)

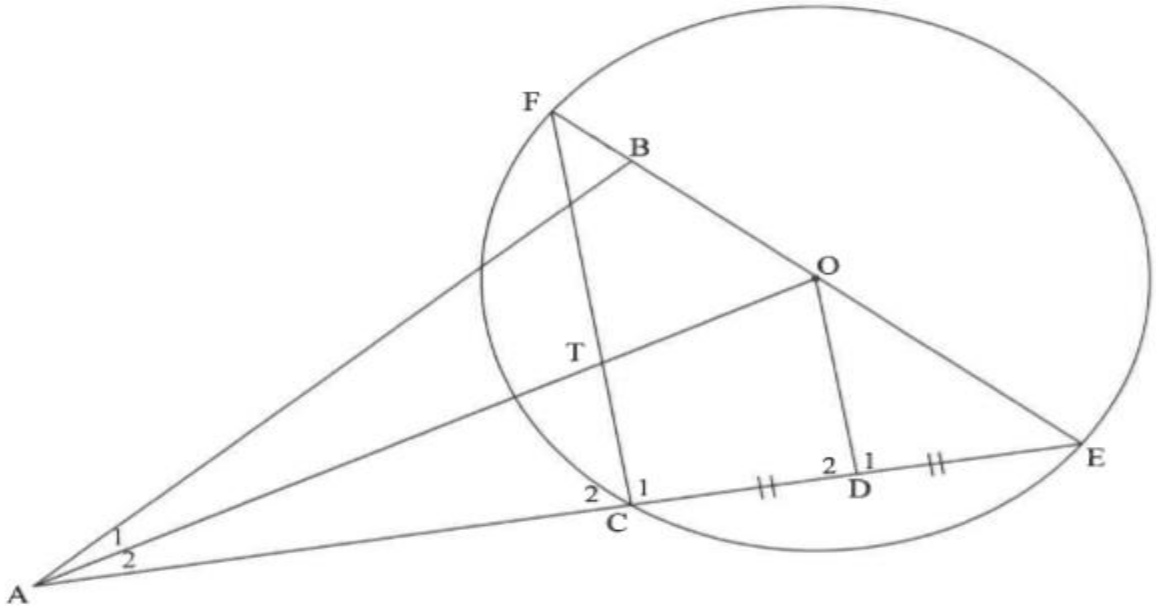
9.2.2 Determine the ratio: $\frac{\text{area } \Delta RFH}{\text{area } \Delta RST}$. (4)

[14]

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QUESTION 10

In the diagram, FBOE is a diameter of a circle with centre O. Chord EC produced meets line BA at A, outside the circle. D is the midpoint of CE. OD and FC are drawn. AFBC is a cyclic quadrilateral.



10.1 Prove, giving reasons, that:

10.1.1 FC ∥ OD (5)

10.1.2 $D\hat{O}E = B\hat{A}E$ (4)

10.1.3 AB × OF = AE × OD (7)

10.2 If it is further given that AT = 3TO, prove that $5CE^2 = 2BE.FE$ (5)

[21]

TOTAL: 150

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SENIOR CERTIFICATE EXAMINATIONS
SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS: 150
PUNTE: 150

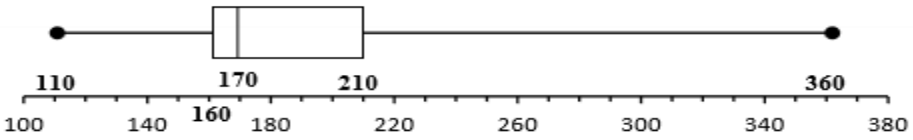
These marking guidelines consist of 21 pages.
Hierdie nasienriglyne bestaan uit 21 bladsye.

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Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 1

110	112	156	164	167	169
171	176	192	228	278	360

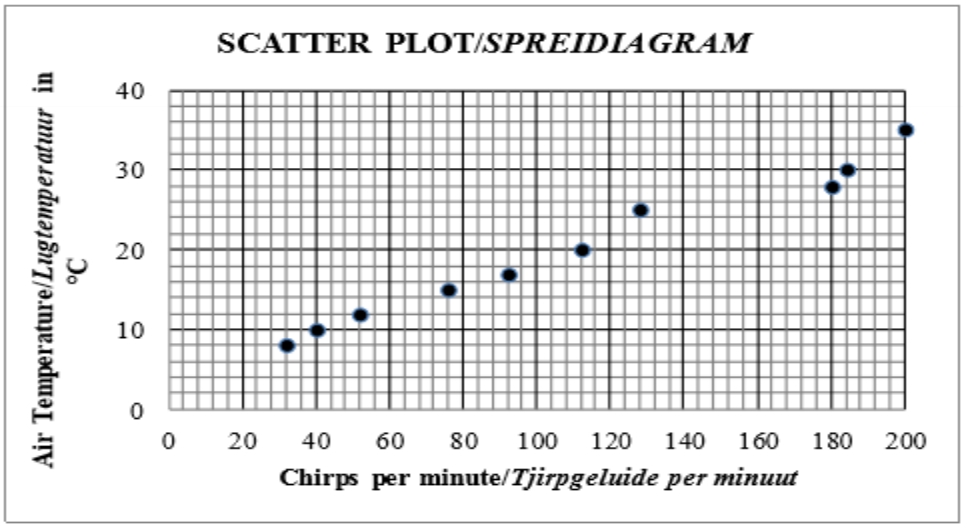
1.1.1	Mean/*Gemiddelde* $= \frac{2283}{12}$ $= 190{,}25$ Mean profit/*Gemiddelde wins* = R190 250,00 or 190,25 thousand rands	✓ sum/*som* ✓ answer ✓ answer in thousands of rands (3)
1.1.2	Median $= \frac{169+171}{2} = 170$ thousand rands = R170 000	✓ answer (1)
1.2		✓ whiskers ✓ quartiles (2)
1.3	IQR $= Q_3 - Q_1$ = 210 – 160 thousand rands = R50 000	✓ answer (1)
1.4	Skewed to the right or positively skewed.	✓ answer (1)
1.5.1	$\sigma = 67{,}04118759$ thousand rands = R67 041,19	✓ answer (1)
1.5.2	$\bar{x} - \sigma = 123{,}21$ thousand rands For **2** months the profit was less than one standard deviation below the mean.	✓ lower limit ✓ answer (2)
		[11]

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QUESTION/*VRAAG* 2

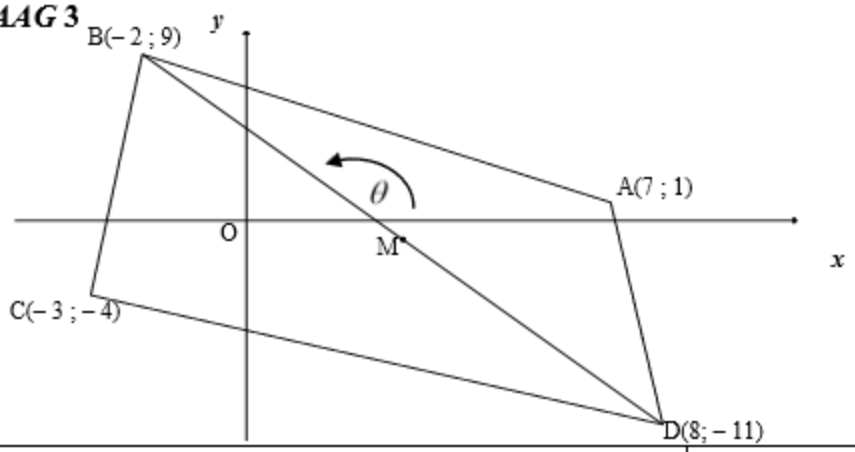
CHIRPS/*TJIRPGELUIDE* PER MINUTE/ *PER MINUUT*	AIR TEMPERATURE/ *LUGTEMPERATUUR* IN °C
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

2.1		3 marks: All points correct 2 marks: 6 – 9 points correct 1 mark: 3 – 5 points correct (3)
2.2	The points lie almost in a straight line. This suggests a very strong positive relationship between the number of chirps per minute and the temperature of the air. *Die punte lê amper in 'n reguitlyn, wat beteken dat daar 'n baie sterk positiewe verband tussen die aantal tjirpgeluide per minuut en die lugtemperatuur is.* **OR/*OF*** $r = 0,99$ so there is a very strong positive relationship between the number of chirps per minute and the temperature of the air. $r = 0,99$, *dus is daar 'n baie sterk positiewe verband tussen die aantal kriekgeluide per minuut en die lugtemperatuur.*	✓ justify with straight line / *Motivering mbv reguitlyn* (1) ✓ link with / *gebruik* $r = 0,99$ *om te motiveer* (1)

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2.3	$a = 3{,}97$ $b = 0{,}15$ $\hat{y} = 3{,}97 + 0{,}15x$	✓ $a = 3{,}97$ ✓ $b = 0{,}15$ ✓ equation (3)
2.4	Air temperature ≈ 15,67°C (calculator) **OR** $\hat{y} \approx 3{,}97 + 0{,}15(80)$ $\approx 15{,}97°C$ **OR** Air temperature ≈ 16°C (graph: Accept between 15°C and 17°C)	✓✓ answer (2) ✓ substitution ✓ answer (2) ✓✓ answer (2)
		[9]

QUESTION/*VRAAG* 3



3.1	$m_{AC} = \frac{1-(-4)}{7-(-3)}$ *OR* $\frac{-4-1}{-3-7}$ $= \frac{5}{10} = \frac{1}{2}$	✓ substitution ✓answer (2)
3.2.1	$y = \frac{1}{2}x + c$ $\quad y - y_1 = \frac{1}{2}(x - x_1)$ $1 = \frac{1}{2}(7) + c$ $\quad y - 1 = \frac{1}{2}(x - 7)$ $c = -\frac{5}{2}$ **OR/*OF*** $y - 1 = \frac{1}{2}x - \frac{7}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$ $\quad y = \frac{1}{2}x - 2\frac{1}{2}$ **OR/*OF*** $y = \frac{1}{2}x + c$ $\quad y - y_1 = \frac{1}{2}(x - x_1)$ $-4 = \frac{1}{2}(-3) + c$ $\quad y - (-4) = \frac{1}{2}(x - (-3))$	✓ substitution M and A(7 ; 1) ✓equation (2) ✓ substitution M and C(–3 ; –4)

3.2.2	$M\left(\frac{-2+8}{2};\frac{9+(-11)}{2}\right)$ $\therefore M(3;-1)$ Equation of AC: $y=\frac{1}{2}x-2\frac{1}{2}$ **OR/OF** $y=\frac{1}{2}x-2\frac{1}{2}$ $y=\frac{1}{2}(3)-2\frac{1}{2}$ $-1=\frac{1}{2}x-2\frac{1}{2}$ $y=-1$ $x=3$ $\therefore$ M lies on AC **OR/OF** $M\left(\frac{-2+8}{2};\frac{9+(-11)}{2}\right)$ $\therefore M(3;-1)$ $m_{CM}=\frac{-4+1}{-3-3}=\frac{1}{2}$ $\therefore$ $m_{CM}=m_{AC}$ and C a common point $\therefore$ M lies on AC	✓x coordinate ✓y coordinate ✓ substitution of x ✓ conclusion (4) ✓ x coordinate ✓ y coordinate ✓ gradient of CM ✓ reasoning & conclusion (4)
3.3	$m_{BD}=\frac{9-(-11)}{-2-8}$ OR $\frac{(-11)-9}{8-(-2)}$ $=-2$ $m_{BD}\times m_{AC}=\frac{1}{2}\times -2$ $=-1$ $\therefore BD\perp AC$	✓ correct substitution ✓ m_{BD} ✓ product of gradients $=-1$ (3)
3.4.1	$\tan\theta=m_{BD}=-2$ $\therefore \theta=116{,}57°$	✓ $\tan\theta=m_{BD}$ ✓ answer (2)
3.4.2	$\tan\beta=m_{BC}$ $m_{BC}=\frac{9-(-4)}{-2-(-3)}$ *OR* $\frac{-4-9}{-3-(-2)}$ $=13$ $\beta=85{,}6°$ $\therefore C\hat{B}D=116{,}57°-85{,}60°$ [ext $\angle$ of Δ] $=30{,}97°$ **OR/OF** BD $=\sqrt{500}$; BC $=\sqrt{170}$ & CD $=\sqrt{170}$ $CD^2=BD^2+BC^2-2BD.BC.\cos C\hat{B}D$ $170=500+170-2\sqrt{500}.\sqrt{170}.\cos C\hat{B}D$ $\cos C\hat{B}D=\frac{\sqrt{500}}{2\sqrt{170}}=0{,}85749...$ $C\hat{B}D=30{,}96°$	✓ $m_{BC}=13$ ✓ value of β ✓ answer (3) ✓ subst into cos rule ✓ value of $\cos C\hat{B}D$ ✓ answer (3)

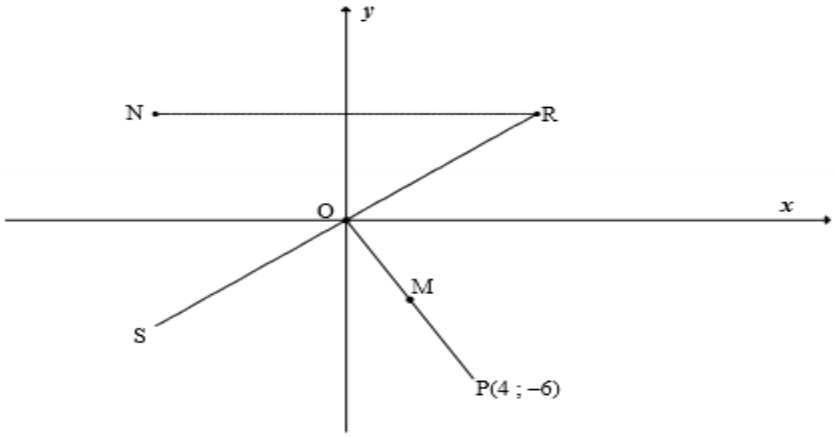
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3.4.3	$AC = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $= \sqrt{(7-(-3))^2 + (1-(-4))^2}$ *OR* $\sqrt{((-3)-7)^2 + ((-4)-1)^2}$ $= \sqrt{100+25}$ $= \sqrt{125} = 5\sqrt{5} = 11,58$	✓ correct substitution into distance formula ✓ answer (2)
3.4.4	$BM = \sqrt{((-2)-3)^2 + (9-(-1)^2}$ *OR* $\sqrt{(3-(-2))^2 + ((-1)-9)^2}$ $= \sqrt{125} = 5\sqrt{5}$ Area of $\Delta ABC = \frac{1}{2}$ base $\times \perp$ height $= \frac{1}{2}(\sqrt{125})(\sqrt{125})$ $= 62,5$ square units Area of ABCD $= 2 \times 62,5$ $= 125$ square units	✓ correct substitution into distance formula ✓ BM ✓ substitution into area formula ✓ 62,5 ✓ $2 \times \Delta ABC$ (5)
		[23]

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QUESTION/*VRAAG* 4



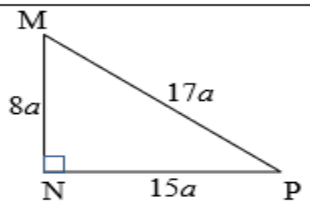
4.1	$M\left(\frac{0+4}{2};\frac{0+(-6)}{2}\right)$ $\therefore M(2;-3)$	✓ 2 ✓ – 3 (2)
4.2.1	$x^2+y^2=4^2+(-6)^2$ $=52$ $\therefore x^2+y^2=52$	✓ substitution ✓ equation (2)
4.2.2	$(x-2)^2+(y+3)^2=\left(\frac{\sqrt{52}}{2}\right)^2=13$ $x^2-4x+4+y^2+6y+9-13=0$ $x^2+y^2-4x+6y=0$	✓ substitution of M ✓ substitution of radius $=\frac{\sqrt{52}}{2}$ ✓ answer (3)
4.2.3	$m_{OP}=\frac{-6}{4}=-\frac{3}{2}$ $m_{RS}\times m_{OP}=-1$ [radius ⊥ tangent / *raaklyn*] $\therefore m_{RS}=\frac{2}{3}$ $\therefore y=\frac{2}{3}x$	✓ m_{OP} ✓ m_{RS} ✓ equation (3)

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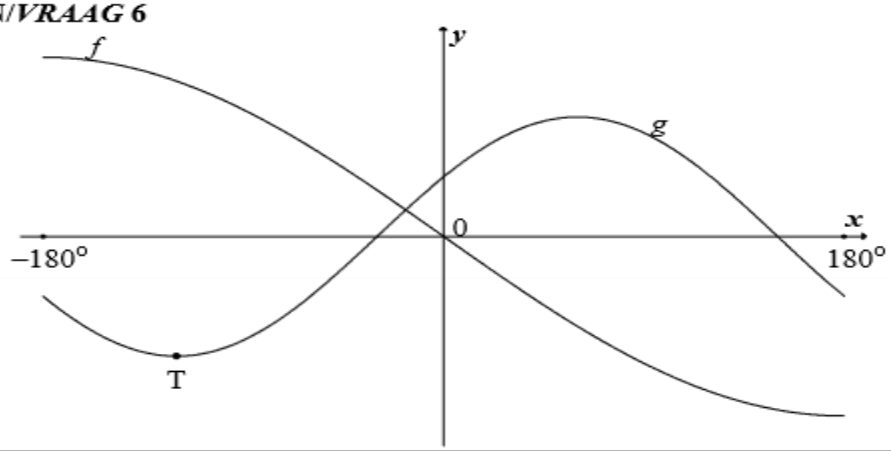
4.3	$x^2 + y^2 = 52$ *and* $y = \frac{2}{3}x$ $x^2 + \left(\frac{2}{3}x\right)^2 = 52$ $x^2 + \frac{4}{9}x^2 = 52$ $1\frac{4}{9}x^2 = 52$ $x^2 = 36$ $x = 6$ $\therefore$ R(6 ; 4) and N(–6 ; 4) $\therefore$ NR = 12 units	✓ substitution ✓ simplification ✓ value of x ✓ length of NR (4)
4.4	Let T(x ; 0) be the other x intercept of the small circle Then OT is the common chord $\therefore (x-2)^2 + (0+3)^2 = 13$ $(x-2)^2 = 13 - 9 = 4$ $x - 2 = \pm 2$ $x = 2 \pm 2$ $x = 4$ *or* 0 OR $x^2 - 4x + 4 + 9 = 13$ $x^2 - 4x = 0$ $x(x-4) = 0$ $x = 0$ or $x = 4$ $\therefore$ length of common chord = OT = 4 units	✓ $y = 0$ ✓ x-values ✓ answer (3) **[17]**

QUESTION/*VRAAG* 5

5.1.1	Given : $\sin M = \frac{15}{17}$ $MN^2 = 17^2 - 15^2$ $= 64$ $MN = 8$ OR  $\therefore \tan M = \frac{15}{8}$	✓ sketch or Pyth ✓ MN = 8 ✓answer (3)
5.1.2	$\sin M = \frac{NP}{MP}$ $\frac{NP}{51} = \frac{15a}{17a}$ $\therefore NP = 45$	✓ equating trig ratios ✓ answer (2)
5.2	$\cos(x - 360°).\sin(90° + x) + \cos^2(-x) - 1$ $= \cos x \,.\, \cos x + \cos^2 x - 1$ $= \cos^2 x + \cos^2 x - 1$ $= 2\cos^2 x - 1$ $= \cos 2x$	✓ $\cos x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ identity (4)
5.3.1	$\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°)$ $= \sin[(2x + 40°) - (x + 30°)]$ $= \sin(x + 10°)$	✓ reduction ✓ answer (2)
5.3.2	$\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°) = \cos(2x - 20°)$ $\therefore \cos(2x - 20°) = \sin(x + 10°)$ $\cos(2x - 20°) = \cos[90° - (x + 10°)]$ $2x - 20° = 80° - x + k.360°$ or $2x - 20° = 360° - (80° - x) + k.360°$ $3x = 100° + k.360°$ or $2x - 20° = 280° + x + k.360°$ $x = 33{,}33° + k.120°$ or $x = 300° + k.360°$; $k \in Z$ **OR/*OF*** $\therefore \cos(2x - 20°) = \sin(x + 10°)$ $\sin[90° - (2x - 20°)] = \sin(x + 10°)$ $110° - 2x = x + 10° + k.360°$ or $110° - 2x = 180° - (x + 10°) + k.360°$ $3x = 100° - k.360°$ or $110° - 2x = 170° - x + k.360°$ $x = 33{,}33° - k.120°$ or $x = -60° - k.360°$; $k \in Z$	✓ equating ✓ co ratio ✓ $80° - x$ ✓$280° + x$ ✓ simplification/*vereenv* ✓ $x = 33{,}33° + k.120°$ ✓ $x = 300° + k.360°$; $k \in Z$ (7) ✓ equating ✓ co ratio ✓ $x + 10°$ ✓$170° - x$ ✓ simplification/*vereenv* ✓ $x = 33{,}33° - k.120°$ ✓ $x = -60° - k.360°$; $k \in Z$ (7)
		[18]

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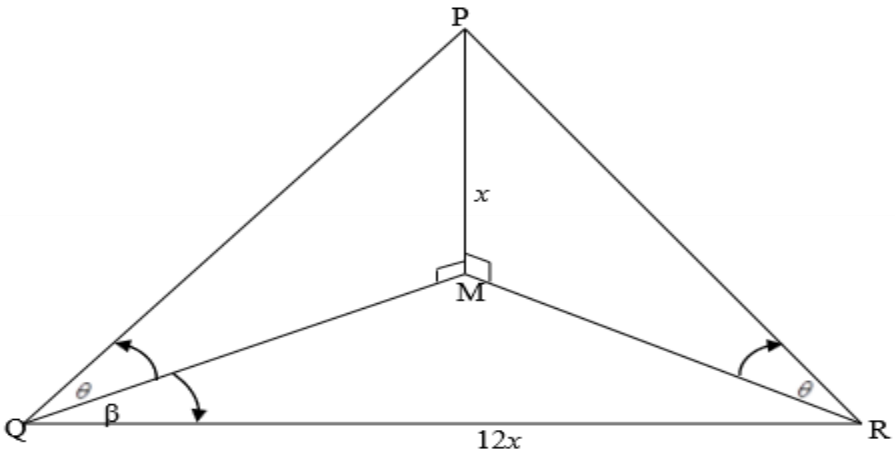
QUESTION/*VRAAG 6*



6.1	Period = 720°	✓ answer (1)
6.2	$y \in [-2\ ;\ 2]$ **OR/*OF*** $-2 \le y \le 2$	✓✓ answer (2) ✓✓ answer (2)
6.3	$f(-120°) - g(-120°)$ $= -3\sin\left(-\frac{120°}{2}\right) - 2\cos(-120° - 60°)$ $= \frac{4+3\sqrt{3}}{2}$ or 4,60 (4,5980...)	✓ $x = -120°$ ✓ substitution ✓ answer (3)
6.4.1	x-intercepts of g at $-90° + 60° = -30°$ and $90° + 60° = 150°$ $\therefore x \in (-30°\ ;\ 150°)$ **OR/*OF*** x-intercepts of g at $-90° + 60° = -30°$ and $90° + 60° = 150°$ $-30° < x < 150°$	✓ value ✓ value ✓ answer (3) ✓ value ✓ value ✓ answer (3)
6.4.2	$x \in [-180°\ ;\ -120°) \cup (-30°\ ;\ 60°) \cup (150°\ ;\ 180°]$ **OR/*OF*** $-180° \le x < -120°$ or $-30° < x < 60°$ or $150° < x \le 180°$	✓ $[-180°\ ;\ -120°)$ ✓ $(-30°\ ;\ 60°)$ ✓ $(150°\ ;\ 180°]$ ✓ notation for inclusive in the first/last interval (4) ✓ $-180° \le x < -120°$ ✓ $-30° < x < 60°$ ✓ $150° < x \le 180°$ 1 mark: each interval ✓ notation for inclusive in the first/last interval (4)
		[13]

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QUESTION/*VRAAG* 7



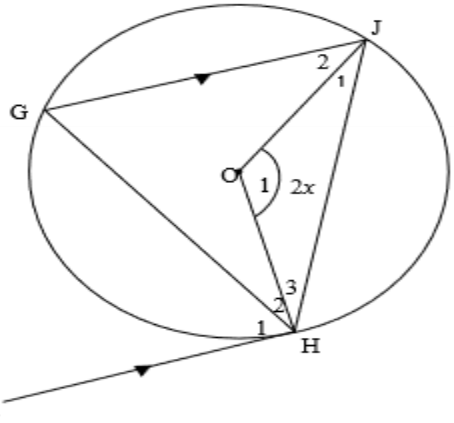
7.1	In PMQ : $\tan\theta = \frac{x}{QM}$ $\therefore QM = \frac{x}{\tan\theta}$ **OR/*OF*** $\frac{x}{\sin\theta} = \frac{MQ}{\sin P}$ $MQ = \frac{x\sin P}{\sin\theta}$ $= \frac{x\cos\theta}{\sin\theta}$ $= \frac{x}{\tan\theta}$	✓ trig ratio ✓ answer (2) ✓ sine rule ✓ answer (2)
7.2	In PMR : $\tan\theta = \frac{x}{MR}$ *OR* PMQ ≡ PMR [AAS/HHS] $\therefore MR = \frac{x}{\tan\theta} = QM$ $\hat{QMR} = 180° - 2\beta$ $\frac{\sin\beta}{MR} = \frac{\sin\hat{QMR}}{12x}$ $\sin\beta \times \frac{\tan\theta}{x} = \frac{\sin(180° - 2\beta)}{12x}$ $\tan\theta = \frac{\sin 2\beta}{12x} \times \frac{x}{\sin\beta}$ $\tan\theta = \frac{2\sin\beta\cos\beta}{12x} \times \frac{x}{\sin\beta}$ $\tan\theta = \frac{\cos\beta}{6}$ **OR**	✓ MR = QM ✓ correct substitution into the sine rule in ΔQMR ✓ reduction ✓ double angle (4)

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	In PMR : $\tan\theta = \frac{x}{MR}$ *OR* PMQ ≡ PMR [AAS/HHS] $MR^2 = QM^2 + QR^2 - 2QM.QR\cos\beta$ $MR^2 = \left(\frac{x}{\tan\theta}\right)^2 + (12x)^2 - 2\left(\frac{x}{\tan\theta}\right)(12x)(\cos\beta)$ $\frac{x^2}{\tan^2\theta} = \frac{x^2}{\tan^2\theta} + 144x^2 - 24\left(\frac{x^2}{\tan\theta}\right)(\cos\beta)$ $24\left(\frac{x^2}{\tan\theta}\right)(\cos\beta) = 144x^2$ $\cos\beta = 6\tan\theta$ $\tan\theta = \frac{\cos\beta}{6}$	✓ correct substitution into the cosine rule in ΔQMR ✓ substitution ✓ MR = QM ✓ simplification (4)
7.3	$\frac{x}{QM} = \frac{\cos\beta}{6}$ [both equal $\tan\theta$] $x = \frac{60\cos 40}{6}$ $x = 7{,}66$ The height of the lighthouse is 8 metres	✓ equating ✓ subst. QM = 60 and $\beta = 40°$ ✓ answer (3)
		[9]

QUESTION/*VRAAG* 8

8.1

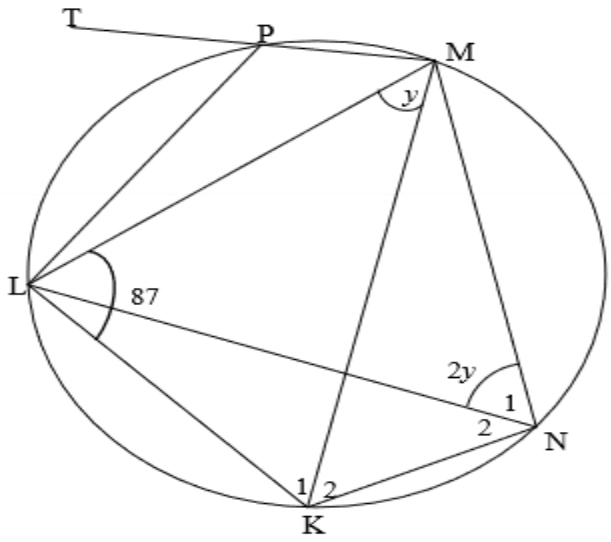


8.1.1	$\hat{G} = x$ [$\angle$ centre $= 2\times$ circumference / *midpts*$\angle = 2\times$*omtreks*$\angle$] $\hat{H}_1 = x$ [alt $\angle$s / *verwiss* $\angle e$; KH $\parallel$ GJ] $G\hat{J}H = x$ [tan chord theorem / *raaklyn koordstelling*]	✓S ✓R ✓S ✓S ✓R (5)
8.1.2	$\hat{J}_1 + \hat{H}_3 = 180° - 2x$ [sum of $\angle$s in Δ / *som van* $\angle e$ *in* Δ] $\therefore \hat{J}_1 = \hat{H}_3 = 90° - x$ [$\angle$s opp equal sides / $\angle e$ *teenoor gelyke sye*] $\therefore x + \hat{H}_2 = 90°$ **OR** [tan $\perp$ radius / *raaklyn* $\perp$ *radius*] $\hat{H}_2 = 90° - x$ $\therefore \hat{H}_2 = \hat{H}_3$	 ✓S ✓S ✓R (3)

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8.2



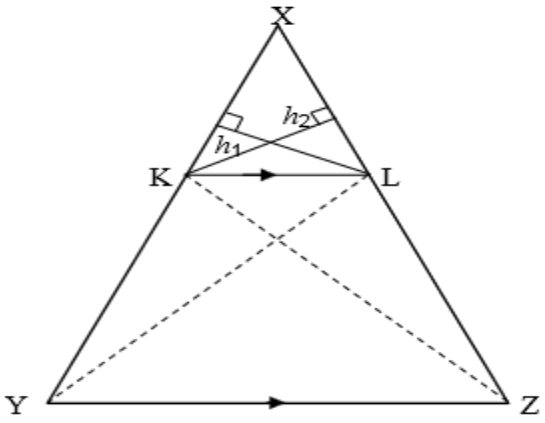
8.2.1	$\hat{N}_2 = y$	[∠s in the same seg / *∠e in dieselfde segment*]	✓S ✓R (2)
8.2.2(a)	$2y + y + 87° = 180°$ $3y = 93°$ $y = 31°$	[opp ∠s of cyclic quad / *teenoorst ∠e v kvh*]	✓S ✓R ✓S (3)
8.2.2(b)	$\hat{TPL} = 62°$	[ext. ∠ of cyclic quad / *buite ∠ v kvh*]	✓S ✓R (2)
			[15]

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QUESTION/*VRAAG* 9

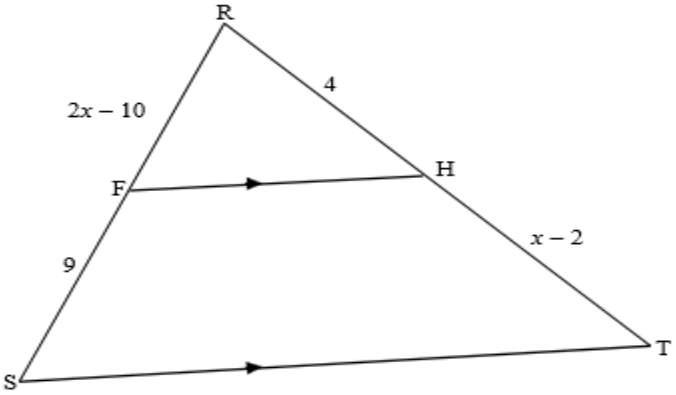
9.1



9.1	Constr: Join KZ and LY and draw h_1 from K ⊥ XL and h_2 from L ⊥ XK *Konstr: Verbind KZ en LY en trek h_1 vanaf* K ⊥ XL *en* h_2 *vanaf* L ⊥ XK Proof / *Bewys*: $\frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\frac{1}{2}XK \times h_1}{\frac{1}{2}KY \times h_1} = \frac{XK}{KY}$ $\frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ} = \frac{\frac{1}{2}XL \times h_2}{\frac{1}{2}LZ \times h_2} = \frac{XL}{LZ}$ area ΔXKL = area ΔXKL [common / *gemeenskaplik*] But area ΔLYK = area ΔKLZ [same base & height ; LK ∥ YZ / *dies basis & hoogte ;* LK ∥ YZ] $\therefore \frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ}$ $\therefore \frac{XK}{KY} = \frac{XL}{LZ}$	✓ constr / *konstr* ✓ $\frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\frac{1}{2}XK \times h_1}{\frac{1}{2}KY \times h_1}$ ✓ S ✓R ✓ S (5)

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

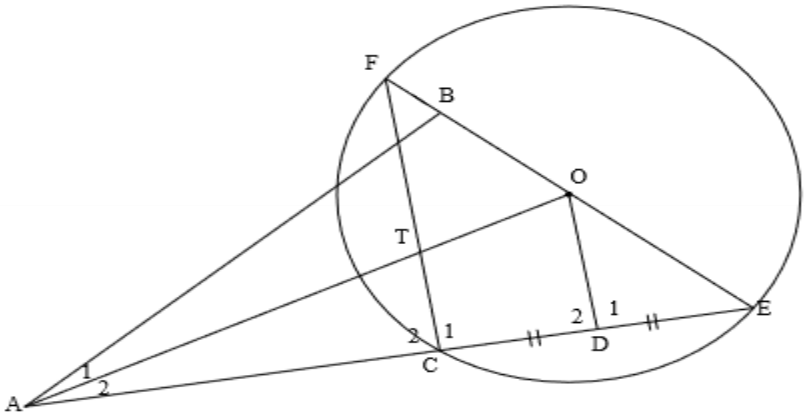
9.2



9.2.1	$\frac{RF}{FS}=\frac{RH}{HT}$ [line ‖ one side of Δ **OR** prop theorem; FH ‖ ST] [*Lyn ‖ een sy van Δ **OF** eweredigh. st;* FH‖ ST] $\frac{2x-10}{9}=\frac{4}{x-2}$ $(2x-10)(x-2)=4\times 9$ $2x^2-14x-16=0$ $x^2-7x-8=0$ $(x-8)(x+1)=0$ $\therefore x=8 \quad (x\neq -1)$ **OR/OF** $\frac{RF}{RS}=\frac{RH}{RT}$ [line ‖ one side of Δ **OR** prop theorem; FH ‖ ST] [*Lyn ‖ een sy van Δ **OF** eweredigh. st;* FH‖ ST] $\frac{2x-10}{2x-1}=\frac{4}{x+2}$ $(2x-10)(x+2)=4(2x-1)$ $2x^2-14x-16=0$ $x^2-7x-8=0$ $(x-8)(x+1)=0$ $\therefore x=8 \quad (x\neq -1)$	✓S/R ✓ substitution ✓ standard form ✓ factors ✓answer with rejection (5) ✓S/R ✓ substitution ✓ standard form ✓ factors ✓answer with rejection (5)
9.2.2	$\frac{\text{area } \Delta RFH}{\text{area } \Delta RST}=\frac{\frac{1}{2}RF\times RH\sin\hat{R}}{\frac{1}{2}RS\times RT\sin\hat{R}}$ $=\frac{\frac{1}{2}\times 6\times 4\times\sin\hat{R}}{\frac{1}{2}\times 15\times 10\times\sin\hat{R}}$ $=\frac{24}{150}=\frac{4}{25}$	✓ numerator/*teller* ✓ denominator/*noemer* ✓ substitution ✓answer (4)
		[14]

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QUESTION/*VRAAG* 10



10.1.1	$\hat{C}_1 = 90°$ [$\angle$ in semi circle / $\angle$ *in halfsirkel*] $\hat{D}_1 = 90°$ [line from centre to midpt of chord / *lyn vanaf midpt na midpt van koord*] $\therefore \hat{C}_1 = \hat{D}_1$ $\therefore$ FC $\parallel$ OD [corresp $\angle$s = / *ooreenkomstige* $\angle e$ =] **OR/OF** FO = OE [radii] CD = DE [given / *gegee*] $\therefore$ FC $\parallel$ OD [midpoint theorem / *middelpuntstelling*]	✓ S ✓ R ✓ S ✓ R ✓ R (5) ✓ S ✓ R ✓ S ✓✓ R (5)
10.1.2	$D\hat{O}E = \hat{F}$ [corresp $\angle$s =; FC $\parallel$ OD] $B\hat{A}E = \hat{F}$ [$\angle$s in the same seg] $\therefore D\hat{O}E = B\hat{A}E$	✓ S ✓ R ✓ S ✓ R (4)
10.1.3	In ΔABE and ΔFCE: $\hat{E}$ is common $B\hat{A}E = \hat{F}$ [proved in 10.1.2] $\therefore A\hat{B}E = \hat{C}_1$ [sum of $\angle$s in Δ] $\therefore \Delta ABE \,|||\, \Delta FCE$ [$\angle\angle\angle$] $\frac{AB}{FC} = \frac{AE}{FE}$ [||| Δs] $AB \times FE = AE \times FC$ But FE = 2 OF [$d = 2r$] And FC =2 OD [midpoint theorem] $AB \times 2OF = AE \times 2OD$ $\therefore AB \times OF = AE \times OD$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ S/R ✓ S (7)

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	OR/*OF* In ΔODE and ΔABE 1. $\hat{E}$ is common 2. $D\hat{O}E = E\hat{A}B$ (proved in 10.1.2) 3. $\hat{D}_1 = A\hat{B}E$ ($\angle$ sum Δ) $\Delta ODE \mid\mid\mid \Delta ABE$ ($\angle\angle\angle$) $\frac{EO}{EA} = \frac{OD}{AB} = \frac{ED}{EB}$ ($\mid\mid\mid$ Δs) $\therefore AB.EO = OD.EA$ but $OE = FO$ (radii) $\therefore AB \times OF = OD \times EA$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ S ✓ R (7)
10.2	$\frac{AT}{TO} = \frac{AC}{CD} = \frac{3}{1}$ [line $\parallel$ one side of Δ **OR** prop theorem; FC $\parallel$ OD] But CD = DE $\frac{AE}{CE} = \frac{5}{2}$ $\therefore AE = \frac{5}{2}CE$ $\frac{BE}{CE} = \frac{AE}{FE}$ [$\mid\mid\mid$ Δs] $\frac{BE}{CE} = \frac{\frac{5}{2}CE}{FE}$ $BE \times FE = \frac{5}{2}CE^2$ $\therefore 5CE^2 = 2BE.FE$	✓ S ✓ R ✓ S ✓ S ✓substitute $AE = \frac{5}{2}CE$ (5)
		[21]

TOTAL/TOTAAL: 150

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

MATHEMATICS P2: JUNE 2018
MARKING GUIDELINES NOTES

QUESTION 1

1.1.1	If left as 190, 25 then penalise 1 mark.
1.1.2	If the position is used: $\left[\frac{1}{4}(n+1)+\frac{3}{4}(n+1)\right]\div 2$ $=\frac{158+219}{2}$ $=\frac{377}{2}$ $=188,5$

QUESTION 2

2.4	Do not accept estimation from the table.

QUESTION 3

3.1	No ca if $\frac{x_2-x_1}{y_2-y_1}$	
3.3	MD^2+AM^2 $=\left[(3-8)^2+(-1+11)^2\right]+\left[(3-7)^2+(-1-1)^2\right]$ $=125+20$ $=145$ AD^2 $=(7-8)^2+(1+11)^2$ $=145$ $MD^2+AM^2=AD^2$	✓ AM^2+MD^2 ✓ AD^2 ✓ $MD^2+AM^2=AD^2$ (3)

QUESTION 4

4.3	Candidates can use the rotation of P through 90° to get to R(6 ; 4) If the candidate assumes that R(4 ; 6) : 1/4 marks

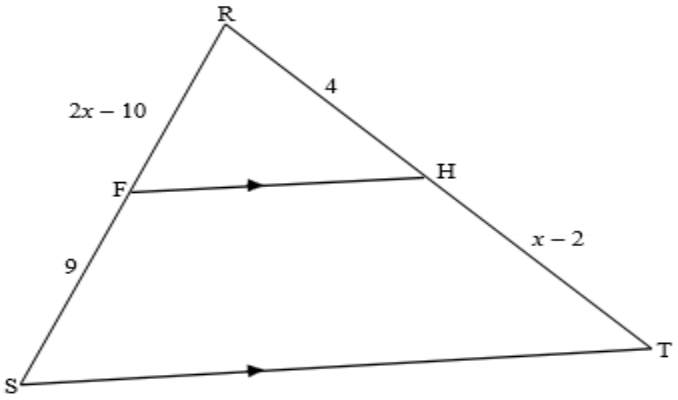
QUESTION 6

6.2	$y\in(-2\ ;\ 2)$	1/2 marks
	$-2<y<2$	1/2 marks

QUESTION 7

7.3	There is NO penalty for incorrect rounding.

QUESTION 9



9.2.2	Join FT. area $\Delta RFH = \frac{4}{10} \times$ (area ΔRFT) But area $\Delta RFT = \frac{6}{15} \times$ (area ΔRST) (common vertex; = heights) area $\Delta RFH = \frac{4}{10} \times \frac{6}{15} \times$(area ΔRST) $\frac{\text{area}\Delta RFH}{\text{area}\Delta RST} = \frac{4}{25}$

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SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2019

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 25 pages.

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QUESTION 1

Each child in a group of four-year-old children was given the same puzzle to complete. The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)	NUMBER OF CHILDREN
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

1.1 How many children completed the puzzle? (1)

1.2 Calculate the estimated mean time taken to complete the puzzle. (2)

1.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency graph (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

1.5 Use the graph to determine the median time taken to complete the puzzle. (2)
[10]

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QUESTION 2

Learners who scored a mark below 50% in a Mathematics test were selected to use a computer-based programme as part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as a percentage) scored by 15 of these learners in both tests is given in the table below.

LEARNER	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15
TEST 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
TEST 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner's mark in the first test was 15 out of a maximum of 50 marks.

2.2.1 Write down the learner's mark for this test as a percentage. (1)

2.2.2 Predict the learner's mark for the second test. Give your answer to the nearest integer. (2)

2.3 For the 15 learners above, the mean mark of the second test is 45,67% and the standard deviation is 13,88%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 50,67%.

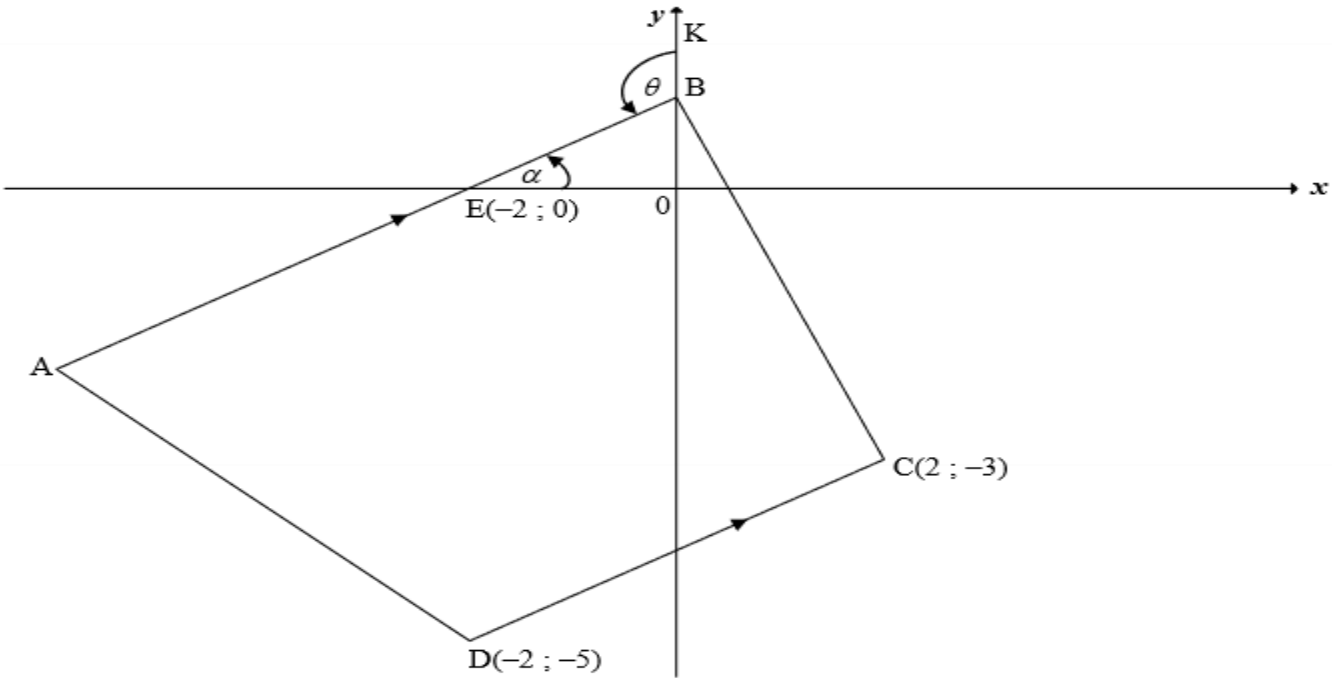
2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)

2.3.2 What is the total mark of the last question? (2)
[10]

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QUESTION 3

In the diagram, A, B, C(2 ; –3) and D(–2 ; –5) are vertices of a trapezium with AB || DC. E(–2 ; 0) is the x-intercept of AB. The inclination of AB is α. K lies on the y-axis and $\hat{KBE} = \theta$.



3.1 Determine:

3.1.1 The midpoint of EC (2)

3.1.2 The gradient of DC (2)

3.1.3 The equation of AB in the form $y = mx + c$ (3)

3.1.4 The size of θ (3)

3.2 Prove that AB $\perp$ BC. (3)

3.3 The points E, B and C lie on the circumference of a circle. Determine:

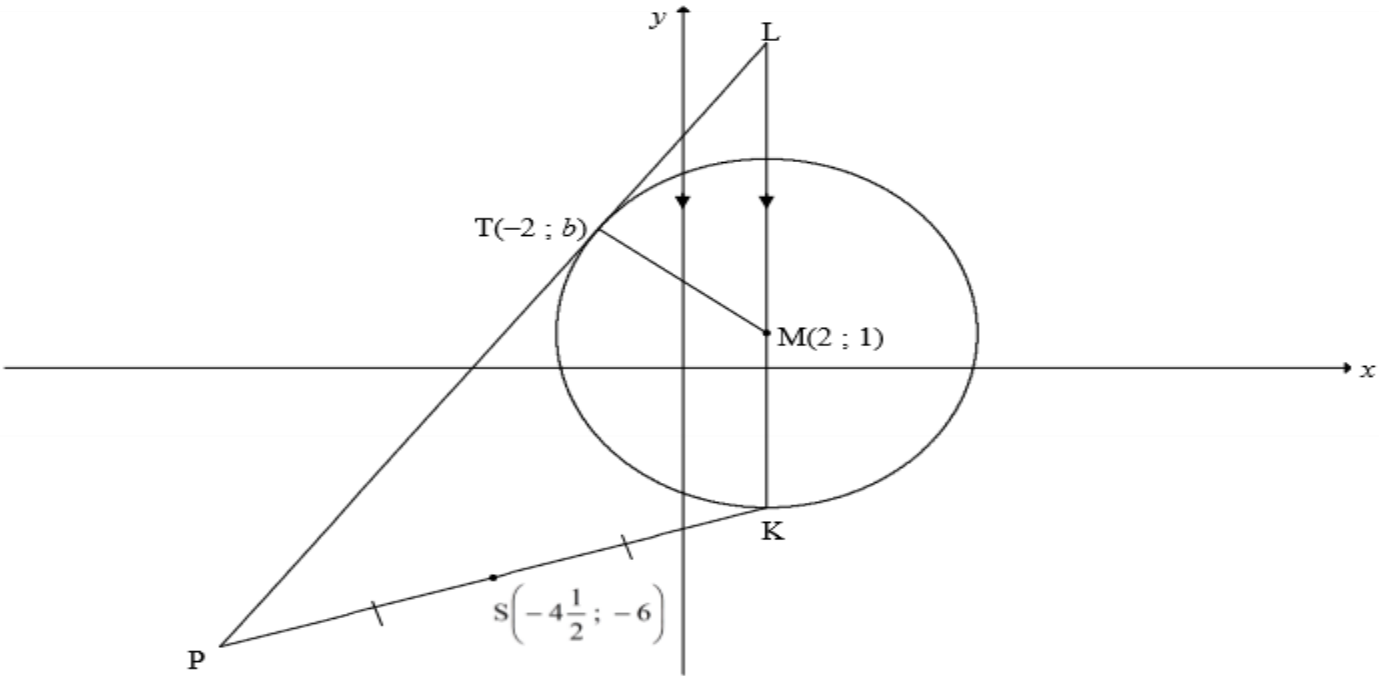
3.3.1 The centre of the circle (1)

3.3.2 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)
[18]

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QUESTION 4

In the diagram, the circle is centred at M(2 ; 1). Radius KM is produced to L, a point outside the circle, such that KML ∥ y-axis. LTP is a tangent to the circle at T(–2 ; b). $S\left(-4\frac{1}{2}\,;\,-6\right)$ is the midpoint of PK.



4.1 Given that the radius of the circle is 5 units, show that $b = 4$. (4)

4.2 Determine:

4.2.1 The coordinates of K (2)

4.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4)

4.2.3 The area of ΔLPK (7)

4.3 Another circle with equation $(x-2)^2 + (y-n)^2 = 25$ is drawn. Determine, with an explanation, the value(s) of n for which the two circles will touch each other externally. (4)
[21]

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QUESTION 5

5.1 **Without using a calculator,** write the following expressions in terms of $\sin 11°$:

5.1.1 $\sin 191°$ (1)

5.1.2 $\cos 22°$ (1)

5.2 Simplify $\cos(x-180°) + \sqrt{2}\sin(x+45°)$ to a single trigonometric ratio. (5)

5.3 Given: $\sin P + \sin Q = \frac{7}{5}$ and $\hat{P} + \hat{Q} = 90°$

Without using a calculator, determine the value of $\sin 2P$. (5)
[12]

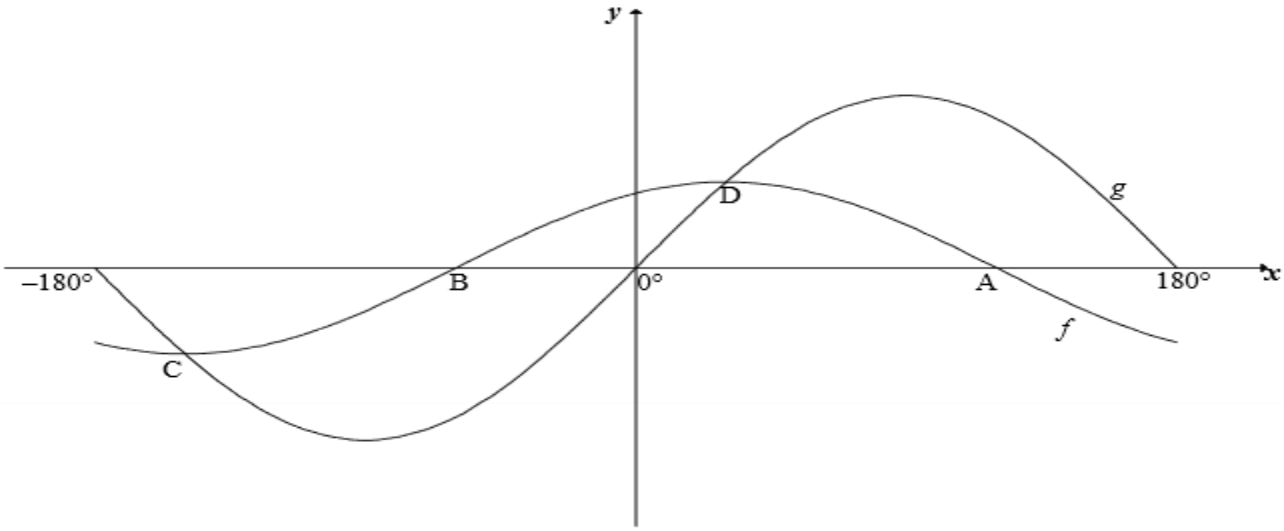
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QUESTION 6

6.1 Determine the general solution of $\cos(x-30°) = 2\sin x$. (6)

6.2 In the diagram, the graphs of $f(x) = \cos(x-30°)$ and $g(x) = 2\sin x$ are drawn for the interval $x \in [-180°; 180°]$. A and B are the x–intercepts of f. The two graphs intersect at C and D, the minimum and maximum turning points respectively of f.



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180°; 180°]$, for which:

(a) Both graphs are increasing (2)

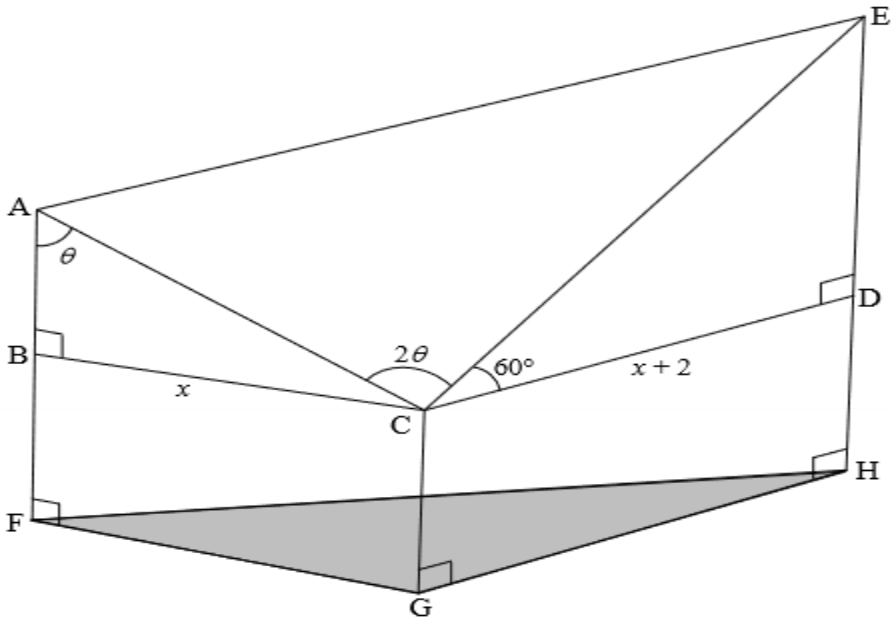
(b) $f(x+10°) > g(x+10°)$ (2)

6.2.3 Determine the range of $y = 2^{2\sin x + 3}$ (5)
[18]

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QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. ΔACE forms the roof of an entertainment centre.
$BC = x$, $CD = x + 2$, $B\hat{A}C = \theta$, $A\hat{C}E = 2\theta$ and $E\hat{C}D = 60°$



7.1 Calculate the length of:

7.1.1 AC in terms of x and θ (2)

7.1.2 CE in terms of x (2)

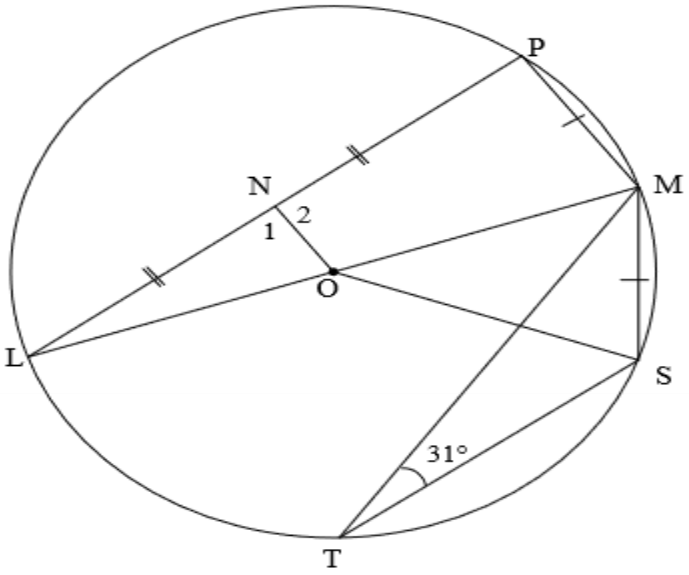
7.2 Show that the area of the roof ΔACE is given by $2x(x+2)\cos\theta$. (3)

7.3 If $\theta = 55°$ and BC = 12 metres, calculate the length of AE. (4)
[11]

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QUESTION 8

8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. PM = MS and $\hat{MTS} = 31°$



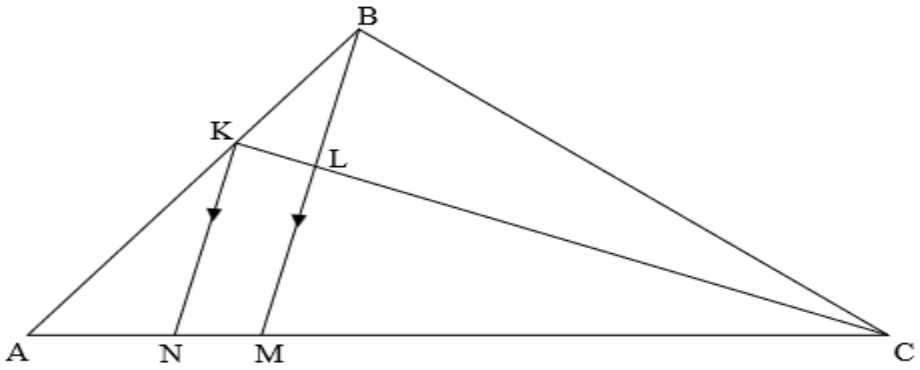
8.1.1 Determine, with reasons, the size of each of the following angles:

(a) $\hat{MOS}$ (2)

(b) $\hat{L}$ (2)

8.1.2 Prove that $ON = \frac{1}{2}MS$. (4)

8.2 In ΔABC in the diagram, K is a point on AB such that AK : KB = 3 : 2. N and M are points on AC such that KN || BM. BM intersects KC at L. AM : MC = 10 : 23.



Determine, with reasons, the ratio of:

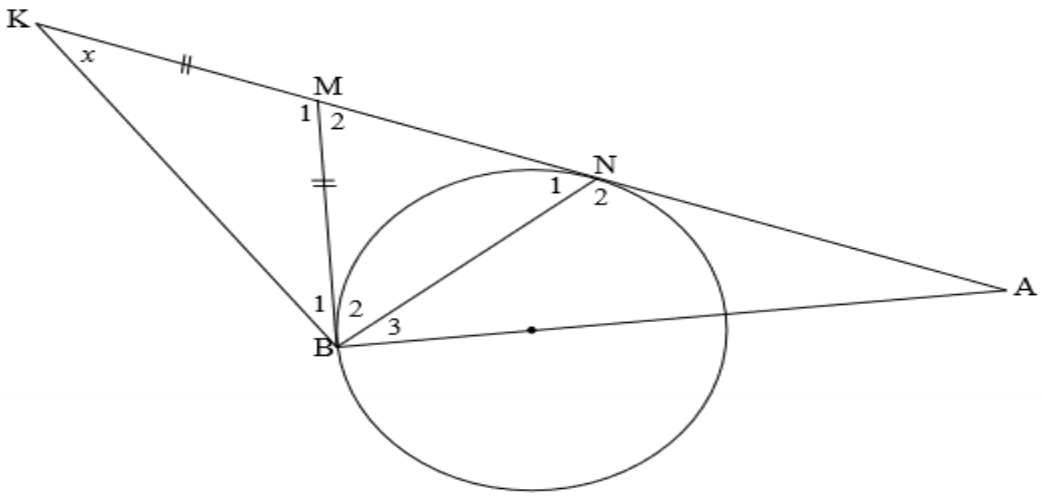
8.2.1 $\frac{AN}{AM}$ (2)

8.2.2 $\frac{CL}{LK}$ (3)

[13]

QUESTION 9

In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N. The straight line from B passing through the centre of the circle meets MN produced in A. NM is produced to K such that BM = MK. BK and BN are drawn.
Let $\hat{K} = x$.

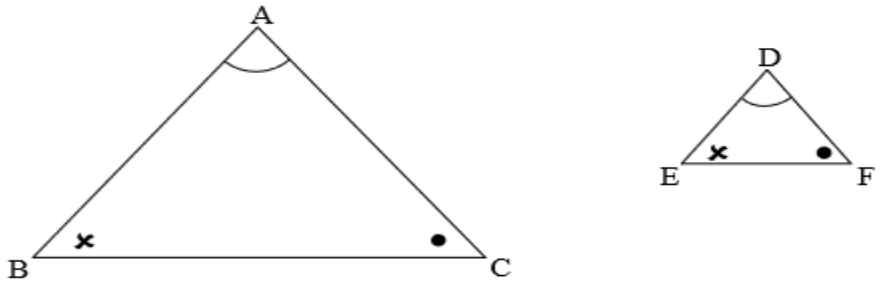


9.1 Determine, with reasons, the size of $\hat{N}_1$ in terms of x. (6)

9.2 Prove that BA is a tangent to the circle passing through K, B and N. (5)
[11]

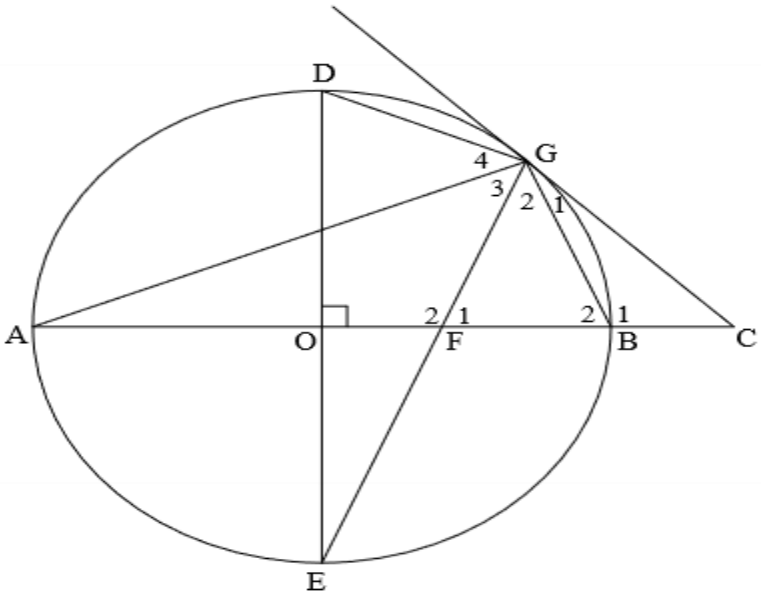
QUESTION 10

10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE}=\frac{AC}{DF}$. (6)

10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G. The straight line from C passing through O cuts the circle at A and B. Diameter DOE is perpendicular to CA. GE and CA intersect at F. Chords DG, BG and AG are drawn.



10.2.1 Prove that:

(a) DGFO is a cyclic quadrilateral (3)

(b) GC = CF (5)

10.2.2 If it is further given that CO = 11 units and DE = 14 units, calculate:

(a) The length of BC (3)

(b) The length of CG (5)

(c) The size of $\hat{E}$. (4)

[26]

TOTAL: 150

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MATHEMATICS P2/
WISKUNDE V2

MARKING GUIDELINES/*NASIENRIGLYNE*

2019

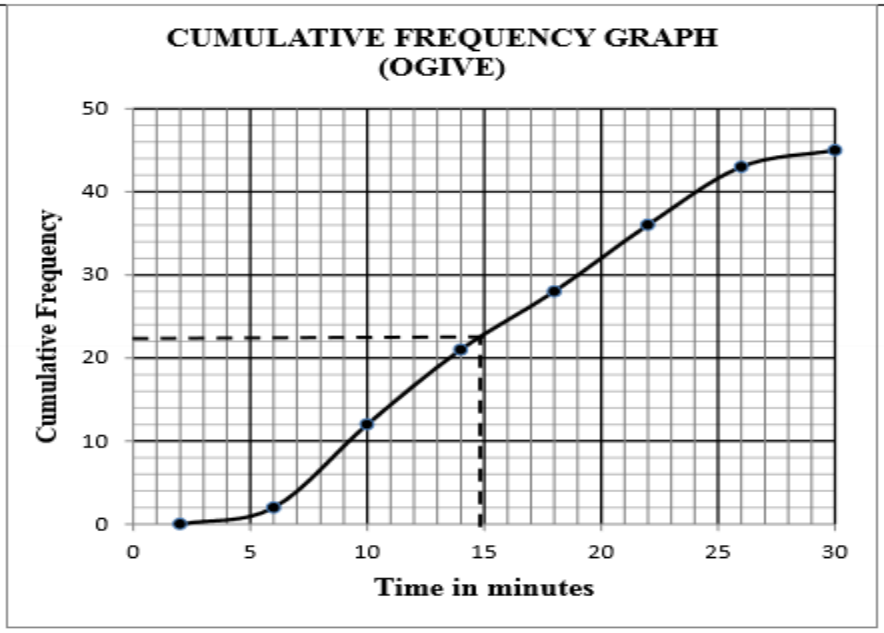
MARKS: 150
PUNTE: 150

These marking guidelines consist of 20 pages.
Hierdie nasienriglyne bestaan uit 20 bladsye..

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QUESTION/*VRAAG* 1

1.1	45 children	✓ answer (1)
1.2	$\bar{x} = \frac{\sum fx}{n} = \frac{(4\times 2)+(8\times 10)+(12\times 9)+(16\times 7)+(20\times 8)+(24\times 7)+(28\times 2)}{45}$ $\bar{x} = \frac{692}{45}$ OR $\bar{x} = 15{,}38$ minutes Answer only: full marks	✓ 692 ✓ answer (2)
1.3		✓ first 4 cum freq correct ✓ last 3 cum freq correct (2)
1.4		✓ plotting cum freq at upper limits correctly (all points) ✓ shape (smooth) ✓ grounding (2;0) (3)
1.5	On graph at the y-value of 22,5 or 23 Median = ± 15 minutes. Answer only: full marks	✓ graph ✓ answer (2)
		[10]

Time taken (t) (in minutes)	Number of children	Cumulative frequency
$2 < t \leq 6$	2	2
$6 < t \leq 10$	10	12
$10 < t \leq 14$	9	21
$14 < t \leq 18$	7	28
$18 < t \leq 22$	8	36
$22 < t \leq 26$	7	43
$26 < t \leq 30$	2	45

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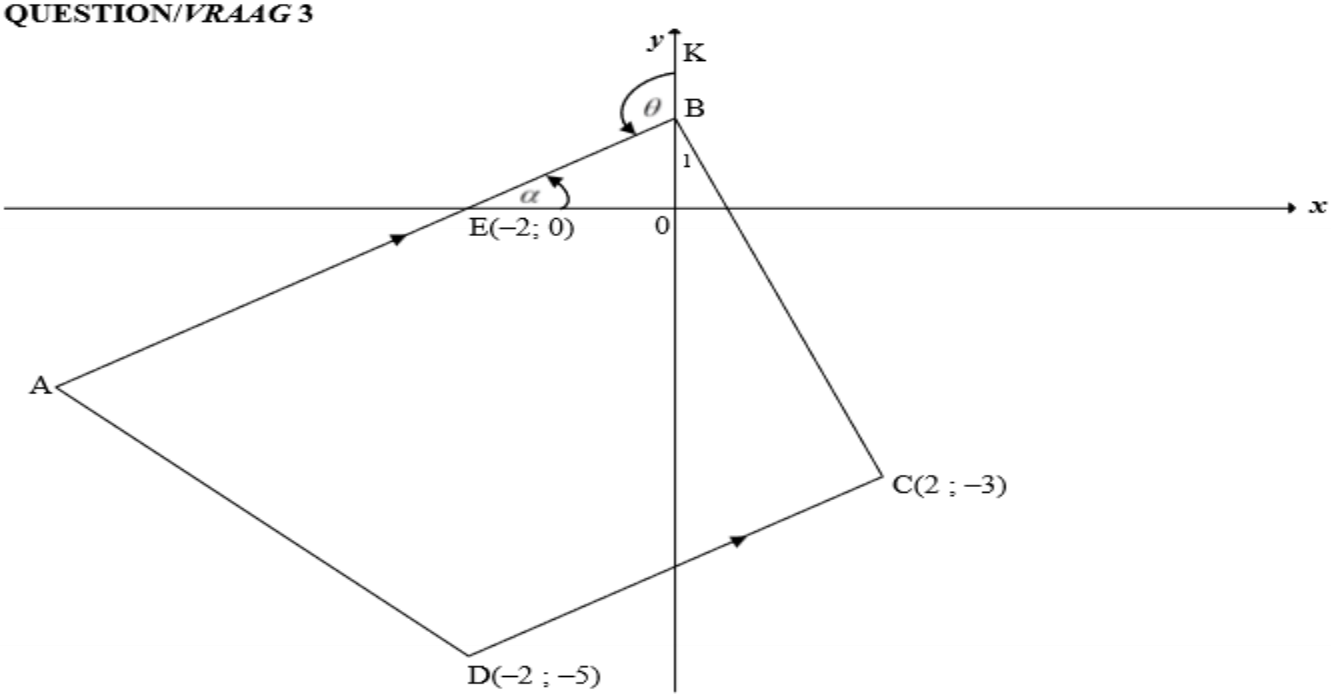
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QUESTION/*VRAAG* 2

2.1	$a = 12{,}44$ $b = 0{,}98$ $y = 12{,}44 + 0{,}98x$ Answer only: full marks	✓ value of a ✓ value of b ✓ equation (3)
2.2.1	Percentage $= \frac{15}{50} \times 100$ $= 30\%$	✓ answer (1)
2.2.2	$\hat{y} = 12{,}44 + 0{,}98x$ $\hat{y} = 12{,}44 + 0{,}98(30)$ $\hat{y} = 41{,}84$ $= 42$ Answer only: full marks **OR** $\hat{y} = 41{,}87$ (if using calculator) $\hat{y} = 42$ **OR** $\hat{y} = \frac{21}{50}$	✓ substitution of 30 ✓ answer as integer (2) ✓ value of y ✓ answer as integer (2) ✓ ✓ answer (2)
2.3.1	standard deviation =13,88	✓ ✓ answer (2)
2.3.2	$x = 50{,}67 - 45{,}67$ $= 5\%$ Answer only: full marks	✓ $50{,}67 - 45{,}67$ ✓ answer (2)
		[10]

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QUESTION/*VRAAG* 3



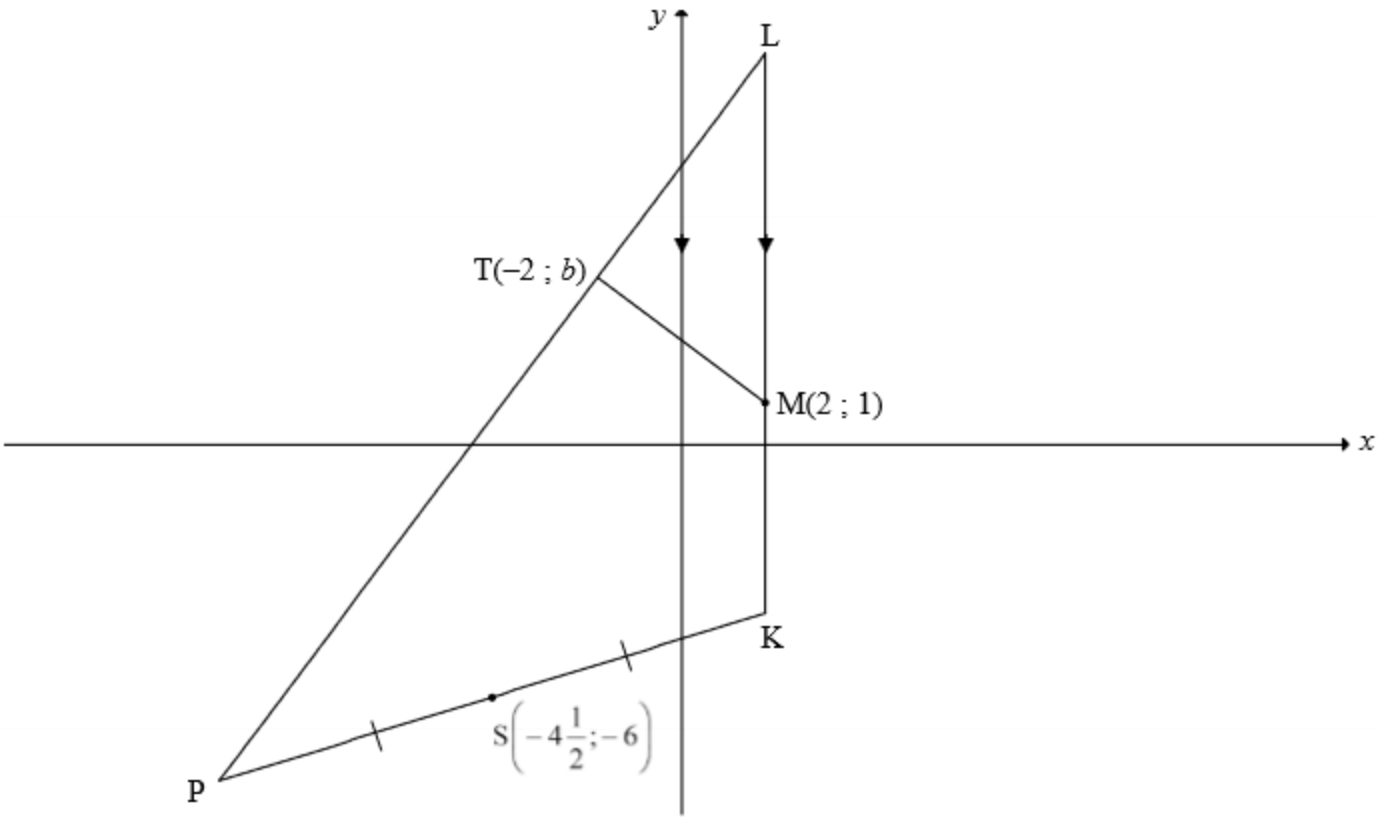
3.1.1	Midpoint of EC: $=\left(\frac{-2+2}{2}\ ;\ \frac{0+(-3)}{2}\right)\ =\left(0\ ;\ \frac{-3}{2}\right)$	✓ x value ✓ y value (2)
3.1.2	$m_{DC}=\frac{-3-(-5)}{2-(-2)}$ OR $\frac{-5-(-3)}{-2-2}$ $=\frac{2}{4}=\frac{1}{2}$ Answer only: full marks	✓ substitution ✓ answer (2)
3.1.3	$m_{AB}=\frac{1}{2}$ [AB ‖ DC] $y=\frac{1}{2}x+c$ OR $y-y_1=\frac{1}{2}(x-x_1)$ $0=\frac{1}{2}(-2)+c$ OR $y-0=\frac{1}{2}(x-(-2))$ $c=1$ $\therefore y=\frac{1}{2}x+1$	✓ $m_{AB}=\frac{1}{2}$ ✓ substitution of $(-2;0)$ ✓ equation (3)
3.1.4	$\tan\alpha=m_{AB}=\frac{1}{2}$ $\alpha=26{,}57°$ $\theta=90°+26{,}57°$ [ext ∠of Δ] $=116{,}57°$	✓ $\tan\alpha=\frac{1}{2}$ ✓ value of α ✓ value of θ (3)

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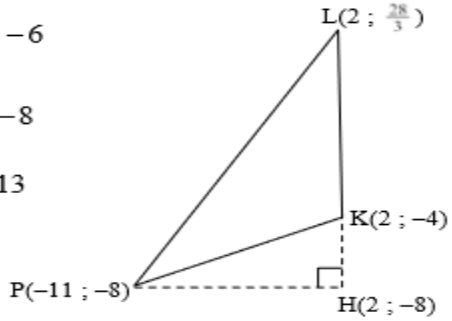
3.2	B(0 ; 1) $m_{BC}=\frac{1-(-3)}{0-2}$ **OR** $m_{BC}=\frac{(-3)-1}{2-0}$ $=-2$ $=-2$ $m_{AB}\times m_{BC}=\frac{1}{2}\times -2$ $=-1$ $\therefore AB\perp BC$	✓ coordinates of B ✓ $m_{BC}=-2$ ✓ product of gradients $=-1$ (3)
3.3.1	$A\hat{B}C=90°$ $\therefore$ EC is diameter [converse: $\angle$ in semi circle] $\therefore$ centre of circle $=\left(0\ ;\ -\frac{3}{2}\right)$	✓ answer (1)
3.3.2	$(x-0)^2+\left(y+\frac{3}{2}\right)^2=r^2$ $(-2-0)^2+\left(0+\frac{3}{2}\right)^2=r^2$ **OR** $(2-0)^2+\left(-3-\left(\frac{-3}{2}\right)\right)^2=r^2$ **OR** $(0-0)^2+\left(1-\left(\frac{-3}{2}\right)\right)^2=r^2$ **OR** $r=\frac{EC}{2}=\frac{\sqrt{(-2-2)^2+(0-(-3))^2}}{2}$ **OR** $r=1-(-\frac{3}{2})$ $\therefore\ r^2=\frac{25}{4}$ or $r=\frac{5}{2}$ $x^2+\left(y+\frac{3}{2}\right)^2=\frac{25}{4}$	✓ substitution of centre ✓ correct substitution of E(–1 ; 0), B(0 ; 1) or C(2 ; –3) to calculate r^2 or r ✓ value of r^2 or r ✓ equation (4)
		[18]

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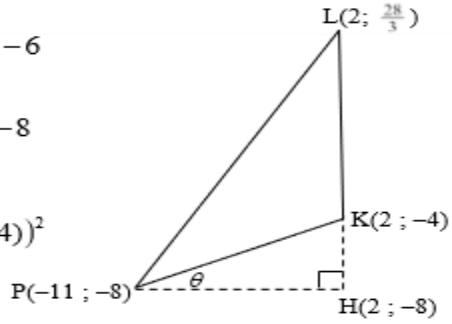
QUESTION/*VRAAG* 4



4.1	$(x-2)^2+(y-1)^2=25$ $(-2-2)^2+(b-1)^2=25$ $(b-1)^2=9$ $b-1=\pm 3$ $\therefore b=4$ or $b\neq -2$ **OF** $(x-2)^2+(y-1)^2=25$ $(-2-2)^2+(b-1)^2=25$ $16+b^2-2b+1=25$ $b^2-2b-8=0$ $\therefore b=4$ or $b\neq -2$	✓ equation of the circle ✓ substitution of point T ✓ simplification ✓ answer (4)
4.2.1	K(2 ; 1 – 5) ∴ K(2 ; –4) Answer only: full marks	✓ x value ✓ y value (2)
4.2.2	$m_{MT}=\frac{4-1}{-2-2}=-\frac{3}{4}$ $m_{PL}=\frac{4}{3}$ [radius ⊥ tangent] $y=\frac{4}{3}x+c$ $4=\frac{4}{3}(-2)+c$ $c=\frac{20}{3}$	✓ m_{MT} ✓ $m_{PL}=\frac{4}{3}$ ✓ substitution of m_{PL} and the point T

	OR $m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3}$ [radius ⊥ tangent] $y - y_1 = \frac{4}{3}(x - x_1)$ $y - 4 = \frac{4}{3}(x + 2)$ $y = \frac{4}{3}x + \frac{20}{3}$ **OR** P(−11 ; −8) $m_{PL} = \frac{4-(-8)}{-2-(-11)}$ $= \frac{4}{3}$ $y = \frac{4}{3}x + c$ $-8 = \frac{4}{3}(-11) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$	✓ m_{MT} ✓ $m_{PL} = \frac{4}{3}$ ✓ substitution of m_{PL} and the point T ✓ equation (4) ✓ coordinates of P ✓ $m_{PL} = \frac{4}{3}$ ✓ substitution of m_{PL} and the point P or T ✓ equation (4)
4.2.3	$y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ $L\left(2 ; \frac{28}{3}\right)$ and $K(2 ; -4)$: $LK = \frac{28}{3} - (-4) = \frac{40}{3}$ Coordinates of P: $\frac{x+2}{2} = -4\frac{1}{2}$ and $\frac{y-4}{2} = -6$ $\therefore x = -11$ $y = -8$ $\therefore$ P(−11; −8) ⊥height (PH) = 2 − (−11) = 13 Area Δ PKL $= \frac{1}{2}(LK)(PH)$ $= \frac{1}{2}\left(\frac{40}{3}\right)(13)$ $= \frac{260}{3}$ **OR** 86,67 square units  	✓ $y_L = \frac{28}{3}$ ✓ length of LK ✓ x_p ✓ y_p ✓ length of ⊥ height ✓ substitution into the area formula ✓ answer (7)

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4.2.3	**OR** $y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ $L\left(2\,;\,\frac{28}{3}\right)$ and $K(2\,;-4)$: $LK = \frac{28}{3} - (-4) = \frac{40}{3}$ <u>Coordinates of P:</u> $\frac{x+2}{2} = -4\frac{1}{2}$ and $\frac{y-4}{2} = -6$ $\therefore\ x = -11 \qquad y = -8$ $\therefore$ P$(-11;-8)$ $PK^2 = (-11-2)^2 + (-8-(-4))^2$ $PK = \sqrt{185}$ units $m_{PK} = \frac{-8-(-4)}{-11-2} = \frac{4}{13}$ $\tan\theta = \frac{4}{13} \quad \therefore \theta = 17{,}1027...°$ $\therefore P\hat{K}L = 90° + 17{,}1027...° = 107{,}1°$ $\text{Area}\,\Delta PKL = \frac{1}{2}(PK)(LK).\sin P\hat{K}L$ $= \frac{1}{2}\left(\sqrt{185}\right)\left(\frac{40}{3}\right)\sin 107{,}10°$ $= 86{,}67$ square units  	✓ $y_L = \frac{28}{3}$ ✓ length of LK ✓ x_p ✓ y_p ✓ $P\hat{K}L$ ✓ substitution into the area rule ✓ answer (7)
4.3	The centres of the two circles lie on the same vertical line $x = 2$. and the sum of the radii = 10 $n - 1 = 10$ or $1 - n = 10$ $n = 11$ $\qquad n = -9$ Answer only: full marks	✓ correct method ✓ sum of radii = 10 ✓ $n = 11$ ✓ $n = -9$ (4)
		[21]

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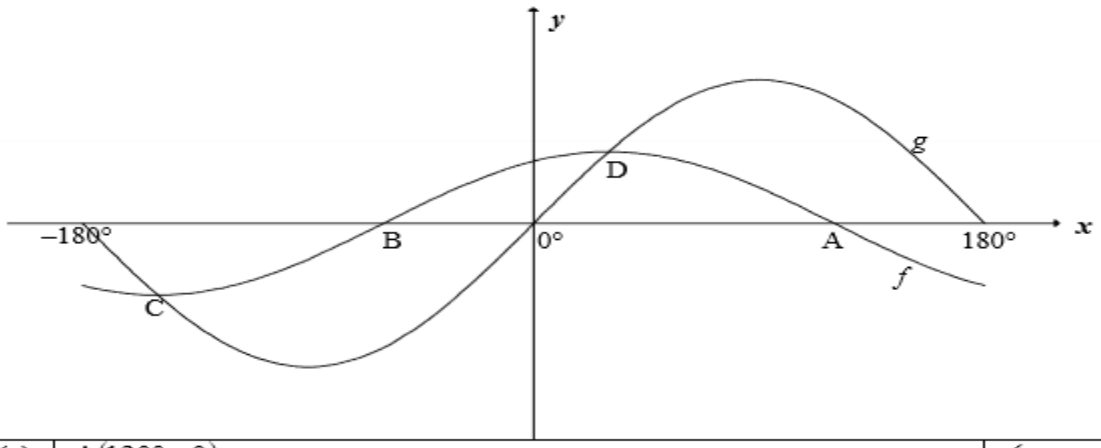
QUESTION/*VRAAG* 5

5.1.1	$\sin 191°$ $=-\sin 11°$	✓ $-\sin 11°$ (1)
5.1.2	$\cos 22°$ $=\cos(2\times 11°)$ $=1-2\sin^2 11°$	✓ answer (1)
5.2	$\cos(x-180°)+\sqrt{2}\sin(x+45°)$ $=-\cos x+\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)$ $=-\cos x+\sqrt{2}\left(\sin x\left(\frac{1}{\sqrt{2}}\right)+\cos x\left(\frac{1}{\sqrt{2}}\right)\right)$ $=-\cos x+\sin x+\cos x$ $=\sin x$ **OR** $\cos(x-180°)+\sqrt{2}\sin(x+45°)$ $=-\cos x+\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)$ $=-\cos x+\sqrt{2}\left(\sin x\left(\frac{\sqrt{2}}{2}\right)+\cos x\left(\frac{\sqrt{2}}{2}\right)\right)$ $=-\cos x+\sin x+\cos x$ $=\sin x$	✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓ simplification of last 2 terms ✓ answer (5) ✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓ simplification of last 2 terms ✓ answer (5)
5.3	$\sin P+\sin Q=\sin P+\cos P$ $(\sin P+\cos P)^2=\left(\frac{7}{5}\right)^2$ $\sin^2 P+2\sin P\cos P+\cos^2 P=\frac{49}{25}$ $2\sin P\cos P=\frac{49}{25}-1$ $\sin 2P=\left(\frac{49}{25}-\frac{25}{25}\right)$ $=\frac{24}{25}$	✓ $\sin Q=\cos P$ ✓ squaring ✓ expansion ✓ $\sin^2 P+\cos^2 P=1$ ✓ answer (5)
		[12]

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QUESTION/*VRAAG* 6

6.1	$\cos(x-30°) = 2\sin x$ $\cos x\cos 30° + \sin x\sin 30° = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x = \frac{3}{2}\sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30° + k.180°; \quad k \in Z$ **OR** $x = 30° + k.360°$ or $x = 210° + k.360°; \quad k \in Z$	✓ expansion ✓ special ∠s ✓ simplification ✓ equation in tan ✓ 30° ✓ $k.180°; k \in Z$ **OR** ✓ 30° and 210° ✓ $k.360°; \quad k \in Z$ (6)

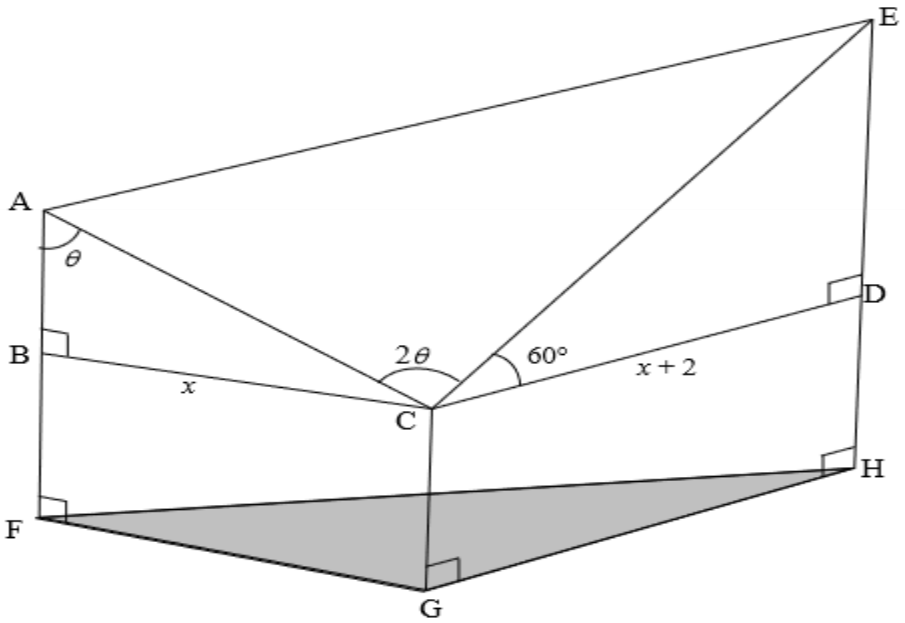


6.2.1(a)	A(120° ; 0)	✓ answer (1)
6.2.1(b)	C(−150° ; −1)	✓ x value ✓ y value (2)
6.2.2(a)	$x \in (-90° ; 30°)$ OR $-90° < x < 30°$	✓ endpoints ✓ correct interval (2)
6.2.2(b)	$x \in (-160° ; 20°)$ OR $-160° < x < 20°$	✓endpoints ✓correct interval (2)
6.2.3	$y = 2^{2\sin x+3}$ Range of $y = 2\sin x$: $y \in [-2 ; 2]$ **OR** $-2 \le y \le 2$ Range of $y = 2\sin x + 3$: $y \in [1 ; 5]$ **OR** $1 \le y \le 5$ Range: $y = 2^{2\sin x+3}$: $y \in [2 ; 32]$ **OR** $2 \le y \le 32$ Answer only: full marks	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)
		[18]

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QUESTION/*VRAAG* 7



7.1.1	$\sin\theta=\frac{x}{AC}$ **OR** $\frac{\sin\theta}{x}=\frac{\sin 90°}{AC}$ $AC=\frac{x}{\sin\theta}$ $AC=\frac{x}{\sin\theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60°=\frac{x+2}{CE}$ **OR** $\frac{\sin 30}{x+2}=\frac{\sin 90°}{CE}$ $CE=\frac{x+2}{\cos 60°}$ $CE=\frac{x+2}{\sin 30°}$ $=\frac{x+2}{\frac{1}{2}}=2(x+2)$ $=2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	Area $\Delta ACE=\frac{1}{2}AC.EC.\sin A\hat{C}E$ $=\frac{1}{2}\left(\frac{x}{\sin\theta}\right)(2(x+2))\sin 2\theta$ $=\frac{x(x+2)\times 2\sin\theta\cos\theta}{\sin\theta}$ $=2x(x+2)\cos\theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin\theta}(2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

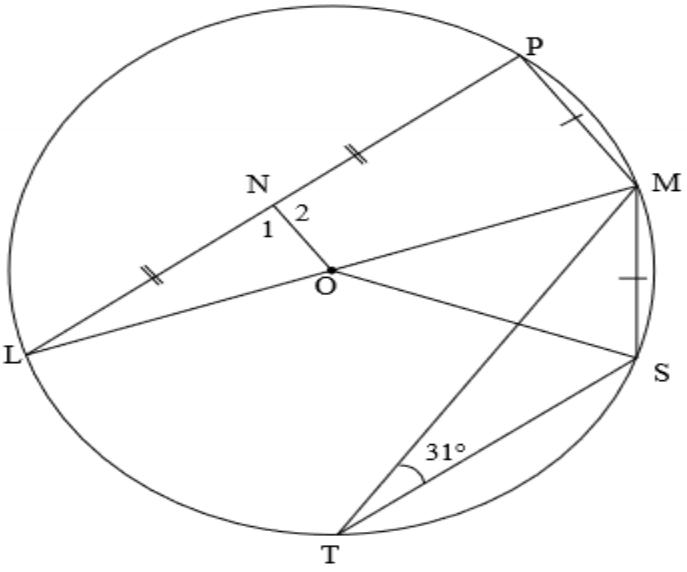
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7.3	$EC = 2(12 + 2) = 28$ $AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos A\hat{C}E$ $= \left(\frac{12}{\sin 55°}\right)^2 + 28^2 - 2\left(\frac{12}{\sin 55°}\right)(28)\cos 110°$ $AE = 35{,}77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		[11]

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QUESTION/*VRAAG* 8

8.1

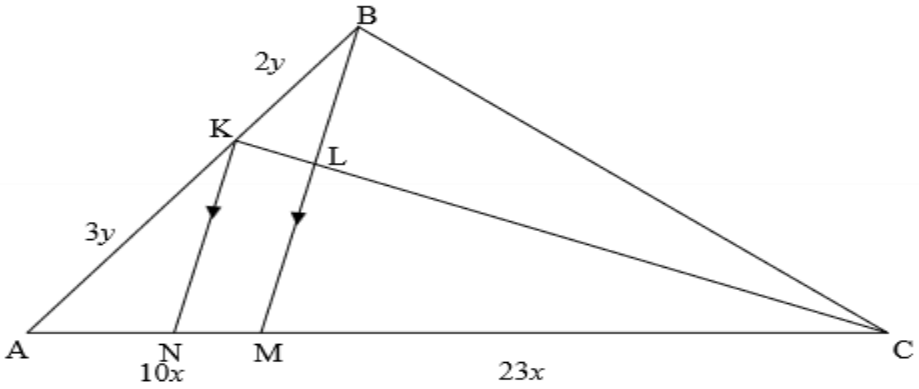


8.1.1(a)	$M\hat{O}S = 62°$ [∠ at centre = 2× ∠ at circumf/*middelpts∠ =2omtreks∠*]	✓ S ✓ R (2)
8.1.1(b)	$\hat{L} = 31°$ [equal chords; equal ∠s /= *koorde; = ∠e*]	✓ S ✓ R (2)
8.1.2	LN = NP and LO = OM $\therefore ON = \frac{1}{2}PM$ [midpoint theorem/*middelpuntstelling*] $\therefore ON = \frac{1}{2}MS$ [PM = MS] **OR** $\hat{N}_1 = 90°$ [line from centre to midpt chord/*lyn v midpt na midpt kd*] $\hat{P} = 90°$ [∠ in semi-circle/*∠ in halfsirkel*] $\hat{L}$ is common/*gemeen* $\therefore \Delta NLO \,\|\|\, \Delta PLM$ (∠∠∠) $\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}$ $\therefore ON = \frac{1}{2}PM$ $\therefore ON = \frac{1}{2}MS$ [PM = MS]	✓ LO = OM ✓ S ✓ R ✓ S (4) ✓ S R ✓ S/R ✓ S ✓ S (4)

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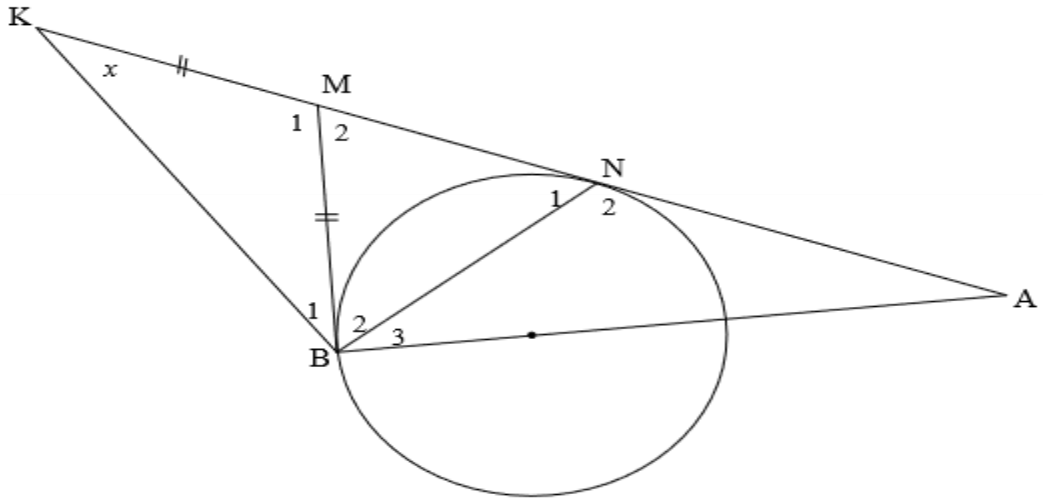
8.2



8.2.1	$\frac{AN}{AM}=\frac{AK}{AB}$ [line ‖ one side of Δ **OR** prop theorem; KN ‖BM/ *lyn ‖ sy van Δ **OR** eweredigheidst; KN‖BM*]	✓ R
	$\frac{AN}{AM}=\frac{3y}{5y}=\frac{3}{5}$	✓ S (2)
8.2.2	$\frac{AM}{MC}=\frac{10x}{23x}$ [given] $AM = 5y = 10x \quad \therefore \quad y=2x$	✓ S
	$\frac{LC}{KL}=\frac{MC}{NM}$ [line ‖ one side of Δ **OR** prop theorem; KN ‖LM/ *lyn ‖ sy van Δ **OR** eweredigheidst; KN‖BM*]	✓ R
	$=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$	✓ S (3)
	OR	
	$\frac{AM}{MC}=\frac{10x}{23x}$ [given]	
	$\frac{AN}{MN}=\frac{3y}{2y}=\frac{6x}{4x}$	✓ S
	$\frac{LC}{KL}=\frac{MC}{NM}$ [line ‖ one side of Δ **OR** prop theorem; KN ‖LM/ *lyn ‖ sy van Δ **OR** eweredigheidst; KN‖BM*]	✓ R
	$=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$	✓ S (3)
		[13]

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QUESTION/*VRAAG* 9

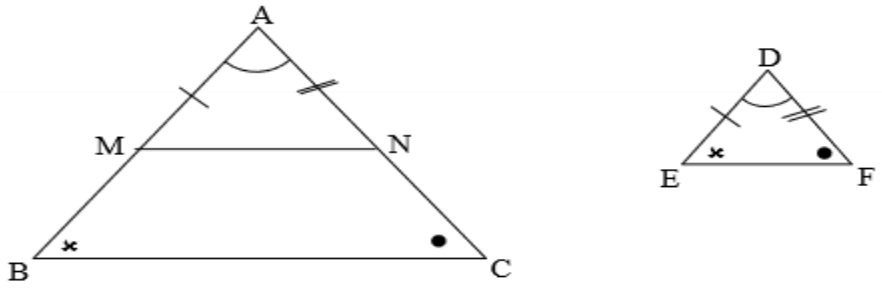


9.1	$\hat{B}_1 = x$ [∠'s opp = sides/*∠e teenoor = sye*] $\hat{M}_2 = 2x$ [ext ∠ of Δ] **OR** $\hat{M}_1 = 180° - 2x$ [∠s of Δ] BM = MN [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{N}_1 = \frac{180° - 2x}{2} = 90° - x$ [∠'s opp = sides/*∠e teenoor = sye*] **OR** NM = BM [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{B}_2 = \hat{N}_1$ [∠'s opp = sides/*∠e teenoor = sye*] $\hat{B}_1 = x$ [∠'s opp = sides/*∠e teenoor = sye*] In Δ KBN: $x + x + \hat{B}_2 + \hat{N}_1 = 180°$ [sum of ∠'s of Δ] $2x + 2\hat{N}_1 = 180°$ $x + \hat{N}_1 = 90°$ $\hat{N}_1 = 90° - x$	✓S ✓S ✓R ✓S ✓R ✓answer (6) ✓S ✓R ✓S ✓R ✓S ✓answer (6)
9.2	$M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent⊥diameter/*raaklyn⊥middellyn*] $\hat{B}_3 = 90° - \hat{B}_2$ $= 90° - (90° - x) = x$ $\hat{B}_3 = \hat{K} = x$ ∴ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ S ✓ R (5)

	OR $\hat{B}_2 = \hat{N}_1$ $\hat{B}_1 + \hat{B}_2 = x + (90° - x) = 90°$ $\therefore$ KN is diameter/*middellyn* [converse $\angle$ in semi-circle/ *omgekeerde $\angle$ in halfsirkel*] $M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent $\perp$ diameter] $\therefore$ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ R ✓ R (5)
		[11]

QUESTION/*VRAAG* 10

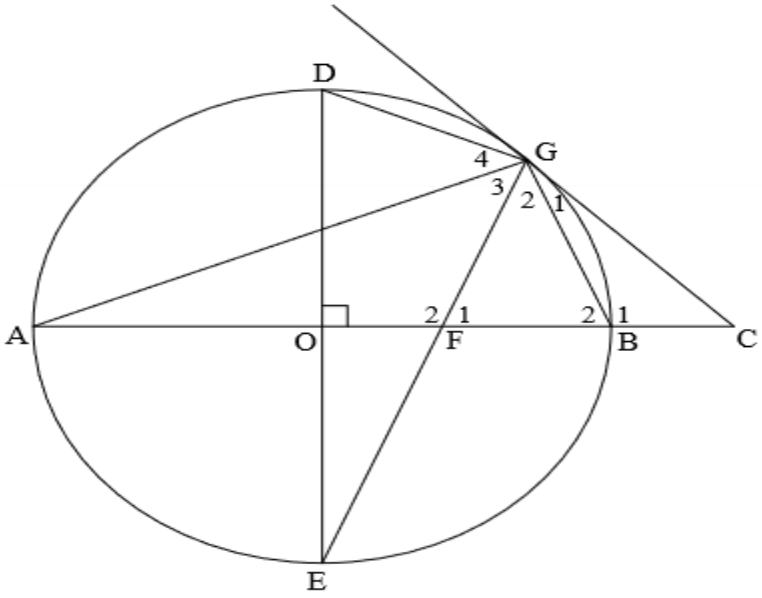
10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. *Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.* Proof: In Δ AMN and Δ DEF AM = DE [Constr] AN = DF [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore \Delta AMN \equiv \Delta DEF$ (SAS) $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ‖ BC [corresp ∠'s are equal/*ooreenkomstige ∠e* =] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ‖ one side of Δ **OR** prop theorem; MN ‖BC] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM = DE and AN = DF]	✓Constr / *Konstr* ✓ $\Delta AMN \equiv \Delta DEF$ ✓SAS ✓MN ‖ BC and R ✓ $\frac{AB}{AM} = \frac{AC}{AN}$ ✓R (6)

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10.2



10.2.1(a)	$D\hat{O}B = 90°$ $D\hat{G}F = \hat{G}_3 + \hat{G}_4 = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $D\hat{O}B + D\hat{G}F = 180°$ ∴ DGFO is a cyclic quad. [converse: opp ∠s of cyclic quad/ *omgekeerde teenoorst ∠e v koordevh*] OR ∠s of quad = 180°/*∠e van koordevh = 180°*] **OR** $E\hat{O}B = 90°$ $D\hat{G}F = \hat{G}_3 + \hat{G}_4 = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $E\hat{O}B = D\hat{G}F$ ∴ DGFO is a cyclic quad. [converse: ext ∠ = opp int ∠/ *omgekeerde buite∠ = teenoorst ∠*] OR ext∠ of quad = opp int ∠/*buite∠ v vh = teenoorst ∠*]	✓ S ✓R ✓ R (3) ✓ S ✓R ✓ R (3)
10.2.1(b)	$\hat{F}_1 = \hat{D}$ [ext ∠of cyclic quad/buite∠ v koordevh] $\hat{G}_1 + \hat{G}_2 = \hat{D}$ [tan-chord theorem/raakl koordst] ∴ $\hat{F}_1 = \hat{G}_1 + \hat{G}_2$ ∴ GC = CF [sides opp equal ∠s/sye teenoor = ∠e]	✓ S ✓ R ✓ S ✓ R ✓ R (5)

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10.2.2(a)	$AB = DE = 14$ [diameters/*middellyne*] $\therefore OB = 7$ units $\therefore BC = OC - OB = 11 - 7$ $= 4$ units Answer only: full marks	✓ S ✓ S ✓ S (3)
10.2.2(b)	In ΔCGB and ΔCAG $\hat{G}_1 = \hat{A} = x$ [tan-chord theorem/*raakl koordst*] $\hat{C} = \hat{C}$ [common] $\Delta CGB \ ||| \ \Delta CAG$ [$\angle, \angle, \angle$] $\frac{CG}{CA} = \frac{CB}{CG}$ $\frac{CG}{18} = \frac{4}{CG}$ $CG^2 = 72$ $CG = \sqrt{72}$ or $6\sqrt{2}$ *or* 8,49 units	✓ S/R ✓ S ✓ S ✓ CA = 18 ✓answer (5)
10.2.2(c)	$OF = OC - FC$ $= 11 - \sqrt{72}$ $\tan E = \frac{OF}{OE}$ $= \frac{11-\sqrt{72}}{7} = 0,36$ $\hat{E} = 19,76°$ **OR** $OF = OC - FC$ $= 11 - \sqrt{72}$ $FE^2 = OE^2 + OF^2$ $= 7^2 + (11-\sqrt{72})^2$ $FE = 7,437.. = 7,44$ $\cos E = \frac{OE}{FE}$ *OR* $\sin E = \frac{OF}{FE}$ $= \frac{7}{7,44} = 0,94$ $= \frac{11-\sqrt{72}}{7,44} = 0,338$ $\hat{E} = 19,76°$ $\hat{E} = 19,76°$	✓ OF ✓ trig ratio ✓ substitution ✓ answer (4) ✓ OF ✓ trig ratio ✓ substitution ✓ answer (4)
		[26]

	TOTAL/*TOTAAL*:	**150**

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2021

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages pages, 1 information sheet and an answer book of 24 pages.

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QUESTION 1

1.1 Sam recorded the amount of data (in MB) that she had used on each of the first 15 days in April. The information is shown in the table below.

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

1.1.1 Calculate the:

(a) Mean for the data set (2)

(b) Standard deviation for the data set (1)

1.1.2 Determine the number of days on which the amount of data used was greater than one standard deviation above the mean. (2)

1.1.3 Calculate the maximum total amount of data that Sam must use for the remainder of the month if she wishes for the overall mean of April to be 80% of the mean for the first 15 days. (3)

1.2 The wind speed (in km per hour) and temperature (in °C) for a certain town were recorded at 16:00 for a period of 10 days. The information is shown in the table below.

WIND SPEED IN km/h (x)	2	6	15	20	25	17	11	24	13	22
TEMPERATURE IN °C (y)	28	26	22	22	16	20	24	19	26	19

1.2.1 Determine the equation of the least squares regression line for the data. (3)

1.2.2 Predict the temperature at 16:00 if, on a certain day, the wind speed of this town was 9 km per hour. (2)

1.2.3 Interpret the value of b in the context of the data. (1)
[14]

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QUESTION 2

The number of days that Grade 8 learners were absent at a certain high school during a year was recorded. This information is represented in the table below.

NUMBER OF DAYS ABSENT	NUMBER OF LEARNERS
$0 \leq x < 5$	34
$5 \leq x < 10$	45
$10 \leq x < 15$	98
$15 \leq x < 20$	43
$20 \leq x < 25$	7
$25 \leq x < 30$	3

2.1 Write down the modal class for the data. (1)

2.2 How many learners were absent from school for less than 15 days? (1)

2.3 How many Grade 8 learners are at the school? (1)

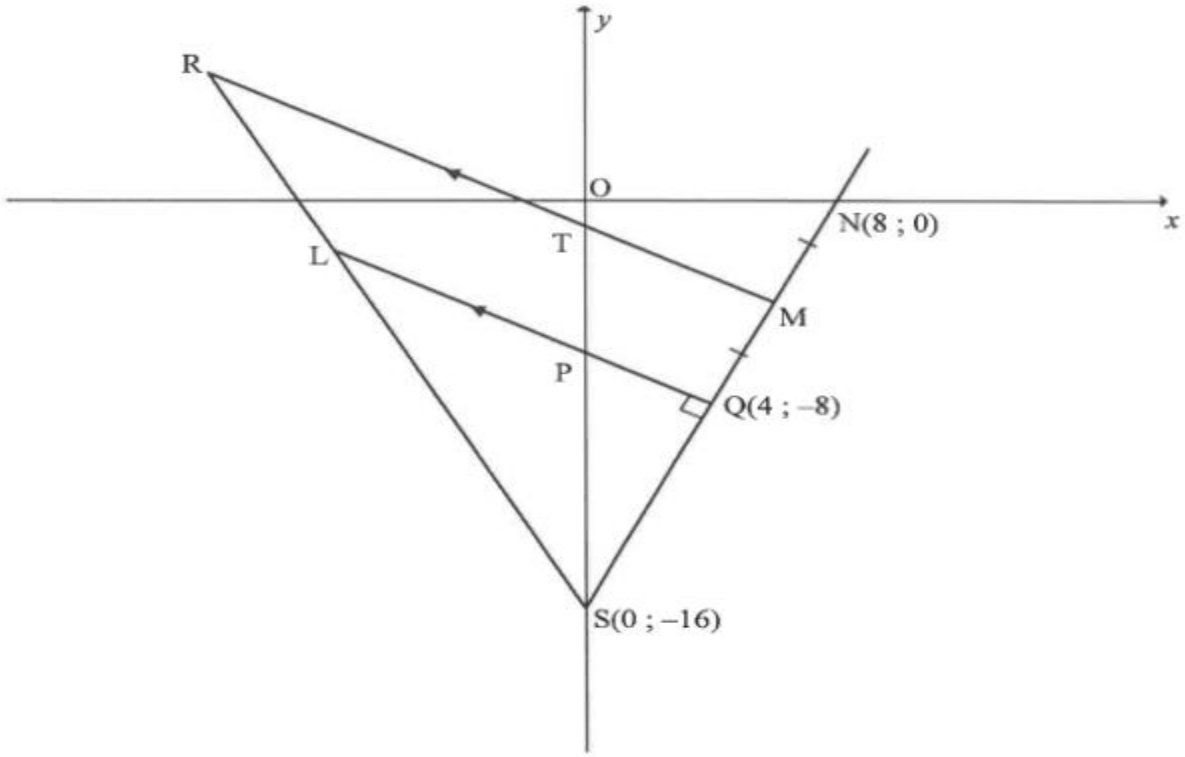
2.4 Draw a cumulative frequency graph (ogive) to represent the data above on the grid provided in the ANSWER BOOK. (4)

2.5 Use the cumulative frequency graph to determine the median number of days the Grade 8 learners were absent. (2)
[9]

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QUESTION 3

In the diagram, S(0 ; –16), L and Q(4 ; –8) are the vertices of ΔSLQ having LQ perpendicular to SQ. SL and SQ are produced to points R and M respectively such that RM ∥ LQ. SM produced cuts the x-axis at N(8 ; 0). QM = MN. T and P are the y-intercepts of RM and LQ respectively.



3.1 Calculate the coordinates of M. (2)

3.2 Calculate the gradient of NS. (2)

3.3 Show that the equation of line LQ is $y=-\frac{1}{2}x-6$. (3)

3.4 Determine the equation of a circle having centre at O, the origin, and also passing through S. (2)

3.5 Calculate the coordinates of T. (3)

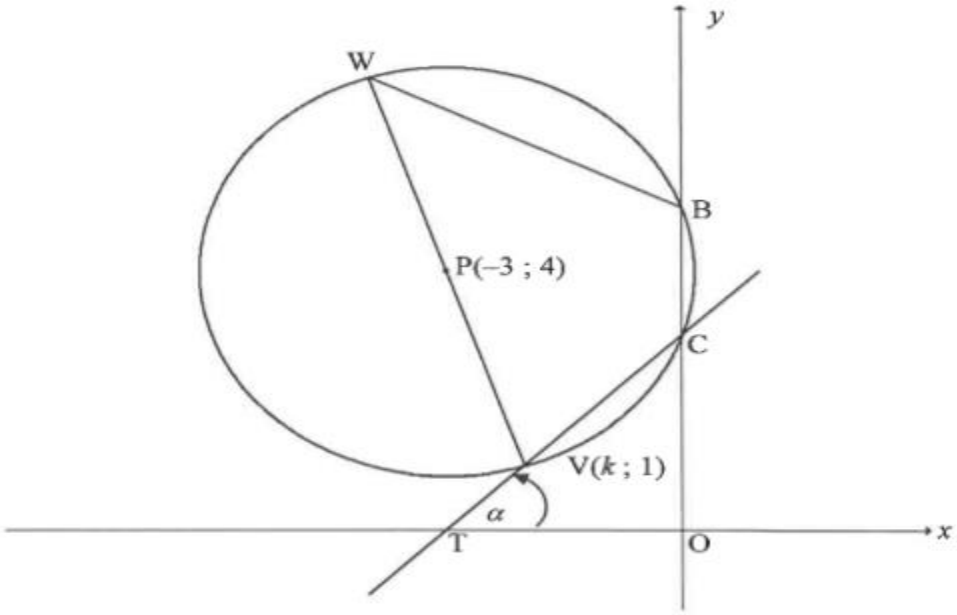
3.6 Determine $\frac{LS}{RS}$. (3)

3.7 Calculate the area of PTMQ. (4)
[19]

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QUESTION 4

In the diagram, P(–3 ; 4) is the centre of the circle. V(k ; 1) and W are the endpoints of a diameter. The circle intersects the y-axis at B and C. BCVW is a cyclic quadrilateral. CV is produced to intersect the x-axis at T. $O\hat{T}C = \alpha$.



4.1 The radius of the circle is $\sqrt{10}$. Calculate the value of k if point V is to the right of point P. Clearly show ALL calculations. (5)

4.2 The equation of the circle is given as $x^2 + 6x + y^2 - 8y + 15 = 0$. Calculate the length of BC. (4)

4.3 If $k = -2$, calculate the size of:

4.3.1 α (3)

4.3.2 $V\hat{W}B$ (2)

4.4 A new circle is obtained when the given circle is reflected about the line $y = 1$.

Determine the:

4.4.1 Coordinates of Q, the centre of the new circle (2)

4.4.2 Equation of the new circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4.3 Equations of the lines drawn parallel to the y-axis and passing through the points of intersection of the two circles (2)

[20]

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QUESTION 5

5.1 Simplify the expression to a **single trigonometric term:**

$$\tan(-x).\cos x.\sin(x-180°)-1$$ (5)

5.2 Given: $\cos 35° = m$

Without using a calculator, determine the value of EACH of the following in terms of m:

5.2.1 $\cos 215°$ (2)

5.2.2 $\sin 20°$ (3)

5.3 Determine the general solution of:

$$\cos 4x.\cos x + \sin x.\sin 4x = -0,7$$ (4)

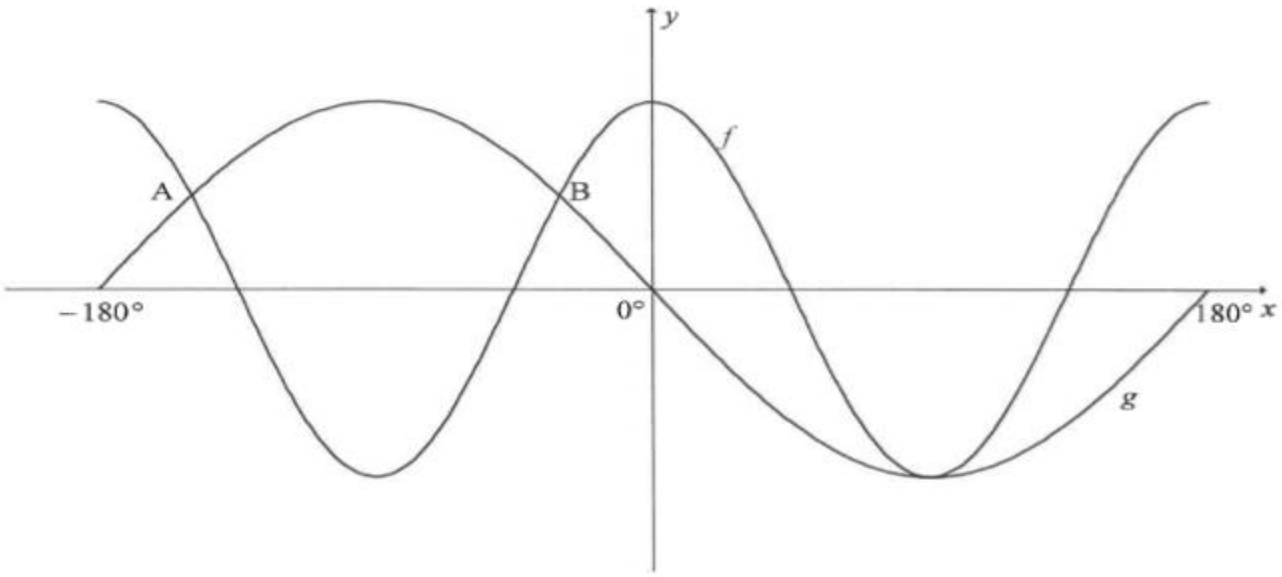
5.4 Prove the identity: $\dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x} = \cos^2 x - \sin^2 x$ (4)

[18]

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QUESTION 6

In the diagram below, the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x$ are drawn for the interval $x \in [-180° ; 180°]$ A and B are two points of intersection of f and g.



6.1 **Without using a calculator**, determine the values of x for which $\cos 2x = -\sin x$ in the interval $x \in [-180° ; 180°]$. (6)

6.2 Use the graphs above to answer the following questions:

6.2.1 How many degrees apart are points A and B from each other? (2)

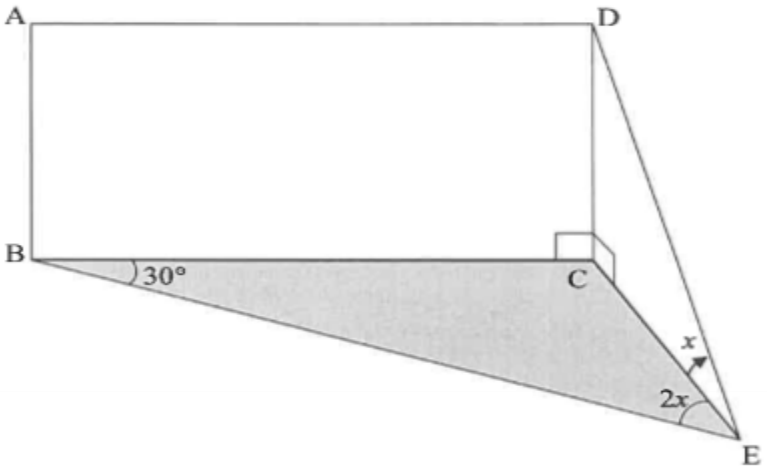
6.2.2 For which values of x in the given interval will $f'(x).g'(x) > 0$? (2)

6.2.3 Determine the values of k for which $\cos 2x + 3 = k$ will have no solution. (3)
[13]

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QUESTION 7

Points B, C and E lie in the same horizontal plane. ABCD is a rectangular piece of board. CDE is a triangular piece of board having a right angle at C. Each piece of board is placed perpendicular to the horizontal plane and joined along DC, as shown in the diagram. The angle of elevation from E to D is x. $B\hat{E}C = 2x$ and $E\hat{B}C = 30°$.

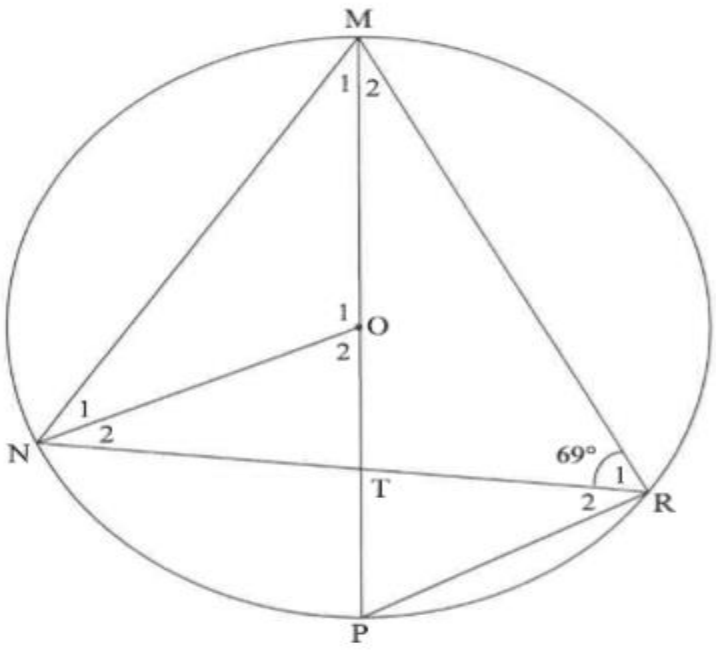


7.1 Show that $DC = \dfrac{BC}{4\cos^2 x}$ (6)

7.2 If $x = 30°$, show that the area of $ABCD = 3AB^2$. (3)
[9]

QUESTION 8

8.1 In the diagram, MP is a diameter of a circle centered at O. MP cuts the chord NR at T. Radius NO and chords PR, MN and MR are drawn. $\hat{R}_1 = 69°$.



Determine, giving reasons, the size of:

8.1.1 $\hat{R}_2$ (2)

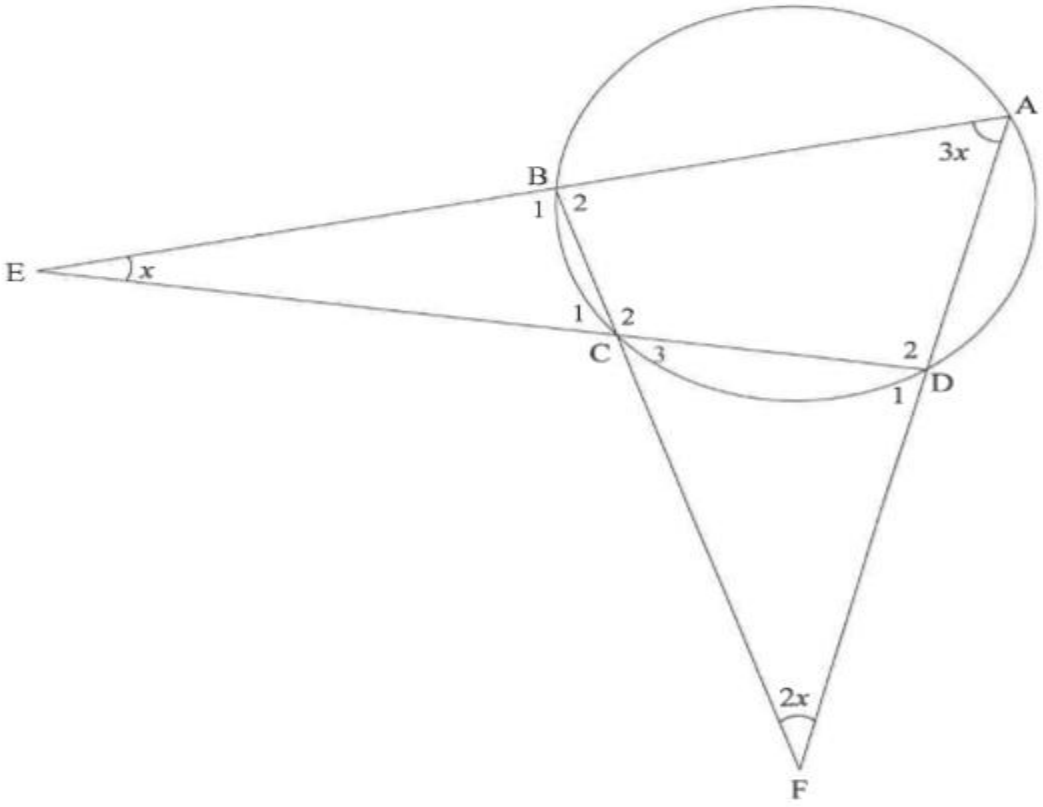
8.1.2 $\hat{O}_1$ (2)

8.1.3 $\hat{M}_1$ (2)

8.1.4 $\hat{M}_2$, if it is further given that NO || PR (4)

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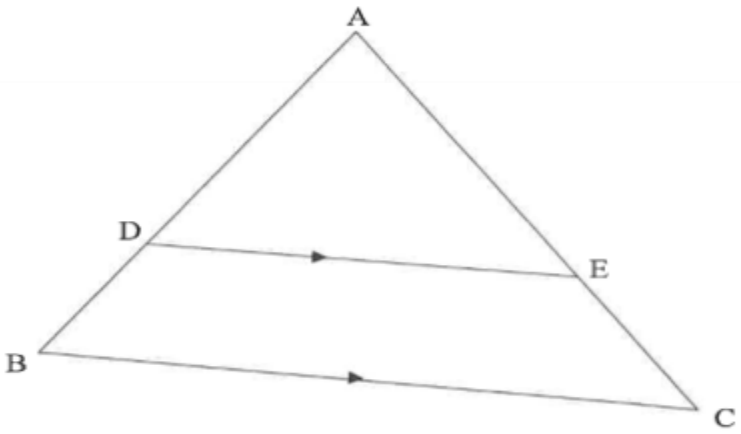
8.2 In the diagram below, ABCD is a cyclic quadrilateral. AB and DC are produced to meet at E. AD and BC are produced to meet at F. $A\hat{F}B = 2x$, $D\hat{A}B = 3x$ and $A\hat{E}D = x$.



Determine, giving reasons, the value of x. (6)
[16]

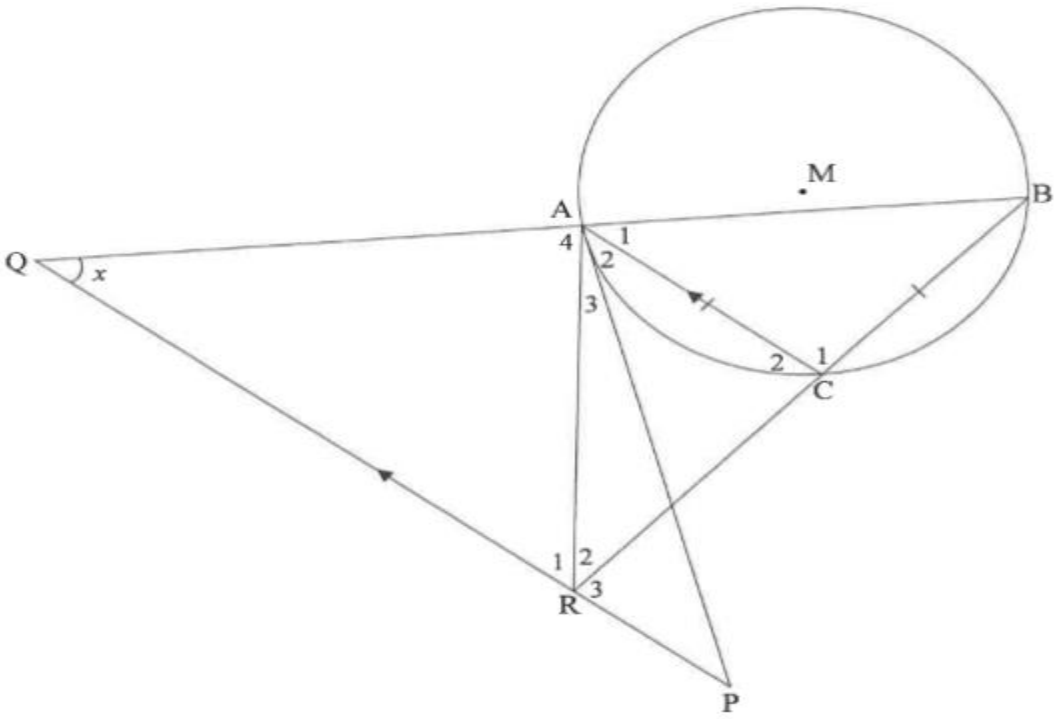
QUESTION 9

9.1 In the diagram, ABC is a triangle. D and E are points on sides AB and AC respectively such that DE ∥ BC .



Use the diagram above to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. prove that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)

9.2 In the diagram, M is the centre of the circle. A, B and C are points on the circle such that AC = BC. PA is a tangent to the circle at A. PQ is drawn parallel to CA to meet BA produced at Q. BC produced meets PQ at R and AR is drawn.
Let $\hat{Q} = x$.



9.2.1 Determine, giving reasons, FOUR other angles EACH equal to x. (6)

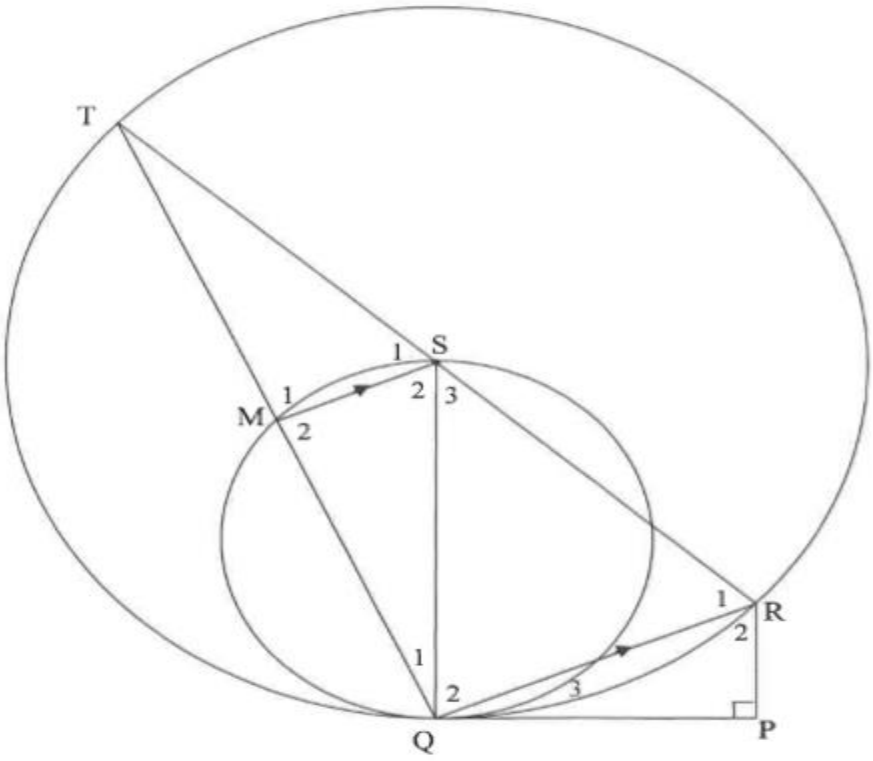
9.2.2 Prove that ABPR is a cyclic quadrilateral. (2)

9.2.3 Prove that $\frac{BA}{BQ} = \frac{BC}{QR}$. (3)
[17]

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QUESTION 10

In the diagram, TSR is a diameter of the larger circle having centre S. Chord TQ of the larger circle cuts the smaller circle at M. PQ is a common tangent to the two circles at Q. SQ is drawn.
RP ⊥ PQ and MS || QR.



10.1 Prove, giving reasons that:

10.1.1 SQ is the diameter of the smaller circle (3)

10.1.2 $RT = \frac{RQ^2}{RP}$ (6)

10.2 If MS = 14 units and $PQ = \sqrt{640}$ units, calculate, giving reasons, the length of the radius of the larger circle. (6)
[15]

TOTAL: 150

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MATHEMATICS P2/
WISKUNDE V2

MARKING GUIDELINES/*NASIENRIGLYNE*

2021

MARKS: 150
PUNTE: 150

These marking guidelines consist of 23 pages.
Hierdie nasienriglyne bestaan uit 23 bladsye.

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QUESTION/VRAAG 1

1.1

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

1.1.1(a)	$\bar{x} = \frac{375}{15}$ $\bar{x} = 25\,\text{MB}$ Answer only: Full marks	✓ 375 ✓ answer (2)
1.1.1(b)	$\sigma = 17{,}65$ MB	✓answer (1)
1.1.2	$25 + 17{,}65 = 42{,}65$ $\therefore$ 2 days	✓ 42,65 ✓ 2 (2)
1.1.3	Overall $\bar{x} = \frac{80}{100} \times 25$ $= 20$ MB $\frac{375+x}{30} = 20$ $x = 600 - 375$ $= 225$ maximum total amount of data that Sam must use for the remainder of the month: **225** MB	✓ Overall $\bar{x} = 20$ ✓ $\frac{375+x}{30} = 20$ ✓ answer (3)

1.2

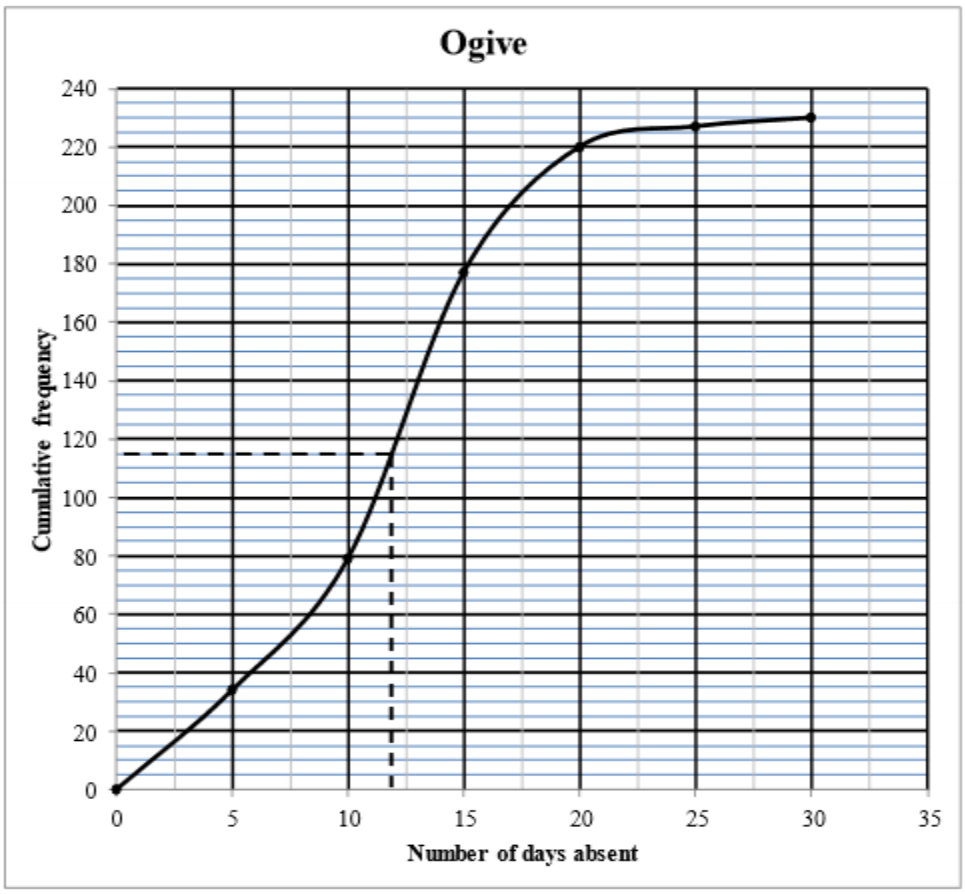
Wind speed in km/h (x)	2	6	15	20	25	17	11	24	13	22
Temperature in °C (y)	28	26	22	22	16	20	24	19	26	19

1.2.1	$a = 29{,}35$ $b = -0{,}46$ $\hat{y} = 29{,}35 - 0{,}46x$	✓ a ✓ b ✓ equation (3)
1.2.2	$y = 25{,}20$ °C (calculator) OR $\hat{y} = 29{,}35 - 0{,}46(9)$ $y = 25{,}21$ °C	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.2.3	$b < 0$, indicating that as the wind speed increases the temperature decreases.	✓ interpretation (1)
		[14]

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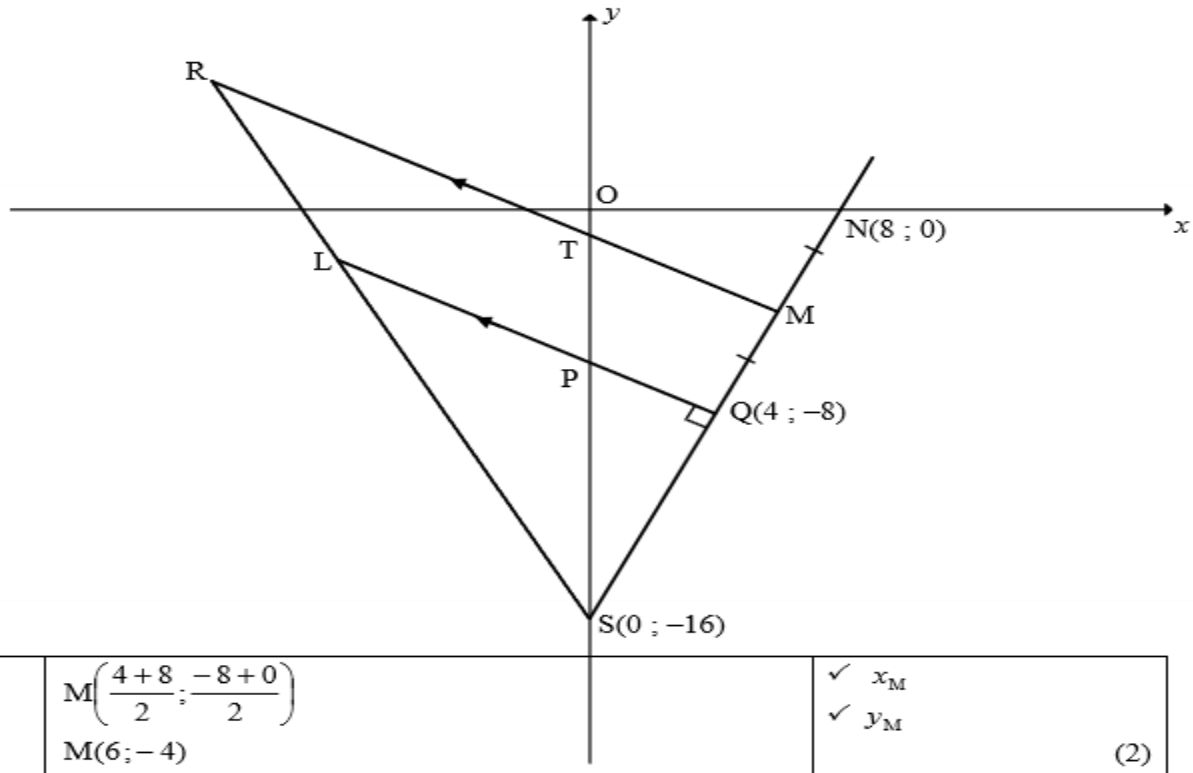
QUESTION/VRAAG 2

Number of days absent	Number of learners	Cumulative frequency
$0 \leq x < 5$	34	34
$5 \leq x < 10$	45	79
$10 \leq x < 15$	98	177
$15 \leq x < 20$	43	220
$20 \leq x < 25$	7	227
$25 \leq x < 30$	3	230

2.1	Modal class: $10 \leq x < 15$	✓ answer (1)
2.2	177 learners	✓ answer (1)
2.3	230 learners	✓ answer (1)
2.4		✓ grounding at (0; 0) ✓ shape ✓ upper limits ✓ All other points correct (4)
2.5	The median is at position 115. □value of median is 12 days (accept 11 – 14) Answer only: Full marks	✓ reading off at 115 ✓ answer (2)
		[9]

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QUESTION/VRAAG 3



3.1	$M\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $M(6;-4)$	✓ x_M ✓ y_M (2)
3.2	$m_{NS}=\frac{0-(-16)}{8-0}$ or $m_{NQ}=\frac{0-(-8)}{8-4}$ or $m_{QS}=\frac{-8-(-16)}{4-0}$ $=2$	✓ subst N and Q or N and Q or Q and S into gradient formula ✓ answer (2)
3.3	$m_{LQ}\times 2=-1$ [LQ ⊥ NS] $\therefore m_{LQ}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$	✓ m_{LQ} ✓ substitution of Q ✓ calculation of c or simplification (3)
3.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$ Answer only: Full marks	✓ identifying radius = 16 ✓ Equation of circle (2)

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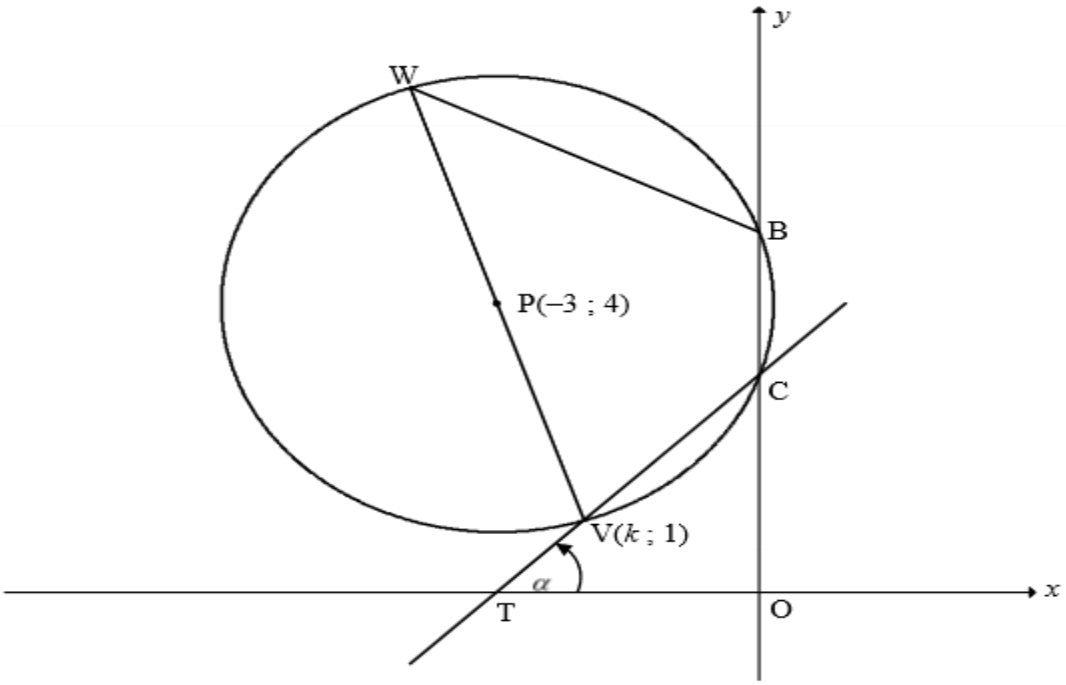
3.5	$m_{RM} = m_{LQ} = -\frac{1}{2}$ [RM ∥ LQ] $-4 = -\frac{1}{2}(6) + c$ **OR** $y + 4 = -\frac{1}{2}(x - 6)$ $c = -1$ $y + 4 = -\frac{1}{2}x + 3$ $\therefore y = -\frac{1}{2}x - 1$ T(0;−1)	✓ m_{RM} ✓ substitution of M(6;−4) ✓ coordinates of T (3)
3.6	T(0;−1), P(0;−6) and S(0;−16) ∴ PS = 10 units and TS = 15 units $\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$ [prop theorem; RM ∥ LP] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks **OR** M(6 ; −4), Q(4 ; −8) and S(0 ; −16) MS = $\sqrt{180} = 6\sqrt{5}$ and QS = $\sqrt{80} = 4\sqrt{5}$ $\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$ [prop theorem; RM ∥ LQ] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks	✓ PS = 10 units ✓ TS = 15 units ✓ answer (3) ✓ MS = $6\sqrt{5}$ units ✓ QS = $4\sqrt{5}$ units ✓ answer (3)
3.7	area of PTMQ = area of ΔTSM − area of ΔPSQ $= \frac{1}{2}.ST.\perp h_M - \frac{1}{2}.PS.\perp h_Q$ $= \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4)$ $= 45 - 20$ = 25 square units **OR** TM = $\sqrt{45} = 3\sqrt{5} = 6{,}71$ MQ = $\sqrt{20} = 2\sqrt{5} = 4{,}47$ PQ = $\sqrt{20} = 2\sqrt{5} = 4{,}47$ area of trapezium PTMQ = $\frac{1}{2}(3\sqrt{5} + 2\sqrt{5})(2\sqrt{5})$ $= \frac{1}{2}(5\sqrt{5})(2\sqrt{5})$ = 25 square units	✓ area of ΔTSM − area of ΔPSQ ✓ area ΔTSM = 45 ✓ area ΔPSQ = 20 ✓ answer (4) ✓ TM = $3\sqrt{5}$ MQ = $2\sqrt{5}$ PQ = $2\sqrt{5}$ ✓ area of trapezium = $\frac{1}{2}$ (sum of ∥sides)(height) ✓ substitute into formula ✓ answer (4)

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	OR $MQ = \sqrt{20} = 2\sqrt{5}$ $PQ = \sqrt{20} = 2\sqrt{5}$ $TP = 5$ area of PTMQ= area of ΔMTP+ area of ΔPQM $\text{area of PTMQ} = \frac{1}{2}\text{TP} \times \perp h_M + \frac{1}{2}\text{MQ} \times \text{PQ}$ area of PTMQ= $10+15=25$	✓ area of ΔMTP + area of ΔPQM area of PTMQ= $\frac{1}{2}(5)\times 6+\frac{1}{2}\left(2\sqrt{5}\right)\left(2\sqrt{5}\right)$ ✓ area ΔMTP = 10 ✓ area ΔPQM = 15 ✓ answer (4)
		[19]

QUESTION 4



4.1	$PV = r = \sqrt{10}$ $PV = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$ $(PV)^2 = (k-(-3))^2 + (1-4)^2 = 10$ $k^2 + 6k + 9 + 9 = 10$ **OR** $(k+3)^2 + 9 = 10$ $k^2 + 6k + 8 = 0$ $(k+3)^2 = 1$ $(k+4)(k+2) = 0$ $k+3 = 1$ or $k+3 = -1$ $k = -4$ or $k = -2$ $\therefore k = -2$	✓ $PV = r = \sqrt{10}$ ✓ substitution into distance formula ✓ standard form ✓ factors ✓ answer (5)
4.2	$x^2 + 6x + y^2 - 8y + 15 = 0$ y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$ $y_C = 3$ or $y_B = 5$ $\therefore BC = 2$ units	✓ $x = 0$ ✓ factors ✓ both values ✓ answer (4)

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4.3.1	$m_{TC} = \frac{3-1}{0-(-2)}$ $= 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$ **OR** $y = mx + 3$ $1 = m(-2) + 3$ $m_{TC} = 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$	✓ substitution into gradient formula ✓ $\tan\alpha = 1$ ✓ answer (3) ✓ substitution into equation of a line ✓ $\tan\alpha = 1$ ✓ answer (3)
4.3.2	$B\hat{C}V = 135°$ [ext ∠ of Δ/*buite* ∠ v Δ] $\therefore V\hat{W}B = 45°$ [opp ∠s of cyclic quad/*teenoorst.* ∠*e v kvh*] Answer only: Full marks **OR** $T\hat{C}O = 45°$ [∠s of Δ/∠*e v* Δ] $\therefore V\hat{W}B = 45°$ [ext ∠s of cyclic quad/*buite* ∠ *v kvh*] Answer only: Full marks	✓ $B\hat{C}V = 135°$ ✓ answer (2) ✓ $T\hat{C}O = 45°$ ✓ answer (2)
4.4.1	$Q(-3;-2)$	✓ x_Q ✓ y_Q (2)
4.4.2	$(x+3)^2 + (y+2)^2 = 10$	✓ LHS ✓ RHS (2)
4.4.3	$x = -2$ or $x = -4$	✓ $x = -2$ ✓ $x = -4$ (2)
		[20]

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QUESTION/VRAAG 5

5.1	$\tan(-x).\cos x.\sin(x-180°)-1$ $=-\tan x.\cos x.\sin(-(180°-x))-1$ $=\frac{-\sin x}{\cos x}.\cos x.(-\sin x)-1$ $=\sin^2 x-1$ $=-\cos^2 x$	✓ $-\tan x$ ✓ $-\sin x$ ✓ $\frac{-\sin x}{\cos x}$ ✓ $\sin^2 x-1$ ✓ answer (5)
5.2.1	$\cos 215°$ $=-\cos 35°$ $=-m$	✓ reduction ✓ answer (2)
5.2.2	$\sin 20°$ $=\cos 70°$ $=\cos 2(35°)$ $=2\cos^2 35°-1$ $=2m^2-1$ OR $=\sin(55°-35°)$ $=\sin 55°\cos 35°-\cos 55°\sin 35°$ $=m.m-\sqrt{1-m^2}.\sqrt{1-m^2}$ $=m^2-(1-m^2)$ $=2m^2-1$	✓ co-function ✓ double angle expansion ✓ answer in terms of m (3) ✓ compound angle expansion ✓ $\cos 55°=\sqrt{1-m^2}$ or $\sin 35°=\sqrt{1-m^2}$ ✓ answer in terms of m (3)
5.3	$\cos 4x.\cos x+\sin 4x.\sin x=-0,7$ $\cos(4x-x)=-0,7$ ref $\angle=45,57...°$ $3x=180°-45,57...°+k.360°$ or $3x=180°+45,57...°+k.360°$ $3x=134,43°+k.360°$ or $3x=225,57°+k.360°$ $x=44,81°+k.120°;\ k\in Z$ $x=75,19°+k.120°;\ k\in Z$	✓ compound angle ✓ $3x=134,43°$ or $225,57°$ ✓ $x=44,81°$ or $75,19°$ ✓ $+k.120°;\ k\in Z$ (4)

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5.4	$\text{RHS} = \cos^2 x - \sin^2 x$ $\text{LHS} = \dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x}$ $= \dfrac{\sin 4x.\cos 2x - \cos 4x.\sin 2x}{\dfrac{\sin 2x}{\cos 2x}}$ $= \sin(4x-2x)\left(\dfrac{\cos 2x}{\sin 2x}\right)$ $= \cos 2x$ $= \cos^2 x - \sin^2 x$ $\text{LHS} = \text{RHS}$	✓ $\sin 2x$ ✓ $\dfrac{\sin 2x}{\cos 2x}$ ✓ $\sin(4x-2x)$ ✓ $\cos 2x$ (4)
		[18]

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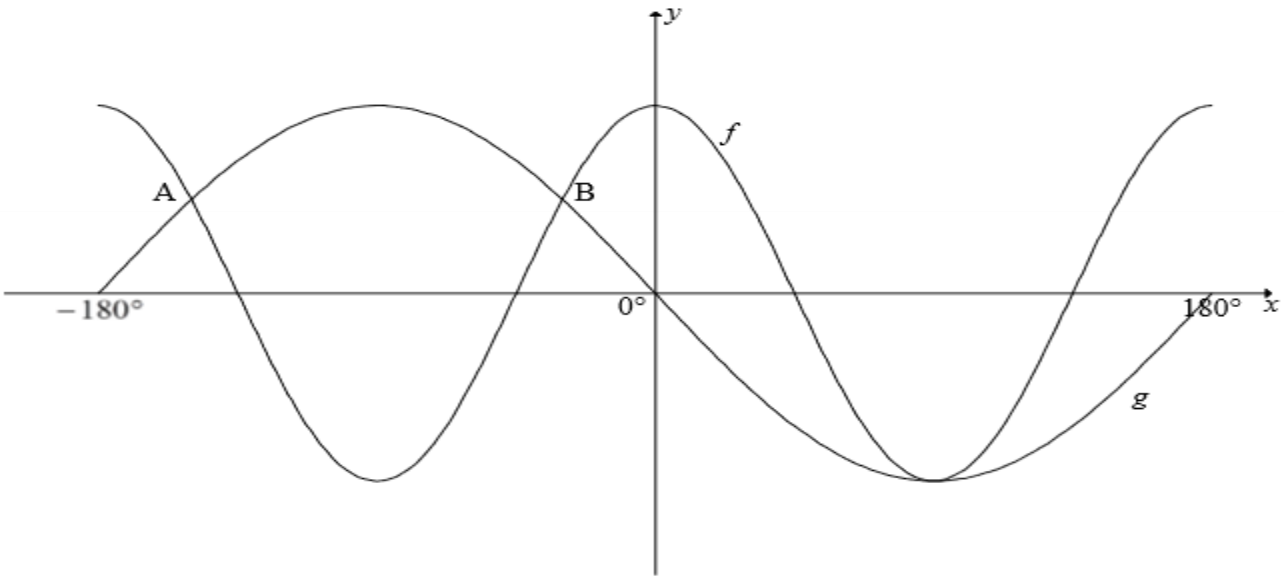
QUESTION/VRAAG 6

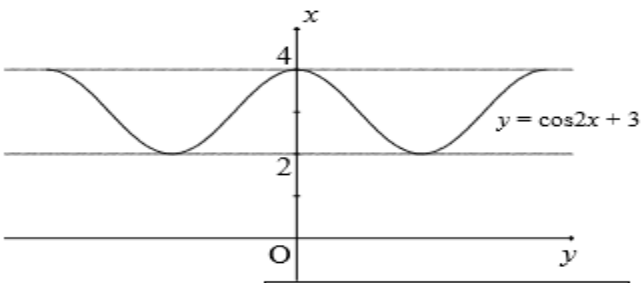
6.1	$1-2\sin^2 x=-\sin x$ $2\sin^2 x-\sin x-1=0$ $(2\sin x+1)(\sin x-1)=0$ $\sin x=-\frac{1}{2}$ or $\sin x=1$ ref $\angle=30°$ ref $\angle=90°$ $x=210°+k.360°$ $x=90°+k.360°$ or $x=330°+k.360°$ $x=-150°$ or $x=-30°$ or $x=90°$ **OR** $\cos 2x=-\sin x$ $\cos 2x=-\cos(90°-x)$ $2x=180°-(90°-x)+k.360°$ or $2x=180°+(90°-x)+k.360°$ $2x=90°+x+k.360°$ or $2x=270°-x+k.360°$ $x=90°+k.360°$ $x=90°+k.120°$ $x=-150°$ or $x=-30°$ or $x=90°$ **OR** $\cos 2x=-\sin x$ $\cos 2x=\cos(90°+x)$ $2x=90°+x+k.360°$ or $2x=360°-(90°+x)+k.360°$ $x=90°+k.360°$ or $3x=270°+k.360°$ $x=90°+k.120°$ $x=-150°$ or $x=-30°$ or $x=90°$ **OR** $\cos 2x=-\sin x$ $\sin(90°-2x)=-\sin x$ $90°-2x=180+x+k.360°$ or $90°-2x=360°-x+k.360°$ $x=-30°+k.120°$ $x=-270°+k.360°$ $x=-150°$ or $x=-30°$ or $x=90°$	✓ identity ✓ factors ✓ $\sin x=-\frac{1}{2}$ ✓ $\sin x=1$ ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6) ✓ co-functions ✓ $2x$ in quadrant 2 ✓ $2x$ in quadrant 3 ✓ both general solutions ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6) ✓ co-functions ✓ $2x$ in quadrant 1 ✓ $2x$ in quadrant 4 ✓ both general solutions ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6) ✓ co-functions ✓ $90°-2x$ in quadrant 3 ✓ $90°-2x$ in quadrant 4 ✓ both general solutions ✓ $-150°$ and $-30°$ ✓ $90°$ (A) (6)

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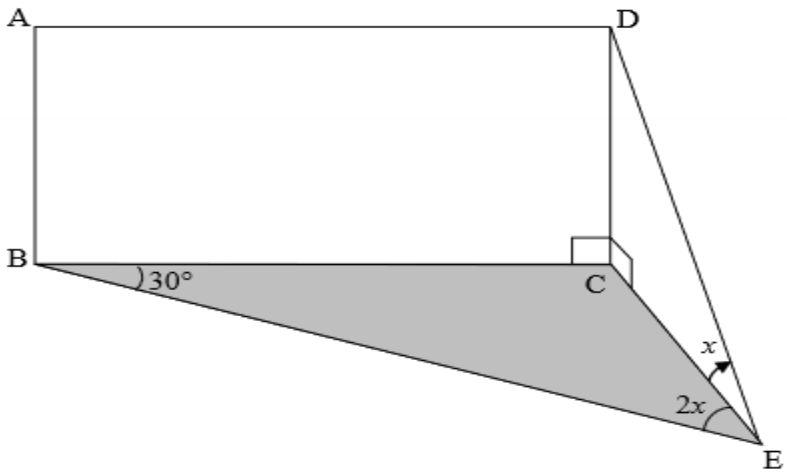
6.2



6.2.1	A(−150°; 0,5) B(−30°; 0,5) AB = −30° − (−150°) AB = 120° Answer only: Full marks	✓ AB = −30° − (−150°) ✓ answer (2)
6.2.2	$x \in (0°; 90°)$ or $x \in (90°; 180°)$ **OR** $0° < x < 90°$ or $90° < x < 180°$	✓ $x \in (0°; 90°)$ ✓ $x \in (90°; 180°)$ (2) ✓ $0° < x < 90°$ ✓ $90° < x < 180°$ (2)
6.2.3	$\cos 2x = k - 3$ $k - 3 < -1$ or $k - 3 > 1$ $k < 2$ or $k > 4$ Answer only: Full marks **OR**  $k < 2$ or $k > 4$ Answer only: Full marks	✓ $k - 3 < -1$ or $k - 3 > 1$ ✓ $k < 2$ ✓ $k > 4$ (3) ✓ graph of $y = \cos 2x + 3$ ✓ $k < 2$ ✓ $k > 4$ (3)
		[13]

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QUESTION/VRAAG 7



7.1	In ΔBCE: $\frac{CE}{\sin\hat{B}} = \frac{BC}{\sin B\hat{E}C}$ $\frac{CE}{\sin 30°} = \frac{BC}{\sin 2x}$ $CE = \frac{BC\sin 30°}{\sin 2x}$	✓ correct use of sine rule ✓ $CE = \frac{BC\sin 30°}{\sin 2x}$
	In ΔCDE: $\frac{DC}{CE} = \tan D\hat{E}C$ $DC = \frac{BC.\sin 30°}{\sin 2x}(\tan x)$ $DC = \frac{BC}{4\sin x\cos x}\left(\frac{\sin x}{\cos x}\right)$ $DC = \frac{BC}{4\cos^2 x}$	✓ correct trig ratio ✓ Subst CE ✓ $2\sin x\cos x$ ✓ $\frac{\sin x}{\cos x}$ (6)

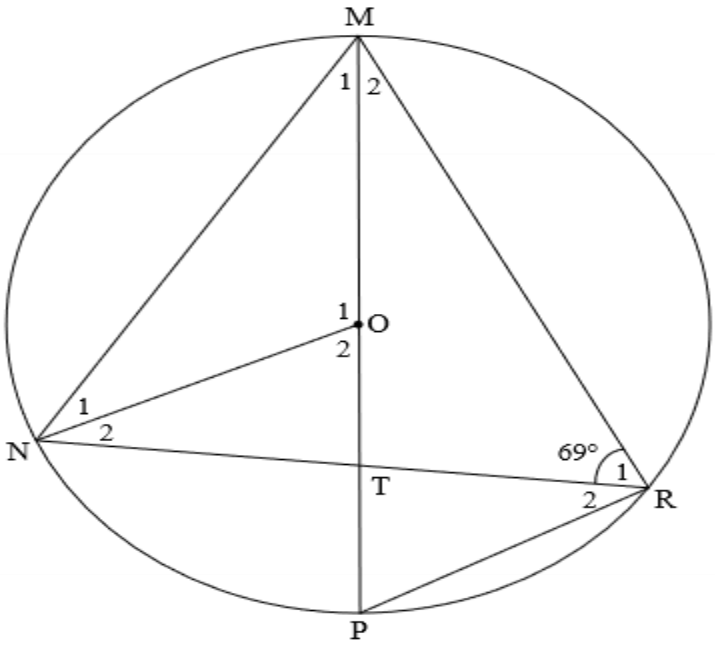
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7.2	$DC = \frac{BC}{4\cos^2 30°}$ $= \frac{BC}{4\left(\frac{\sqrt{3}}{2}\right)^2}$ $= \frac{BC}{3}$ $\therefore BC = 3DC$ But $AB = DC$ [opp sides of rectangle/*teenoorst. sye v reghoek*] $\therefore BC = 3AB$ Area of rectangle $= (AB)(BC)$ $= (AB)(3AB)$ $=3AB^2$	✓ $DC == \frac{BC}{3}$ ✓ $BC = 3AB$ ✓ substitution into area formula (3)
		[9]

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QUESTION/VRAAG 8

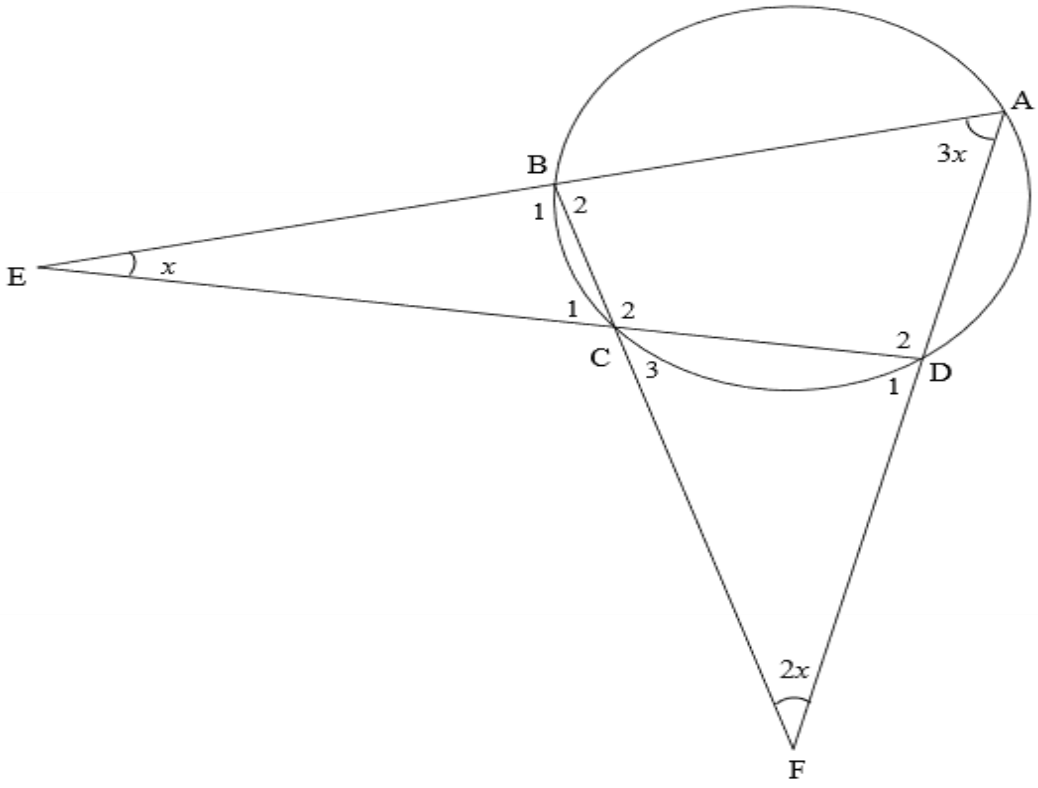
8.1



8.1.1	$M\hat{R}P = 90°$ $\hat{R}_2 = 21°$	[∠ in semi circle/∠ *in halwe sirkel*]	✓ R ✓ S (2)
8.1.2	$\hat{O}_1 = 138°$	[∠ at centre = 2 ×∠ at circumference/ *midpts.* ∠ = 2× *omtreks* ∠]	✓ S ✓R (2)
8.1.3	$\hat{M}_1 = 21°$ **OR** $\hat{M}_1 + N_1 = 180° - 138°$ $\therefore \hat{M}_1 = 21°$	[∠s in the same segment/∠*e in dieselfde sirkel segment*] [sum of ∠s in Δ/∠*e v* Δ] [∠s opp equal sides/∠*e teenoor gelyke sye*]	✓ S ✓R (2) ✓ S ✓R (2)
8.1.4	$\hat{O}_2 = 42°$ $\hat{P} = 42°$ $\hat{M}_2 = 48°$ **OR** $\hat{N}_2 = \hat{R}_2 = 21°$ $\hat{N}_1 = \hat{M}_1 = 21°$ $\hat{M}_2 = 48°$	[∠s on a str line/∠*e op 'n reguitlyn*] [alt ∠s; NO ‖ PR/*Verw.* ∠*e,* NO ‖ PR] [sum of ∠s in Δ/∠*e v* Δ] [alt ∠s; NO ‖ PR/*Verw.* ∠*e,* NO ‖ PR] [∠s opposite equal sides/∠*e teenoor gelyke sye*] [sum of ∠s of Δ NMR//∠*e v* ΔNMR]	✓ S ✓ S ✓R ✓ S (4) ✓ S ✓R ✓ S ✓ S (4)

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8.2



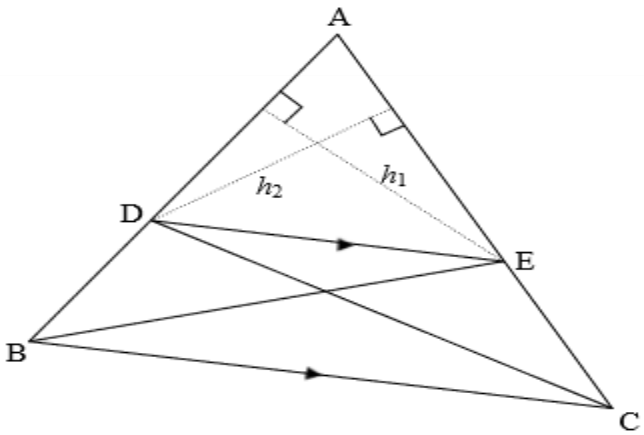
8.2	$\hat{D}_1 = 4x$	[ext ∠ of Δ/*buite ∠ v Δ*]	✓ S/R
	$\hat{D}_2 = 180° - 4x$	[∠s on a str line/*∠e op 'n reguitlyn*]	✓ S
	$\hat{B}_1 = 5x$	[ext ∠ of Δ/*buite ∠ v Δ*]	✓ S
	$\hat{B}_1 = \hat{D}_2$	[ext ∠ of cyclic quad/*buite ∠ v kvh*]	✓S ✓R
	$180° - 4x = 5x$		
	$9x = 180°$		
	$x = 20°$		✓ answer (6)
	OR		
	$\hat{C}_1 = 3x$	[ext ∠ of cyclic quad/*buite ∠ v kvh*]	✓ S ✓ R
	$\hat{B}_2 = 4x$	[ext ∠ of Δ/*buite ∠ v Δ*]	✓ S
	$\hat{C}_1 = \hat{C}_3 = 3x$	[vert opp ∠s]	✓ S
	$\hat{D}_2 = 5x$	[ext ∠ of Δ/*buite ∠ v Δ*]	
	$4x + 5x = 180°$	[opp ∠ of cyclic quad/*teenoorst. ∠e v kvh*]	✓S/R
	$x = 20°$		✓ answer (6)

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	OR $\hat{C}_3 = 3x$ [ext ∠ of cyclic quad/*buite ∠ v kvh*] $\hat{D}_1 = 4x$ [ext ∠ of Δ/*buite ∠ v* Δ] $2x + 3x + 4x = 180°$ [sum of ∠s in Δ/*∠e v* Δ] $9x = 180°$ $x = 20°$	✓ S ✓R ✓ S ✓ S ✓R ✓ answer (6)
		[16]

QUESTION/VRAAG 9

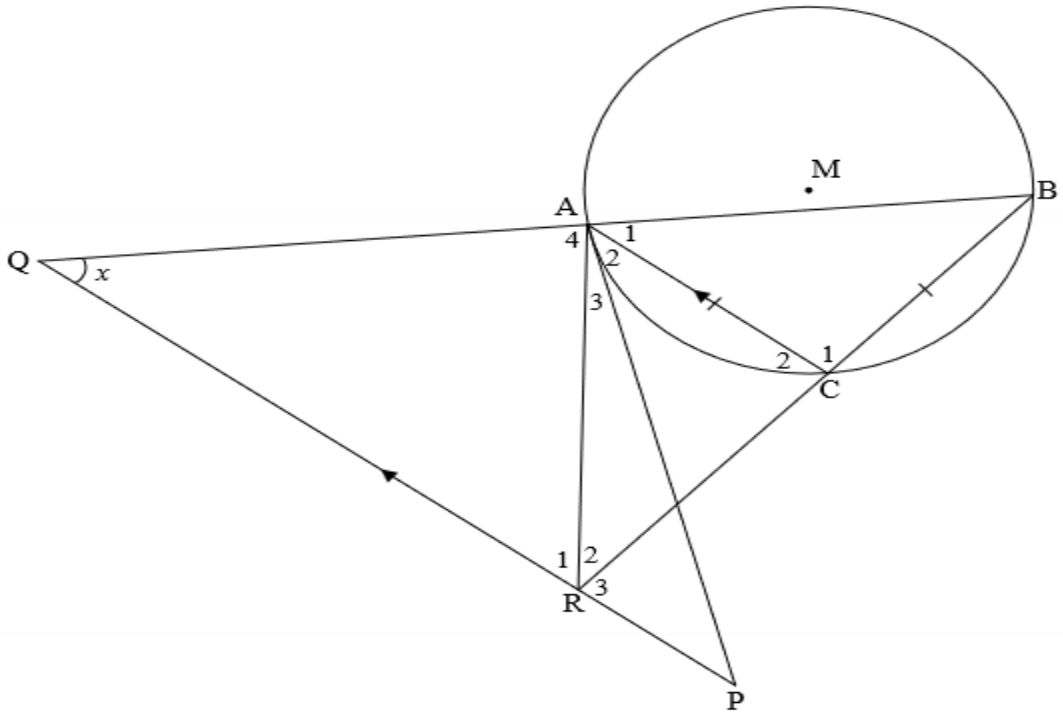
9.1



9.1	Constr: Join BE and CD and draw h_1 from E ⊥ AD and h_2 from D ⊥ AE *Konstr: Verbind BE en CD en trek h_1 vanaf* E ⊥ AD *en* h_2 *vanaf* D ⊥ AE Proof/*Bewys*: $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}BD \times h_1} = \frac{AD}{BD}$ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$ area ΔADE = area ΔADE [common/*gemeenskaplik*] But area ΔBDE = area ΔDEC [same base & height ; DE ∥ BC/ *dies basis & hoogte ;* DE ∥ BC] $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓ constr/*konstr* ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE}$ ✓ $\frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}BD \times h_1}$ **or R** ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{AE}{EC}$ ✓ S ✓R (6)

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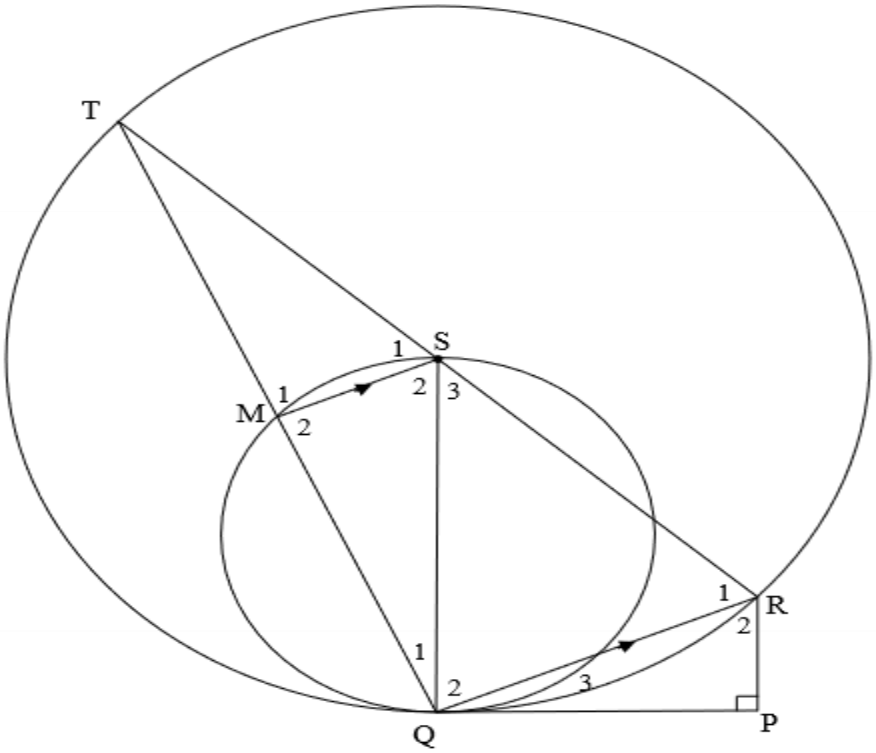
9.2



9.2.1	$\hat{A}_1 = x$ [corresp ∠s; PQ ‖ CA/*ooreenkomstige ∠e*, PQ ‖ CA] $\hat{B} = x$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{A}_2 = x$ [tan-chord theorem/*∠ tussen raaklyn en koord*] $\hat{P} = x$ [alt ∠s; PQ ‖ CA/*verw. ∠e*, PQ ‖ CA]	✓ S ✓ R ✓ S/R ✓ S ✓ R ✓ S/R (6)
9.2.2	$\hat{B} = \hat{P}$ [proved in 9.2.1/*bewys in 9.2.1*] ∴ A, B, P and R are concyclic ∴ ABPR is a cyclic quadrilateral [conv ∠s in the same segment/ *koord onderspan gelyke omtreks ∠e*]	✓ S ✓ R (2)
9.2.3	$\frac{BA}{BQ} = \frac{BC}{BR}$ [prop th; AC ‖ QP] **OR** [line ‖ one side Δ/*lyn ‖ een syn v Δ*] But QR = BR [sides opp = ∠s/*sye teenoor = ∠e*] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ R ✓ S (3)

	OR In Δ ABC and Δ BQR: $\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1] $\hat{B} = \hat{Q} = x$ [proved in 9.2.1] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \mid\mid\mid \Delta BQR$ $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ S (3)
	OR In Δ ABC and Δ BQR: $\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1] $\hat{B} = \hat{Q} = x$ [proved in 9.2.1] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \mid\mid\mid \Delta BQR$ [$\angle\angle\angle$] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ R (3)
	OR In Δ ABC and Δ QBR: $\hat{B}$ is common $\hat{A}_1 = \hat{Q} = x$ [corres $\angle$s; PQ $\parallel$ CA] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \mid\mid\mid \Delta QBR$ [$\angle\angle\angle$] But QR = BR [sides opp = $\angle$s/*sye teenoor* = $\angle$*e*] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ S (3)
		[17]

QUESTION/VRAAG 10



10.1.1	$\hat{Q}_1 + \hat{Q}_2 = 90°$	[∠ in semi circle/∠ *in halwe sirkel*]	✓ S/R
	$\therefore \hat{M}_2 = 90°$	[co-interior ∠, MS ∥ QR/*ko-binne ∠e*, MS ∥ QR]	✓ S/R
	∴ SQ is a diameter	[converse: ∠ in semi circle/ *Omgekeerde: ∠ in halwe sirkel*]	✓ R (3)
	OR		
	MS ∥ QR		
	$\frac{TS}{SR} = \frac{TM}{MQ} = \frac{1}{1}$	[prop theorem; SM ∥ QR] **OR** [line ∥ one side of Δ]/*lyn* ∥ *een sy v*Δ	✓ S/R
	$\therefore TM = MQ$		
	$\therefore \hat{M}_2 = 90°$	[Line from centre bisects chord/*midpt. sirkel; midpt koord*]	✓ S/R
	∴ SQ is a diameter	[converse: ∠ in semi circle/ *Omgekeerde: ∠ in halwe sirkel*]	✓ R (3)
	OR		
	SQ ⊥ QP	[tan ⊥ rad/*raaklyn ⊥ radius*]	✓ S ✓ R
	∴ SQ is a diameter	[converse: tan ⊥ rad/*Omgekeerde: raaklyn ⊥ radius*]	✓ R (3)

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10.1.2	In ΔRTQ and ΔRQP		
	$\hat{T} = \hat{Q}_3$	[tan-chord theorem/$\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$\hat{Q}_1 + \hat{Q}_2 = 90°$	[co-interior $\angle$s, MS $\parallel$ QR/*ko-binne* $\angle e$, MS $\parallel$ QR] **or** [$\angle$ in semi circle/$\angle$ *in halwe sirkel*]	✓ S
	$\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$		✓ S
	$\hat{R}_1 = \hat{R}_2$	[$\angle$s of Δ/$\angle e$ *van* Δ]	✓ S
	$\Delta RTQ \ \|\|\| \ \Delta RQP$		
	$\frac{RT}{RQ} = \frac{RQ}{RP}$		✓ ratio
	$RT = \frac{RQ^2}{RP}$		(6)
	OR		
	In ΔRTQ and ΔRQP		
	$\hat{T} = \hat{Q}_3$	[tan-chord theorem $\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$\hat{Q}_1 + \hat{Q}_2 = 90°$	[co-interior $\angle$s, MS $\parallel$ QR/*ko-binne* $\angle e$, MS $\parallel$ QR] **or** [$\angle$ in semi circle/$\angle$ *in halwe sirkel*]	✓ S
	$\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$		✓ S
	$\Delta RTQ \ \|\|\| \ \Delta RQP$	[$\angle,\angle,\angle$]	✓ R
	$\frac{RT}{RQ} = \frac{RQ}{RP}$		✓ ratio
	$RT = \frac{RQ^2}{RP}$		(6)
10.2	QR = 28 units	[midpoint theorem/*midpt. stelling*]	✓ S ✓ R
	$RP^2 = 28^2 - \left(\sqrt{640}\right)^2$	[Pythagoras/*Pythagoras*]	✓ S
	RP = 12 units		✓ RP = 12
	$RT = \frac{RQ^2}{RP}$		
	$RT = \frac{28^2}{12}$		
	$RT = \frac{196}{3}$		✓ RT
	Radius = $\frac{98}{3}$ units		✓ answer (6)
			[15]

TOTAL/TOTAAL: 150

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basic education

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Basic Education
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SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2022

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages and 1 information sheet.

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QUESTION 1

The table below shows the mass (in kg) of the school bags of 80 learners.

MASS (in kg)	FREQUENCY
$5 < m \leq 7$	6
$7 < m \leq 9$	18
$9 < m \leq 11$	21
$11 < m \leq 13$	19
$13 < m \leq 15$	11
$15 < m \leq 17$	4
$17 < m \leq 19$	1

1.1 Write down the modal class of the data. (1)

1.2 Complete the cumulative frequency column in the table in the ANSWER BOOK. (2)

1.3 Draw a cumulative frequency graph (ogive) for the given data on the grid provided in the ANSWER BOOK. (3)

1.4 Use the graph to determine the median mass for this data. (2)

1.5 The international guideline for the mass of a school bag is that it should not exceed 10% of a learner's body mass.

1.5.1 Calculate the estimated mean mass of the school bags. (2)

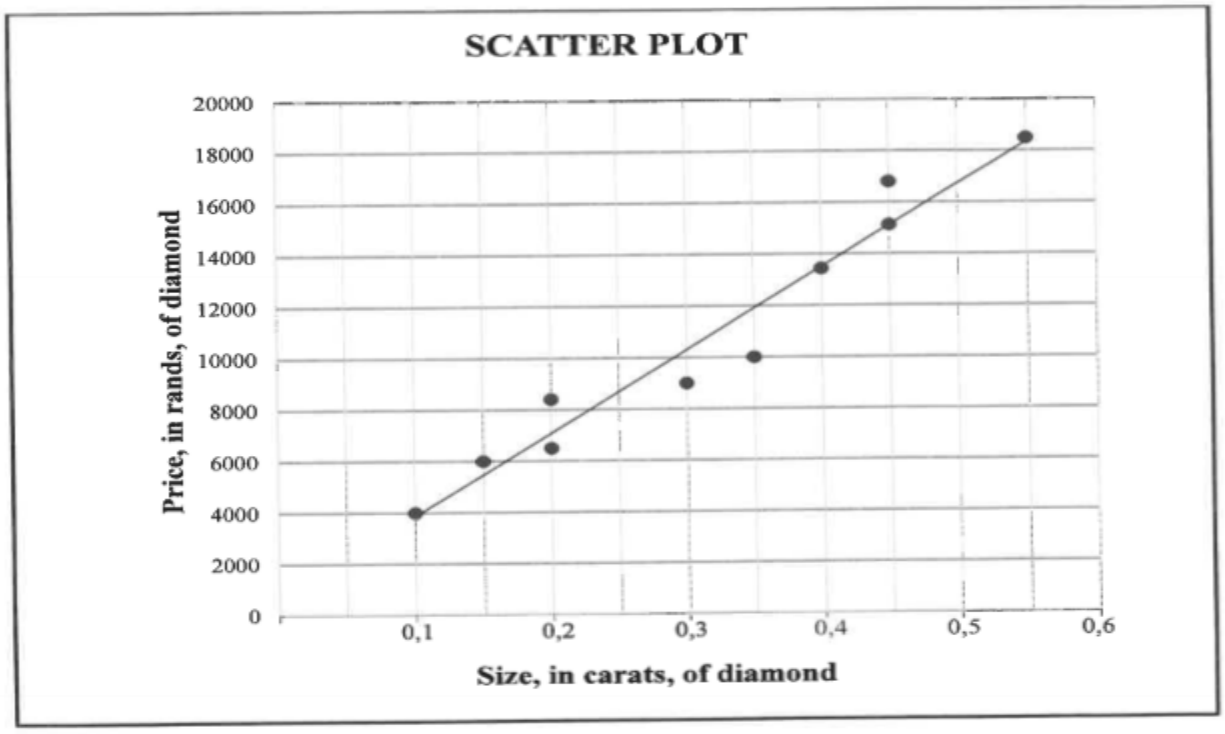
1.5.2 The mean mass of this group of learners was found to be 80 kg. On average, are these school bags satisfying the international guideline with regard to mass? Motivate your answer. (2)
[12]

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QUESTION 2

The table below shows the size (in carats) and the price (in rands) of 10 diamonds that were sold by a diamond trader. This information is also presented in the scatter plot below. The least squares regression line for the data is drawn.

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1 Determine the equation of the least squares regression line for the data. (3)

2.2 If the trader sold a diamond that was 0,25 carats in size, predict the selling price of this diamond in rands. (2)

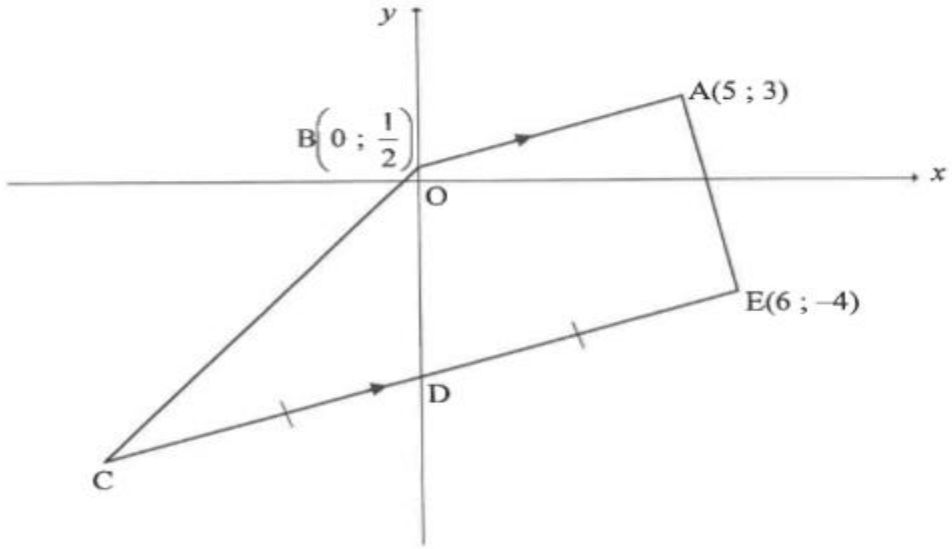
2.3 Calculate the average price increase per 0,05 carat of the diamonds. (2)

2.4 It was later found that the selling price of the 0,35 carat diamond was recorded incorrectly. The correct price is R11 500. When this correction is made to the data set, the correlation between the size and price of these diamonds gets stronger. Explain the reason for this by referring to the given scatter plot. (1)
[8]

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QUESTION 3

In the diagram, A(5 ; 3), $B\left(0 ; \frac{1}{2}\right)$, C and E(6 ; –4) are the vertices of a trapezium having BA ∥ CE. D is the y-intercept of CE and CD = DE.



3.1 Calculate the gradient of AB. (2)

3.2 Determine the equation of CE in the form $y = mx + c$. (3)

3.3 Calculate the:

3.3.1 Coordinates of C (3)

3.3.2 Area of quadrilateral ABCD (4)

3.4 If point K is the reflection of E in the y-axis:

3.4.1 Write down the coordinates of K (2)

3.4.2 Calculate the:

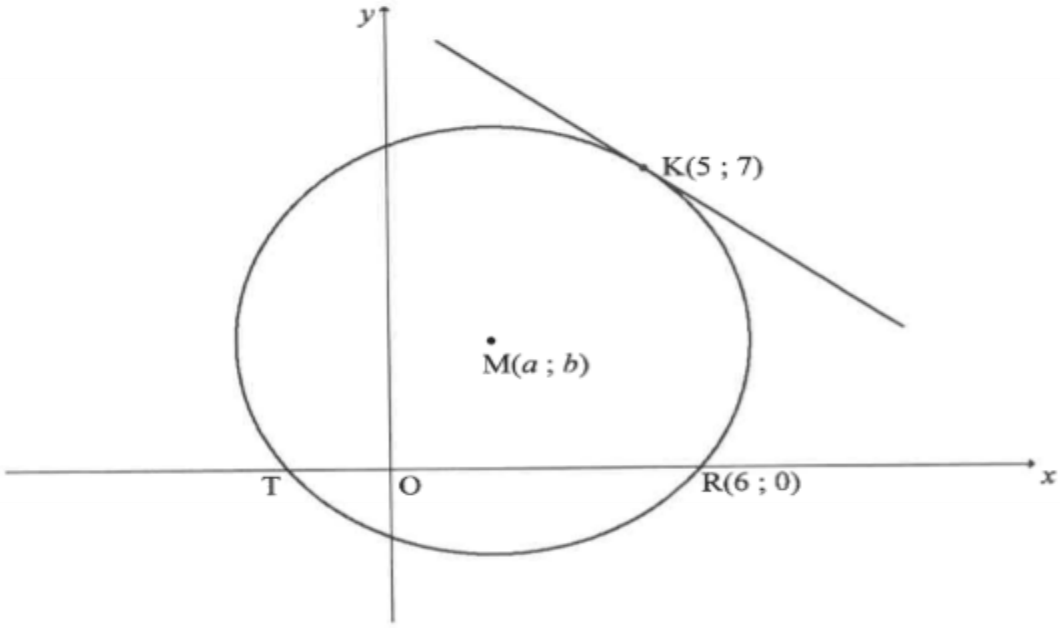
(a) Perimeter of ΔKEC (4)

(b) Size of $K\hat{C}E$ (3)
[21]

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QUESTION 4

In the diagram, the circle centred at M(a ; b) is drawn. T and R(6 ; 0) are the x-intercepts of the circle. A tangent is drawn to the circle at K(5 ; 7).



4.1 M is a point on the line $y = x + 1$.

4.1.1 Write b in terms of a. (1)

4.1.2 Calculate the coordinates of M. (5)

4.2 If the coordinates of M are (2 ; 3), calculate the length of:

4.2.1 The radius of the circle (2)

4.2.2 TR (2)

4.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y = mx + c$. (5)

4.4 A horizontal line is drawn as a tangent to the circle M at the point N(c ; d), where $d < 0$.

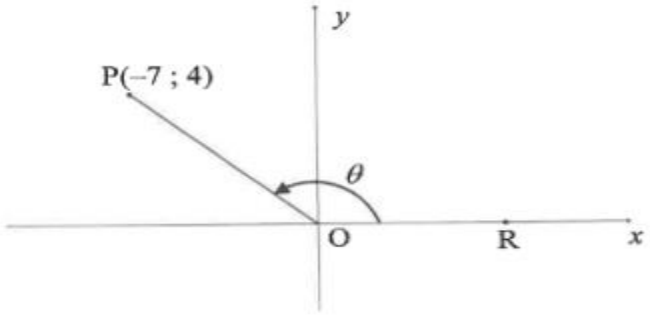
4.4.1 Write down the coordinates of N. (2)

4.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)
[20]

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QUESTION 5

5.1 In the diagram below, P(–7 ; 4) is a point in the Cartesian plane. R is a point on the positive x-axis such that obtuse $P\hat{O}R = \theta$.



Calculate, **without using a calculator**, the:

5.1.1 Length OP (2)

5.1.2 Value of:

(a) $\tan\theta$ (1)

(b) $\cos(\theta - 180°)$ (2)

5.2 Determine the general solution of: $\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$ (7)

5.3 Given the identity: $\dfrac{\sin 3x}{1-\cos 3x} = \dfrac{1+\cos 3x}{\sin 3x}$

5.3.1 Prove the identity given above. (3)

5.3.2 Determine the values of x, in the interval $x \in [0° ; 60°]$, for which the identity will be undefined. (3)
[18]

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QUESTION 6

6.1 **Without using a calculator**, simplify the following expression to a single trigonometric term:

$$\frac{\sin 10^\circ}{\cos 440^\circ} + \tan(360^\circ - \theta).\sin 2\theta$$ (6)

6.2 Given: $\sin(60^\circ + 2x) + \sin(60^\circ - 2x)$

6.2.1 Calculate the value of k if $\sin(60^\circ + 2x) + \sin(60^\circ - 2x) = k\cos 2x$. (3)

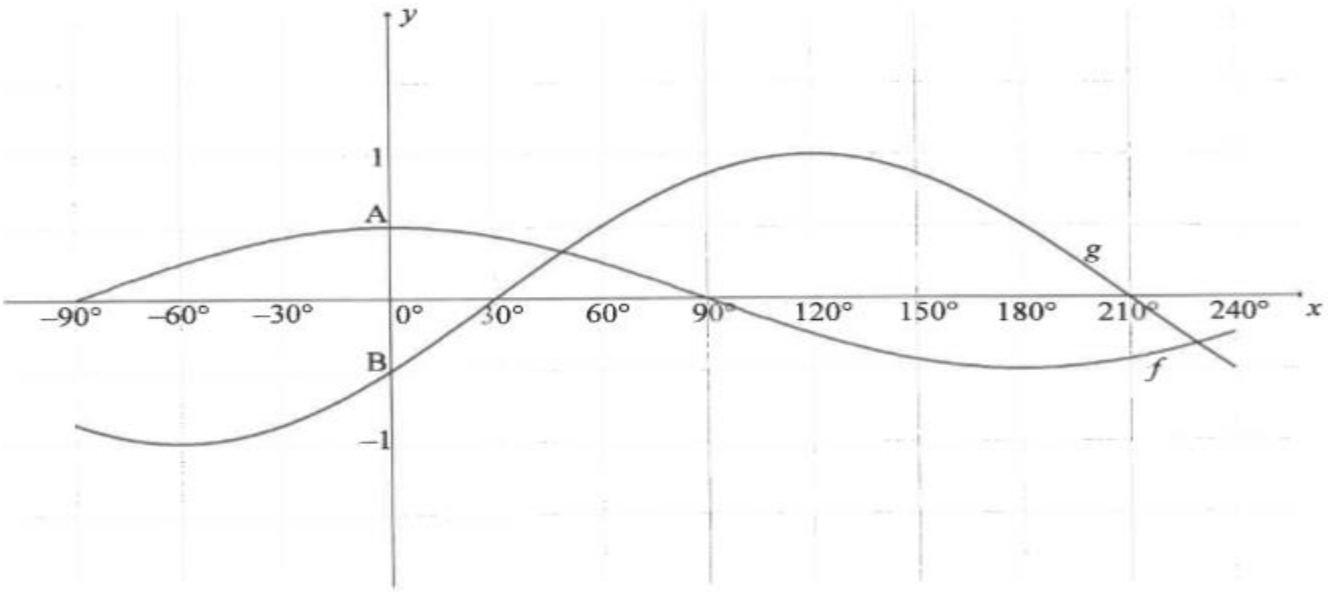
6.2.2 If $\cos x = \sqrt{t}$, **without using a calculator**, determine the value of $\tan 60^\circ[\sin(60^\circ + 2x) + \sin(60^\circ - 2x)]$ in terms of t. (3)

[12]

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QUESTION 7

In the diagram below, the graphs of $f(x)=\frac{1}{2}\cos x$ and $g(x)=\sin(x-30°)$ are drawn for the interval $x\in[-90°;240°]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB. (2)

7.2 Write down the range of $3f(x)+2$. (2)

7.3 Read off from the graphs a value of x for which $g(x)-f(x)=\frac{\sqrt{3}}{2}$. (2)

7.4 For which values of x, in the interval $x\in[-90°;240°]$, will:

7.4.1 $f(x).g(x)>0$ (2)

7.4.2 $g'(x-5°)>0$ (2)
[10]

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QUESTION 8

FIGURE I shows a ramp leading to the entrance of a building. B, C and D lie on the same horizontal plane. The perpendicular height (AC) of the ramp is 0,5 m and the angle of elevation from B to A is 15°. The entrance of the building (AE) is 0,915 m wide.

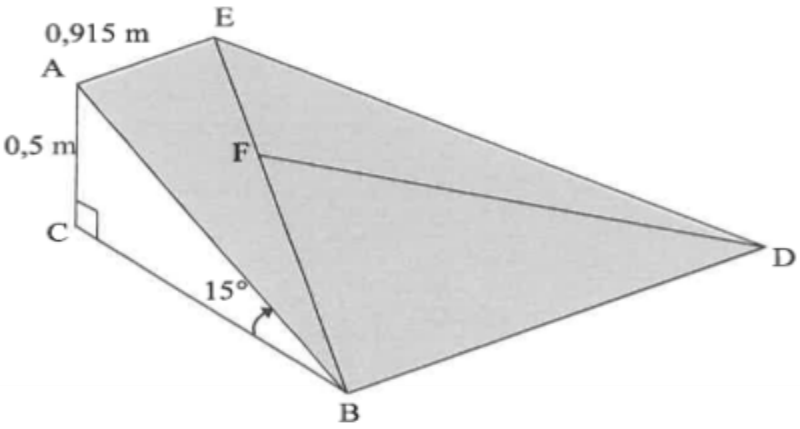


FIGURE I

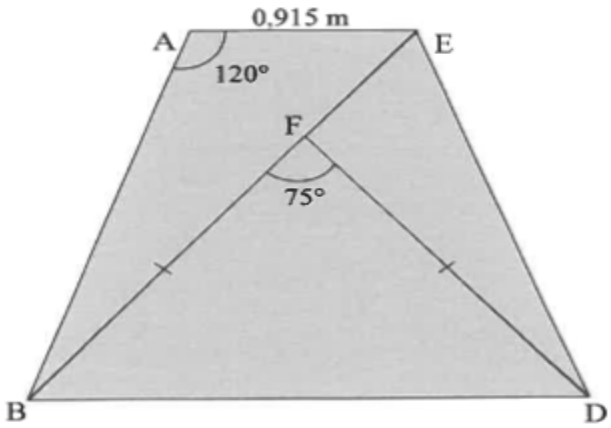


FIGURE II (top view)

8.1 Calculate the length of AB. (2)

8.2 Figure II shows the top view of the ramp. The area of the top of the ramp is divided into three triangles, as shown in the diagram.

If $B\hat{A}E = 120°$, calculate the length of BE. (3)

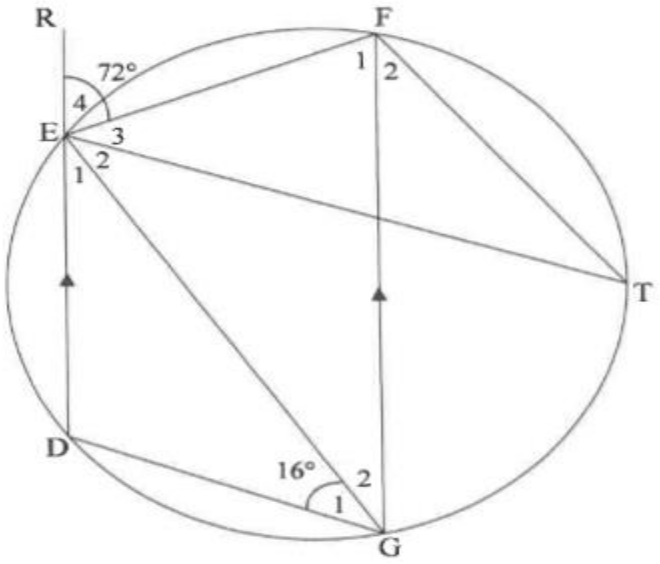
8.3 Calculate the area of ΔBFD if $B\hat{F}D = 75°$, $BF = FD$ and $BF = \frac{5}{7}BE$. (3)

[8]

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QUESTION 9

9.1 In the diagram, DEFG is a cyclic quadrilateral with DE ∥ GF. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72°$ and $\hat{G}_1 = 16°$.



Determine, with reasons, the size of the following angles:

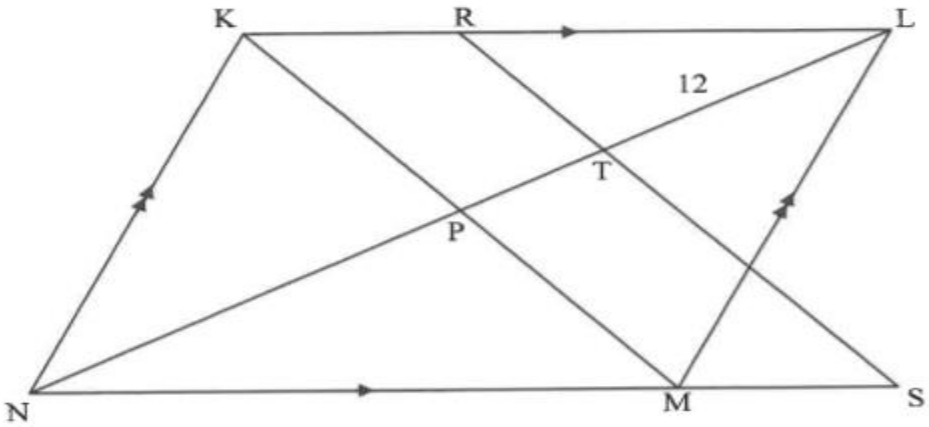
9.1.1 $D\hat{G}F$ (2)

9.1.2 $\hat{T}$ (2)

9.1.3 $G\hat{E}F$ (2)

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9.2 In the diagram, the diagonals of parallelogram KLMN intersect at P. NM is produced to S. R is a point on KL and RS cuts PL at T.
NM : MS = 4 : 1, NL = 32 units and TL = 12 units.



9.2.1 Determine, with reasons, the value of the ratio NP : PT in simplest form. (4)

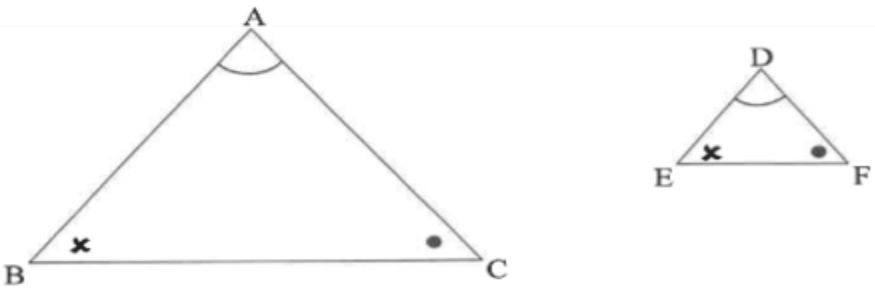
9.2.2 Prove, with reasons, that KM || RS. (2)

9.2.3 If NM = 21 units, determine, with reasons, the length of RL. (4)
[16]

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QUESTION 10

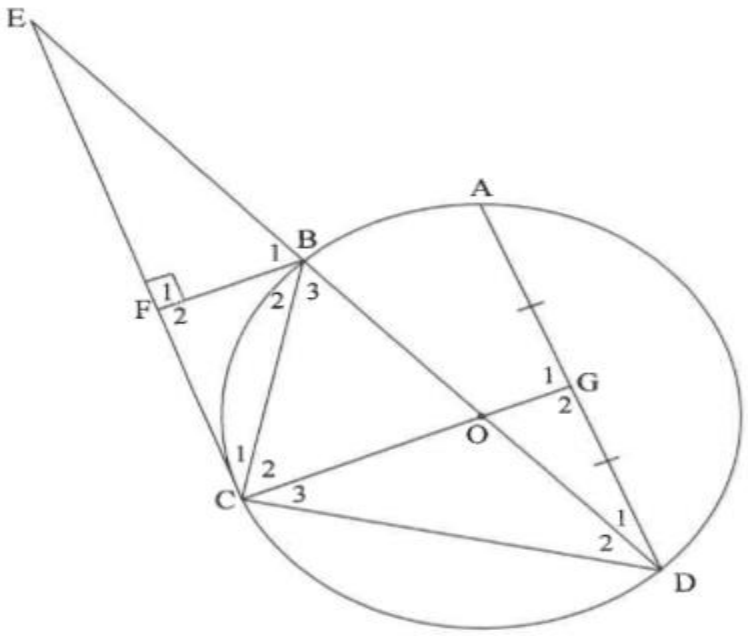
10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. $\frac{AB}{DE}=\frac{AC}{DF}$. (6)

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10.2 In the diagram, O is the centre of a circle passing through A, B, C and D. EC is a tangent to the circle at C. Diameter DB produced meets tangent EC at E. F is a point on EC such that BF ⊥ EC. Radius CO produced bisects AD at G. BC and CD are drawn.



10.2.1 Prove, with reasons, that:

(a) FB || CG (3)

(b) ΔFCB ||| ΔCDB (5)

10.2.2 Give a reason why $\hat{G}_1 = 90°$. (1)

10.2.3 Prove, with reasons, that $CD^2 = CG.DB$. (5)

10.2.4 Hence, prove that DB = CG + FB. (5)
[25]

TOTAL: 150

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MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

2022

MARKS: 150
PUNTE: 150

These marking guidelines consist of 20 pages./
Hierdie nasienriglyne bestaan uit 20 bladsye.

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QUESTION/*VRAAG* 1

1.1 Modal class: $9 < m \leq 11$ ✓ answer (1)

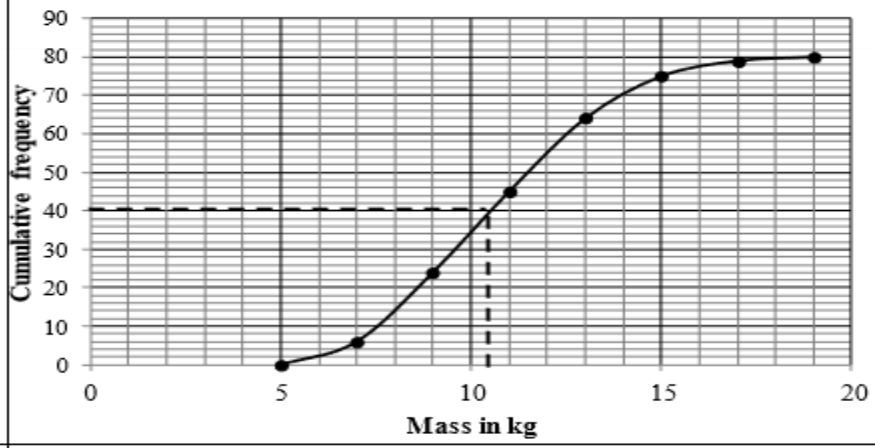
1.2

Mass (in kg)	Frequency	Cumulative frequency
$5 < m \leq 7$	6	6
$7 < m \leq 9$	18	24
$9 < m \leq 11$	21	45
$11 < m \leq 13$	19	64
$13 < m \leq 15$	11	75
$15 < m \leq 17$	4	79
$17 < m \leq 19$	1	80

✓ adding

✓ 80 (2)

1.3



✓ grounding (5 ; 0)

✓ points

✓ shape (3)

1.4 Median mass: 10,5 kg ✓✓ answer (2)

1.5.1

$$\bar{x} = \frac{(6\times6+18\times8+21\times10+19\times12+11\times14+4\times16+1\times18)}{80}$$

$$= \frac{854}{80}$$

$$= 10{,}68$$

Answer only 2/2

✓ 854

✓ answer (2)

1.5.2 Learners' bags are heavier than the stipulated international guideline.
Estimated mean = 10,68 kg
10% of 80 kg
= 8 kg
10,68 kg > 8 kg

✓ answer

✓ 8 kg (2)

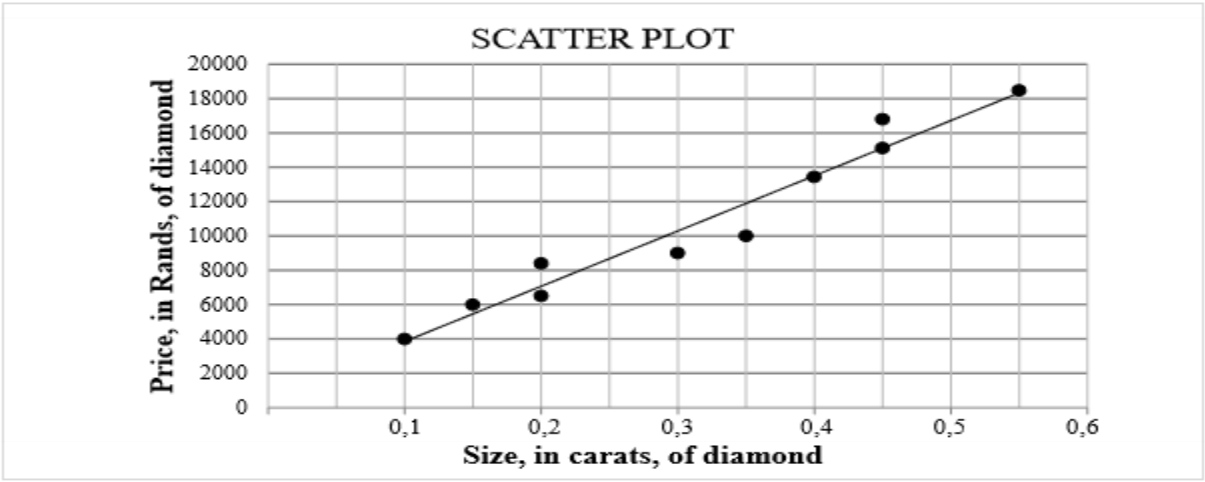
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	OR/ *OF*	
	Learners' bags are heavier than the stipulated international guideline.	✓ answer
	Estimated mean $= \frac{10,68}{80} \times 100$	
	$= 13,35\%$	✓ 13,35%
	$13,35\% > 10\%$	
		(2)
		[12]

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QUESTION/*VRAAG* 2

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480

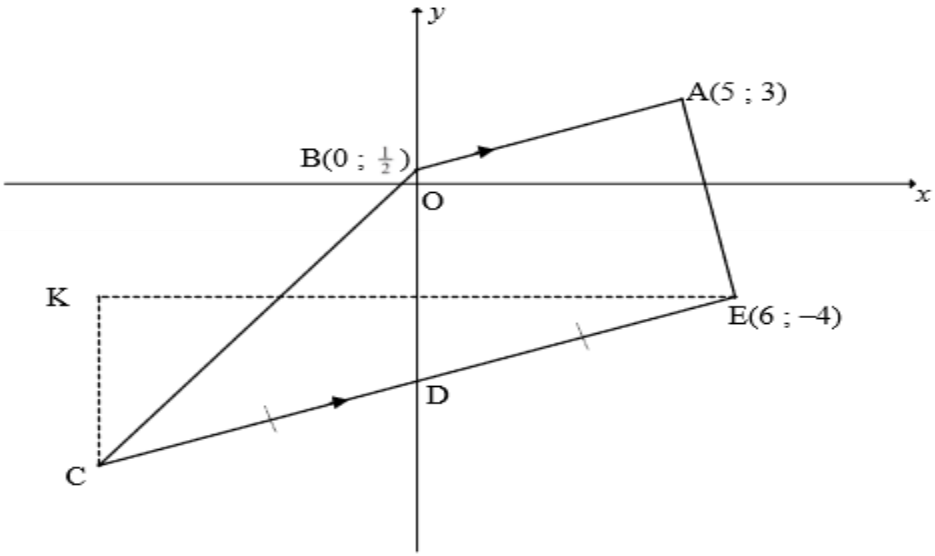


2.1	$a = 634{,}382\ldots$ $b = 32\ 189{,}263\ldots$ $\hat{y} = 634{,}38 + 32189{,}26x$ Answer only 3/3	✓ a ✓ b ✓ equation (3)
2.2	$\hat{y} = 634{,}38 + 32189{,}26(0{,}25)$ $= \text{R}8\ 681{,}70$ **OR/*OF*** $\hat{y} = \text{R}8\ 681{,}70$ (if using calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
2.3	Average price increase $= \text{R}\dfrac{32189{,}26}{20}$ per 0,05 carat $= \text{R}1\ 609{,}46$ per 0,05 carat **OR/*OF*** Average price increase $= 0{,}05 \times 32189{,}26$ $= \text{R}1\ 609{,}46$ per 0,05 carat **OR/*OF*** at 0,3: $\hat{y} = \text{R}10\ 291{,}16$ $\therefore$ Average price increase $= 10\ 291{,}16 - 8\ 681{,}70$ $= \text{R}1\ 609{,}46$ per 0,05 carat Answer only 2/2	✓ divide gradient by 20 ✓ answer (2) ✓ multiply gradient by 0,05 ✓ answer (2) ✓ Estimated price of a 0,3 carat diamond ✓ answer (2)
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.	✓ reason (1)
		[8]

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QUESTION/*VRAAG* 3



3.1	$m_{AB} = \frac{3-\frac{1}{2}}{5-0}$ $m_{AB} = \frac{1}{2}$ Answer only 2/2	✓ substitution ✓ answer (2)
3.2	$m_{CE} = m_{BA} = \frac{1}{2}$ $-4 = \frac{1}{2}(6)+c$ **OR/*OF*** $y-(-4) = \frac{1}{2}(x-6)$ $c = -7$ $y = \frac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; –7) $\frac{x_C+6}{2} = 0$ $\frac{y_C+(-4)}{2} = -7$ $x_C = -6$ $y_C = -10$ C(–6 ; –10) Answer only 3/3	✓ D(0 ; –7) ✓ $x_C = -6$ ✓ $y_C = -10$ (3)
3.3.2	Area $\Delta BCD = \frac{1}{2}(7,5)(6)$ $= 22,5$ Area $\Delta ABD = \frac{1}{2}(7,5)(5)$ $= 18,75$ Area ABCD $= 22,5 + 18,75 = 41,25$ units^2	✓ subst of correct base and height into the area formula ✓ area $\Delta BCD = 22,5$ ✓ area $\Delta ABD = 18,75$ ✓ answer (4)

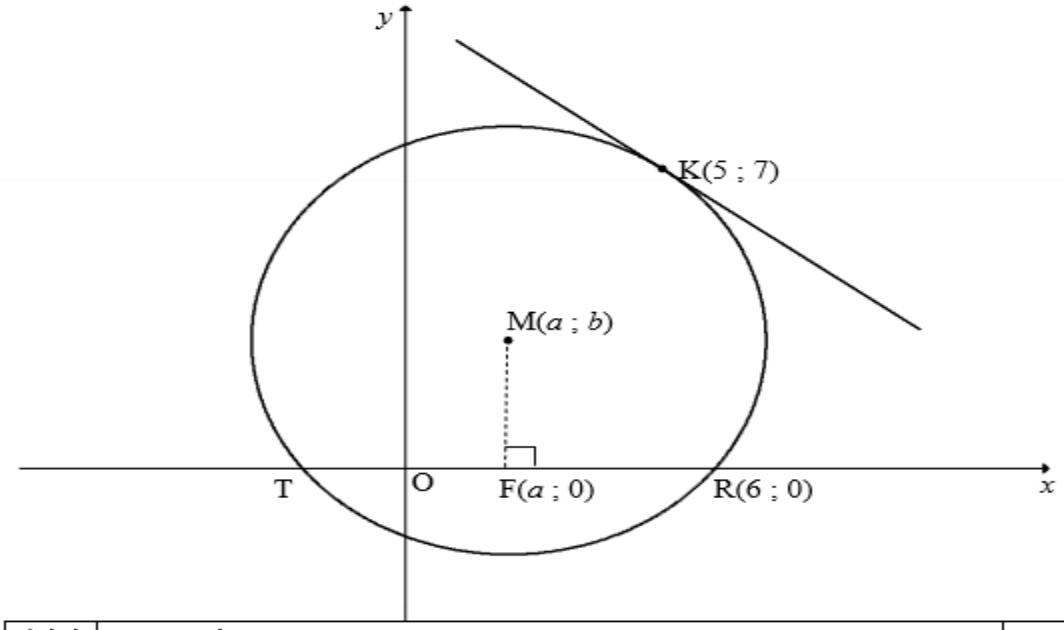
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3.4.1	K(–6 ; –4)	✓ $x_K = -6$ ✓ $y_K = -4$ (2)
3.4.2a	KC = 6 units; KE = 12 units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter $\Delta KEC = 6 + 12 + \sqrt{180}$ $= 31,42$ units	✓ KC = 6 units ✓ KE = 12 units ✓ CE ✓ answer (4)
3.4.2b	$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$ $K\hat{C}E = 63,43°$ **OR/*OF*** $\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$ **OR/*OF*** $m_{CE} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26,57°$ $K\hat{C}E = 90° - 26,57°$ $K\hat{C}E = 63,43°$ **OR/*OF*** $KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$ $(12)^2 = (6)^2 + \left(\sqrt{180}\right)^2 - 2(6)\left(\sqrt{180}\right)(\cos K\hat{C}E)$ $\cos K\hat{C}E = \frac{\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$	✓ trig ratio ✓ $\tan K\hat{C}E = 2$ ✓ answer (3) ✓ trig ratio ✓ $\sin K\hat{C}E = \frac{12}{\sqrt{180}}$ ✓ answer (3) ✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26,57°$ ✓ answer (3) ✓ substitution into cosine rule ✓ trig ratio ✓ answer (3)
		[21]

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QUESTION/*VRAAG* 4



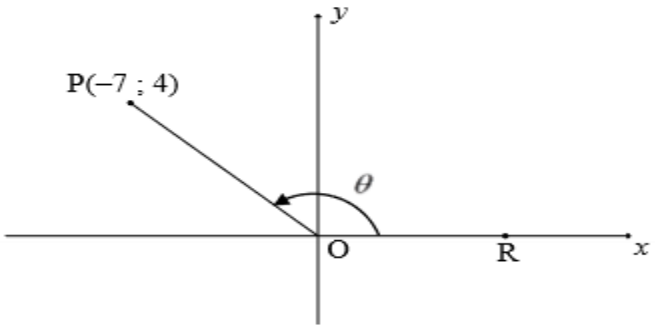
4.1.1	$y = x + 1$ $b = a + 1$	✓ $b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$ $(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore$M(2 ; 3)	✓ equating radii / solving simultaneously ✓ substitution $b = a + 1$ ✓ $12a = 24$ ✓ $a = 2$ ✓ $b = 3$ (5)
4.2.1	$(6-2)^2 + (0-3)^2 = r^2$ $r = 5$ **OR/*OF*** $(2-5)^2 + (3-7)^2 = r^2$ $r = 5$ Answer only 2/2	✓ substitution R and M ✓ $r = 5$ (2) ✓ substitution K and M ✓ $r = 5$ (2)

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4.2.2	T(–2 ; 0) TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** $(x-2)^2+(0-3)^2=25$ $x^2-4x+4+9=25$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ TR = 8 units Answer only 2/2	✓ T(–2 ; 0) ✓ answer (2) ✓ 4 units ✓ answer (2) ✓ x values ✓ answer (2)
4.3	$m_{\text{radius}}=\frac{7-3}{5-2}$ $m_{\text{radius}}=\frac{4}{3}$ $m_{\text{tangent}}=-\frac{3}{4}$ $7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$ $c=\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$	✓ substitution ✓ $m_{\text{radius}}=\frac{4}{3}$ ✓ $m_{\text{tangent}}=-\frac{3}{4}$ ✓ substitution ✓ answer (5)
4.4.1	N(2 ; –2)	✓ $x_N=2$ ✓ $y_N=-2$ (2)
4.4.2	$(-2-2)^2+(0+2)^2=r^2$ $r^2=20$ $(x-2)^2+(y+2)^2=20$	✓ substitution ✓ $r^2=20$ ✓ answer (3)
		[20]

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QUESTION/*VRAAG* 5



5.1.1	$OP=\sqrt{(-7)^2+(4)^2}$ $=\sqrt{65}$ Answer only 2/2	✓ substitution ✓ answer (2)
5.1.2(a)	$\tan\theta=\frac{4}{-7}$	✓ answer (1)
5.1.2(b)	$\cos(\theta-180°) = -\cos\theta$ $= \frac{7}{\sqrt{65}}$	✓ reduction ✓ answer (2)
5.2	$\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.180°$; $k\in Z$ **OR/*OF*** $\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.360°$ or $x=251{,}57°+k.360°;\ k\in Z$	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ ✓ $+k.180°;\ k\in Z$ (7) ✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ and $251{,}57°$ ✓ $+k.360°;\ k\in Z$ (7)

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5.3.1	$\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{1+\cos 3x}{1+\cos 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 3x)}$ $= \frac{(\sin 3x)(1+\cos 3x)}{1-\cos^2 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{\sin^2 3x}$ $= \frac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$ **OR/*OF*** $\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{\sin 3x}{\sin 3x}$ $= \frac{\sin^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{1-\cos^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{(1-\cos 3x)(1+\cos 3x)}{\sin 3x(1-\cos 3x)}$ $= \frac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$	✓ multiply by "1" ✓ $1-\cos^2 3x$ ✓ square identity (3) ✓ multiply by "1" ✓ square identity ✓ factors (3)
5.3.2	undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$ $3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$ $x = 0°$ or $x = 60°$	✓ $\sin 3x = 0$ and $1-\cos 3x = 0$ ✓ $0°$ ✓ $60°$ (3)
		[18]

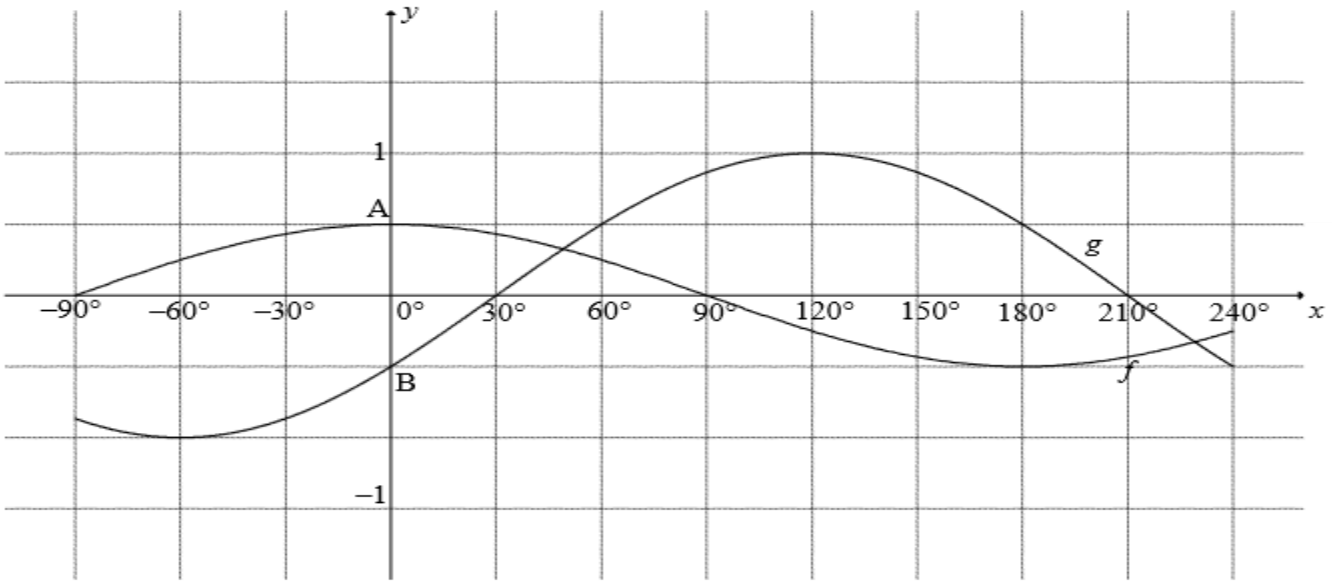
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QUESTION/*VRAAG* 6

6.1	$\frac{\sin 10^\circ}{\cos 440^\circ} + \tan(360^\circ - \theta).\sin 2\theta$ $= \frac{\cos 80^\circ}{\cos 80^\circ} - \tan\theta(2\sin\theta\cos\theta)$ $= 1 - \frac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)$ $= 1 - 2\sin^2\theta$ $= \cos 2\theta$	✓ $-\tan\theta$ ✓ $\cos 80^\circ$ ✓ co-ratio ✓ double angle ✓ quotient identity ✓ answer (6)
6.2.1	$\sin(60^\circ + 2x) + \sin(60^\circ - 2x) = k\cos 2x$ $(\sin 60^\circ\cos 2x + \cos 60^\circ\sin 2x) + (\sin 60^\circ\cos 2x - \cos 60^\circ\sin 2x) = k\cos 2x$ $2\sin 60^\circ\cos 2x = k\cos 2x$ $2\left(\frac{\sqrt{3}}{2}\right)\cos 2x = k\cos 2x$ $\therefore k = \sqrt{3}$	✓ both expansions correct ✓ special ∠s ✓ answer (3)
6.2.2	$\tan 60^\circ[\sin(60^\circ + 2x) + \sin(60^\circ - 2x)]$ $= \tan 60^\circ[k\cos 2x]$ $= \sqrt{3}\left(\sqrt{3}\cos 2x\right)$ $= 3(2\cos^2 x - 1)$ $= 3\left(2\left(\sqrt{t}\right)^2 - 1\right)$ $= 6\left(\sqrt{t}\right)^2 - 3$ $= 6t - 3$	✓ special ∠ ✓ double ∠s ✓ answer i.t.o t (3)
		[12]

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QUESTION/*VRAAG* 7



7.1	$A\left(0;\frac{1}{2}\right)$ $B\left(0;-\frac{1}{2}\right)$ $AB=\frac{1}{2}-\left(-\frac{1}{2}\right)$ $=1$ unit Answer only 2/2	✓ y-values ✓ answer (2)
7.2	Range of f: $y\in\left[-\frac{1}{2};\frac{1}{2}\right]$ Range of $3f(x)+2$: $y\in\left[\frac{1}{2};3\frac{1}{2}\right]$ **OR/*OF*** $\frac{1}{2}\le y\le 3\frac{1}{2}$	✓ critical values ✓ answer (2)
7.3	$x=90°$	✓✓ $x=90°$ (2)
7.4.1	$x\in(30°;90°)\cup(210°;240°]$ **OR/*OF*** $30°<x<90°$ or $210°<x\le 240°$	✓ $x\in(30°;90°)$ ✓ $(210°;240°]$ (2) ✓ $30°<x<90°$ ✓ $210°<x\le 240°$ (2)
7.4.2	$x\in(-55°;125°)$ **OR/*OF*** $-55°<x<125°$	✓ critical values ✓ answer (2) ✓ critical values ✓ answer (2)
		[10]

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QUESTION/*VRAAG* 8

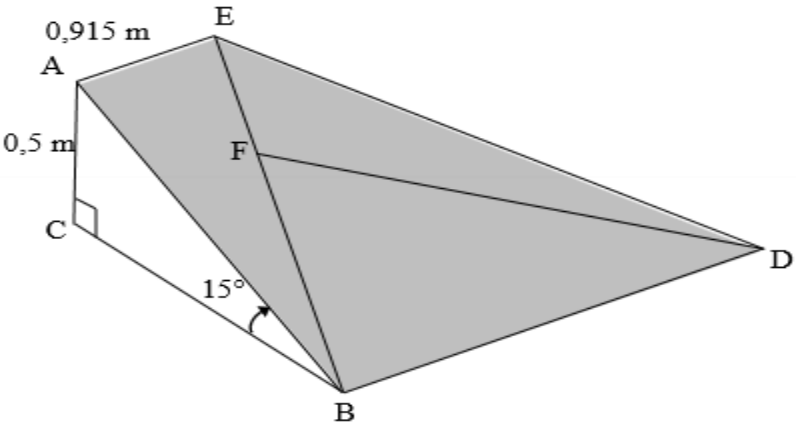


FIGURE I

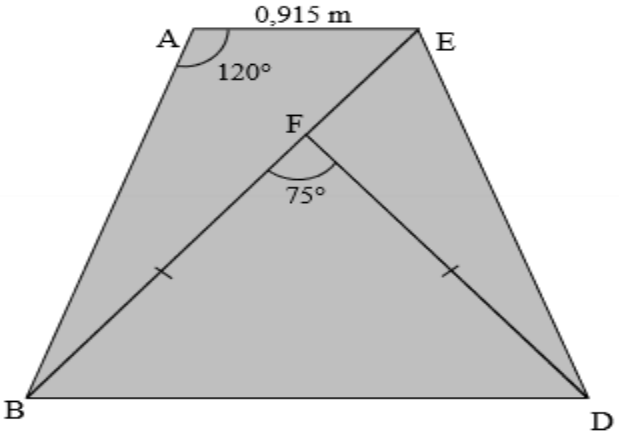


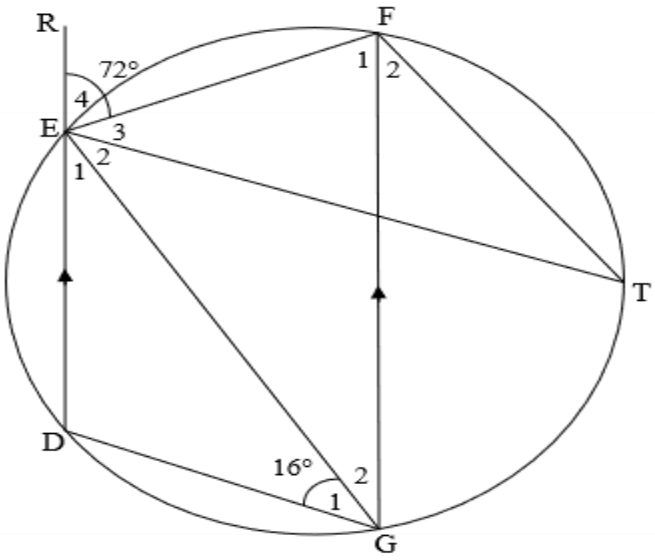
FIGURE II (top view)

8.1	$\frac{0,5}{AB} = \sin 15°$ $AB = \frac{0,5}{\sin 15°}$ $AB = 1,93 \text{ m}$ Answer only 2/2	✓ trig ratio ✓ answer (2)
8.2	$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos B\hat{A}E$ $BE^2 = (1,93)^2 + (0,915)^2 - 2(1,93)(0,915)(\cos 120°)$ $BE = 2,52 \text{ m}$	✓ correct use of cosine rule ✓ substitution ✓ answer (3)
8.3	$BF = FD = \frac{5}{7}(2,52) = 1,80 \text{ m}$ $\text{Area } \Delta BFD = \frac{1}{2}(BF)(FD)\sin B\hat{F}D$ $= \frac{1}{2}(1,8)(1,8)(\sin 75°)$ $= 1,56 \text{ m}^2$	✓ BF ✓ correct substitution into the area rule ✓ answer (3)
		[8]

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QUESTION/*VRAAG* 9

9.1

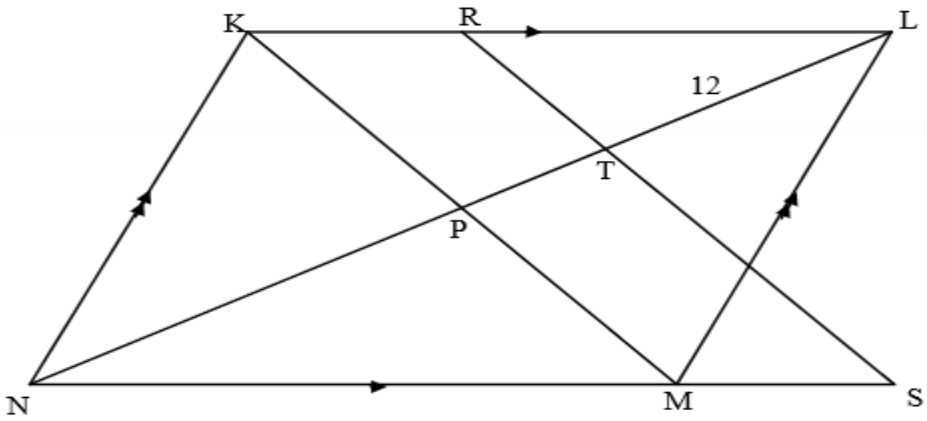


9.1.1	$D\hat{G}F = \hat{E}_4 = 72°$ [ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S ✓ R (2)
9.1.2	$\hat{G}_2 = 72° - 16° = 56°$ $\hat{T} = \hat{G}_2 = 56°$ [∠s in the same seg/ *∠e in dies.* ⊙ *segment*]	✓ S ✓ S / R (2)
9.1.3	$\hat{F}_1 = \hat{E}_4 = 72°$ [alt ∠s; DE ‖ GF / *verw. ∠e*; DE ‖ GF] $\therefore G\hat{E}F = 52°$ [sum of ∠s in Δ / *∠e van* Δ] **OR/*OF*** $\hat{E}_1 = 56°$ [alt ∠s; DE ‖ GF / *verw. ∠e*; DE ‖ GF] $\therefore G\hat{E}F = 52°$ [∠s on a str. line/ *∠e op 'n reguitlyn*]	✓ S / R ✓ S (2) ✓ S / R ✓ S (2)

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9.2



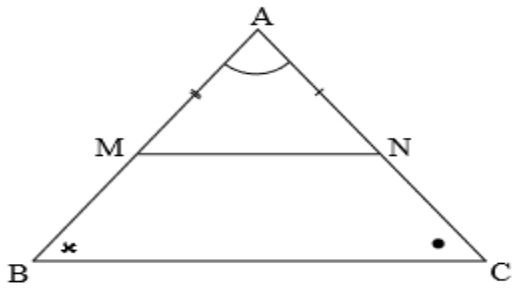
9.2.1	NP = PL = 16 [diag of ||m / *hoeklyne van ||m*] PT = 4 NP : PT = 16 : 4 = 4 : 1	✓ S ✓ R ✓ S ✓ answer (4)
9.2.2	NM : MS = 4 : 1 NP : PT = NM : MS KM ‖ RS [line divides two sides of Δ in prop / *Lyn verdeel 2 sye v Δ eweredig*] **OR/*OF*** [converse prop theorem / *omgekeerde lyn ‖ een sy v Δ*]	 ✓ S ✓ R (2)
9.2.3	$\frac{RL}{KL} = \frac{TL}{LP}$ [prop theorem; KM ‖ RS **OR** line ‖ one side of Δ / *Lyn ‖ een sy v Δ*] $RL = \frac{12 \times 21}{16}$ = 15,75	✓ S ✓ R ✓ S ✓ answer (4)

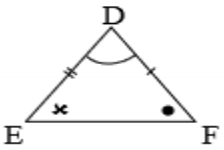
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	OR / *OF*		
	NM : MS = 4 : 1		
	KR = MS = 5,25	[opp side of $\|\|^{m}$ / *teenoorst. sye van* $\|\|^{m}$]	✓ S ✓ R
	KL = NM = 21		
	RL + 5,25 = 21		✓ S
	RL = 15,75		✓ answer
			(4)
			[16]

QUESTION/*VRAAG* 10

10.1

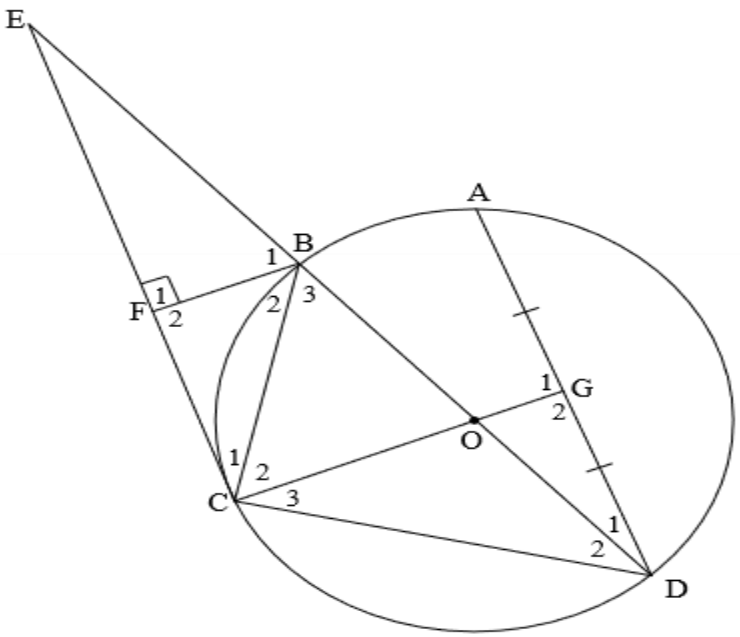




10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. Proof: In Δ AMN and Δ DEF AM = DE [Constr / *Konstruksie*] AN = DF [Constr / *Konstruksie*] $\hat{A} = \hat{D}$ [Given /*Gegee*] $\therefore \Delta AMN \equiv \Delta DEF$ [*s*, ∠, *s*] $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ‖ BC [corresp ∠'s are equal/ *ooreenk. ∠e gelyk*] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ‖ one side of Δ **OR/*OF*** prop theorem; MN ‖BC / *Lyn ‖ een sy v Δ*] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM=DE and AN=DF]	✓Constr ✓S ✓R ✓S /R ✓S ✓R (6)

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10.2



10.2.1(a)	$F\hat{C}O = 90°$ [tan ⊥ radius / *raaklyn ⊥ radius*] $\hat{F}_1 = 90°$ [BF ⊥ EC] $\therefore F\hat{C}O = \hat{F}_1 = 90°$ FB ∥ CG [corresp ∠s = / *ooreenk. ∠ gelyk*]	✓ S / R ✓ S ✓ R (3)
10.2.1(b)	In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / $\angle \frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] $\hat{B}_2 = \hat{B}_3$ [sum of ∠s in Δ / ∠*e van* Δ] $\therefore$ ΔFCB ||| ΔCDB **OR/*OF*** In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / $\angle \frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] $\therefore$ ΔFCB ||| ΔCDB [∠,∠,∠]	✓ S /R ✓ S ✓ S ✓ R ✓ S ✓ S / R ✓ S ✓ S ✓ R ✓ R (5)

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10.2.2	$\hat{G}_1 = 90°$ [line from centre to midpt of chord / *midpt.* ⊙; *midpt. koord*]	✓ R (1)
10.2.3	In ΔGCD and ΔCDB $\hat{G}_2 = B\hat{C}D = 90°$ $\hat{C}_3 = \hat{D}_2$ [∠s opp equal sides /∠*e teenoor gelyke sye*] $G\hat{D}C = \hat{B}_3$ [sum of ∠s in Δ /∠*e van* Δ] $\therefore \Delta GCD \,|||\, \Delta CDB$ [∠,∠,∠] $\therefore \frac{CD}{DB} = \frac{CG}{CD}$ [||| Δs] $\therefore CD^2 = CG.DB$	✓ identifying Δs ✓ S ✓ S / R ✓ S **OR** ✓ R ✓ S (5)
10.2.4	$\frac{BC}{DB} = \frac{FB}{BC}$ [ΔFCB ||| ΔCDB] $\therefore BC^2 = DB.FB$ $CD^2 + BC^2 = CG.DB + DB.FB$ $DB^2 = DB(CG + FB)$ $DB = CG + FB$	✓ S ✓ R ✓ S ✓ sum ✓ $DB^2 = CD^2 + BC^2$ (5)
		[25]

TOTAL/*TOTAAL*: 150

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SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

MAY/JUNE 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.

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QUESTION 1

1.1 The owner of a small company wishes to establish whether advertising in a regional newspaper is effective. The table below shows the amount spent on advertising and the corresponding sales figures for the last 9 years.

Amount spent on advertising (in rands) (x)	21 300	23 700	24 800	30 540	24 100	40 680	22 400	35 250	29 110
Sales (in rands) (y)	311 500	326 700	349 200	470 000	316 100	564 200	314 000	487 300	392 900

1.1.1 Determine the equation of the least squares regression line for the data. (3)

1.1.2 Predict the sales for a year in which the company will spend R28 500 on advertising. (2)

1.1.3 Write down the correlation coefficient of the data. (1)

1.1.4 Describe the association between the amount spent on advertising in the regional newspaper and the sales of this company. (1)

1.2 The profit that the small company made over the same 9 years is given in the table below.

Profit (in rands)	110 750	107 376	152 338	244 480	144 021	275 994	121 900	207 636	187 700

1.2.1 Calculate the mean profit made over the 9 years. (2)

1.2.2 Write down the standard deviation for the data. (1)

1.2.3 Determine the number of years in which the company made a profit that was greater than one standard deviation above the mean. (2)
[12]

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QUESTION 2

The ages of the people who attended a music concert was summarised in the table below.

AGE	NUMBER OF PEOPLE
$5 < x \leq 15$	20
$15 < x \leq 25$	25
$25 < x \leq 35$	60
$35 < x \leq 45$	90
$45 < x \leq 55$	55
$55 < x \leq 65$	40
$65 < x \leq 75$	30

2.1 Write down the modal class of the data. (1)

2.2 How many people attended the music concert? (1)

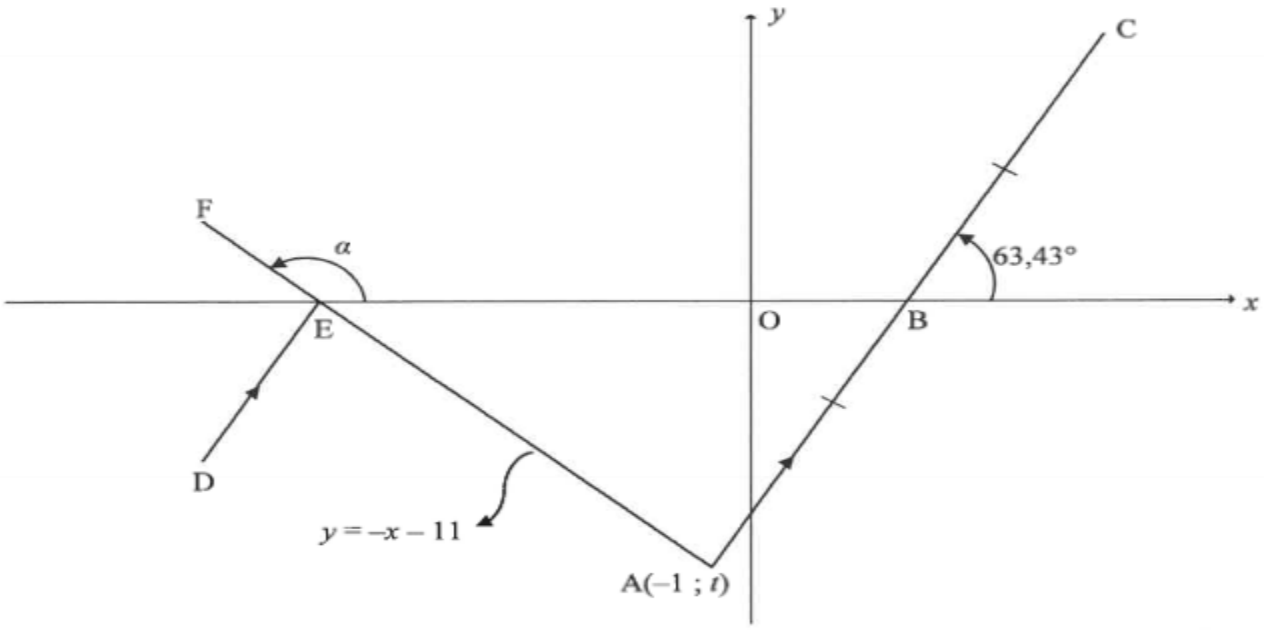
2.3 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (4)

2.4 Use the cumulative frequency graph to determine the median age of the people who attended the music concert. (2)
[8]

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QUESTION 3

In the diagram, the equation of line AF is $y = -x - 11$. B, a point on the x-axis, is the midpoint of the straight line joining A$(-1 ; t)$ and C. The angles of inclination of AF and AC are α and 63,43° respectively. AF cuts the x-axis in E. D is a point such that DE ∥ AC.



3.1 Calculate the:

3.1.1 Value of t (2)

3.1.2 Size of α (2)

3.1.3 Gradient of AC, to the nearest whole number (2)

3.2 Determine the equation of AC in the form $y = mx + k$. (2)

3.3 Calculate the:

3.3.1 Coordinates of C (3)

3.3.2 Size of FÊD (3)

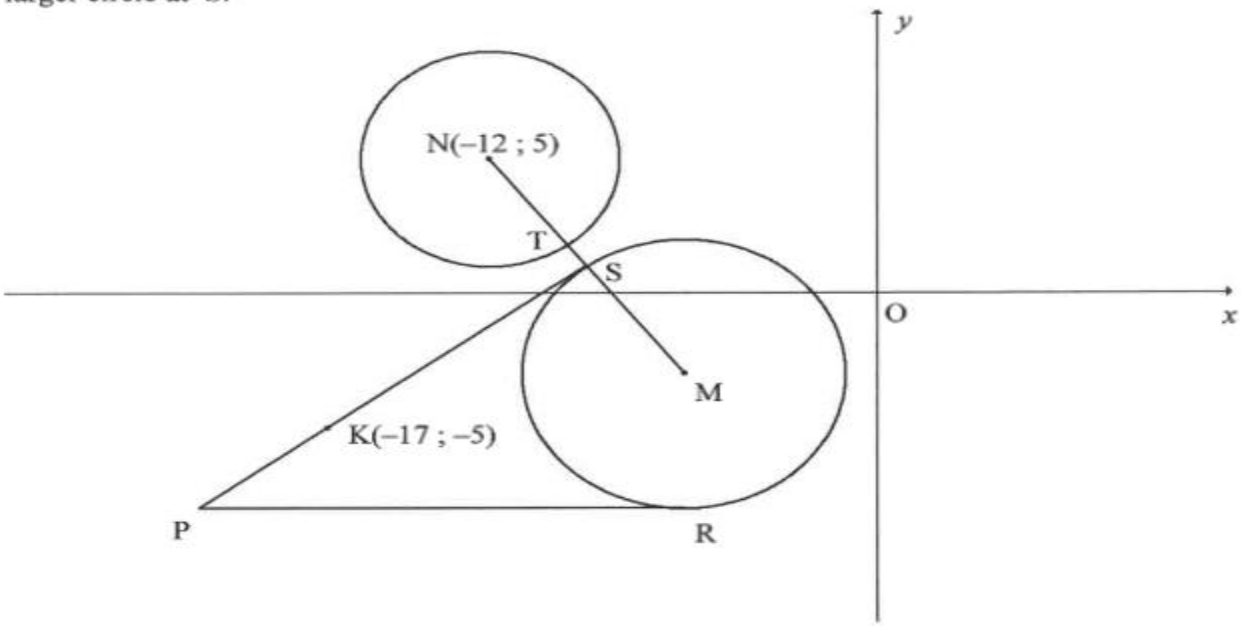
3.4 G is a point such that EAGC, in that order, is a parallelogram.

Determine the equation of a circle centred at G and passing through the point B.
Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)
[18]

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QUESTION 4

In the diagram, the equation of the circle centred at N(–12 ; 5) is $x^2 + y^2 + 24x - 10y + 153 = 0$. The equation of the circle centred at M is $(x+6)^2 + (y+3)^2 = 25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x-axis. K(–17 ; –5) is a point on PS. The straight line joining N and M cuts the smaller circle at T and the larger circle at S.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the:

4.2.1 Length of the radius of the smaller circle (2)

4.2.2 Length of TS (4)

4.3 Determine the equation of the tangent:

4.3.1 PR (2)

4.3.2 PS, in the form $y = mx + c$ (5)

4.4 Quadrilateral PSMR is drawn. Calculate the:

4.4.1 Perimeter of PSMR (5)

4.4.2 Ratio of $\dfrac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ (2)

[22]

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QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to a single trigonometry ratio:

$$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$$ (5)

5.2 Given that $\cos 20° = p$

Without using a calculator, write EACH of the following in terms p:

5.2.1 $\cos 200°$ (2)

5.2.2 $\sin(-70°)$ (2)

5.2.3 $\sin 10°$ (3)

5.3 Determine, **without using a calculator**, the value of:

$$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$$ (3)

5.4 Consider: $\frac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}=-\sin x$

5.4.1 Prove the above identity. (3)

5.4.2 Determine the value of $\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$ (3)

5.5 Given: $3\tan 4x=-2\cos 4x$

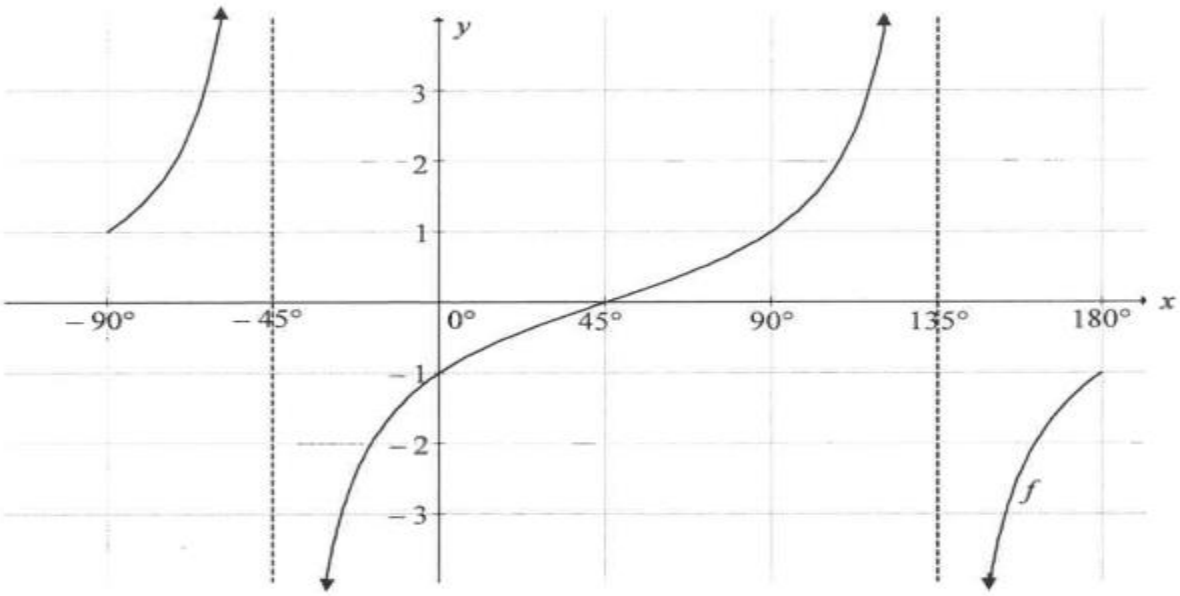
5.5.1 **Without using a calculator**, show that $\sin 4x=-0,5$ is the only solution to the above equation. (4)

5.5.2 Hence, determine the general solution of x in the equation $3\tan 4x=-2\cos 4x$ (3)
[28]

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QUESTION 6

In the diagram below, the graph of $f(x)=\tan(x-45°)$ is drawn for $x\in[-90°;180°]$.



6.1 Write down the period of f. (1)

6.2 Draw the graph of $g(x)=-\cos 2x$ for the interval $x\in[-90°;180°]$ on the grid given in the ANSWER BOOK. Show ALL intercepts with the axes, as well as the minimum and maximum points of the graph. (3)

6.3 Write down the range of g. (1)

6.4 The graph of g is shifted $45°$ to the left to form the graph of h. Determine the equation of h in its simplest form. (2)

6.5 Use the graph(s) to determine the values of x in the interval $x\in[-90°;90°]$ for which:

6.5.1 $f(x)>1$ (2)

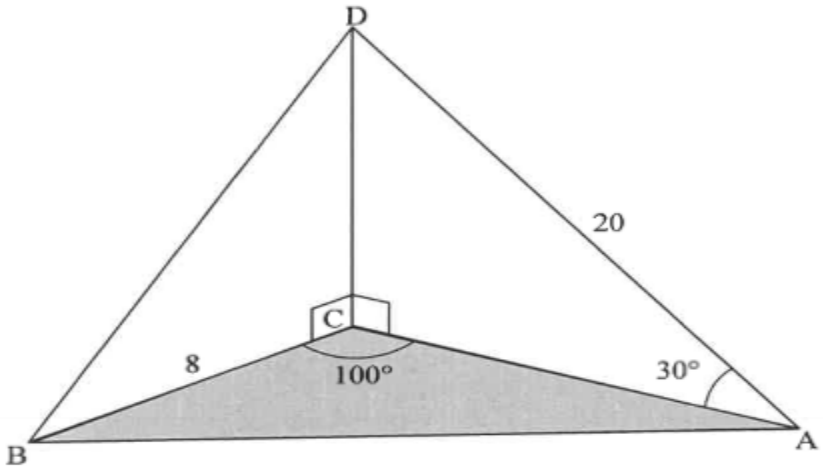
6.5.2 $2\cos 2x-1>0$ (4)

[13]

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QUESTION 7

In the diagram, A, B and C are points in the same horizontal plane. D is a point directly above C, that is DC ⊥ AC and DC ⊥ BC. It is given that $A\hat{C}B=100°$, $C\hat{A}D=30°$, AD = 20 units and BC = 8 units.



7.1 Calculate the length of:

7.1.1 AC (2)

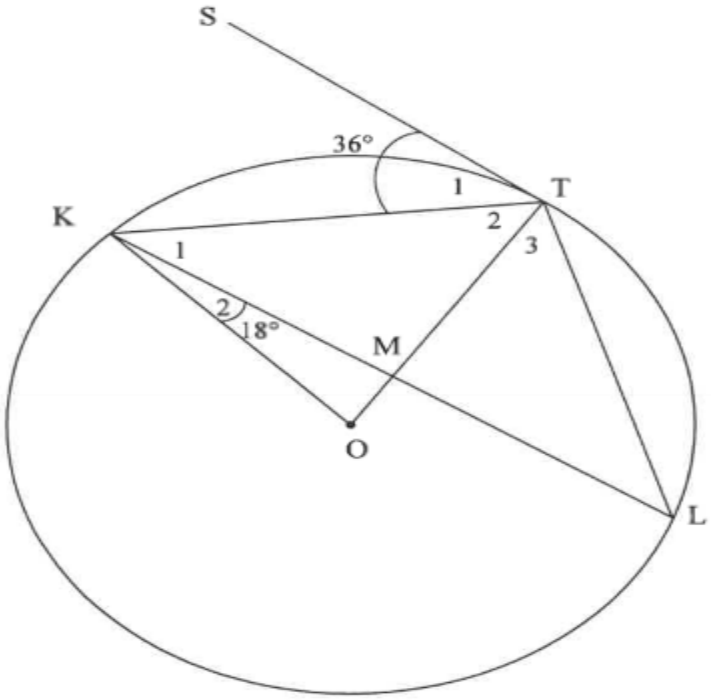
7.1.2 AB (3)

7.2 If it is further given that $A\hat{B}D = 73,4°$, calculate the size of $A\hat{D}B$. (3)
[8]

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QUESTION 8

8.1 In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersects KL at M. ST is a tangent to the circle at T. $S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.



8.1.1 Determine, giving reasons, the size of:

(a) $\hat{T}_2$ (2)

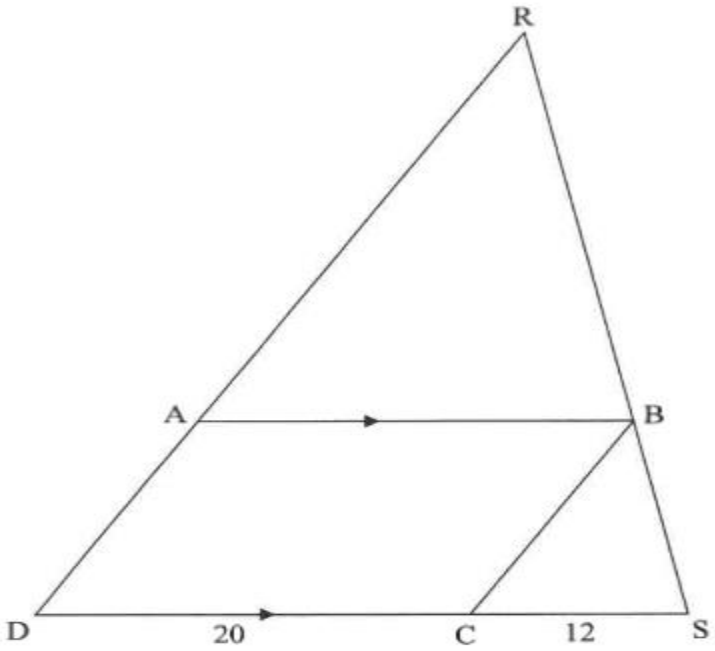
(b) $\hat{L}$ (2)

(c) $K\hat{O}T$ (2)

8.1.2 Prove, giving reasons, that KM = ML. (3)

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8.2 In the diagram, ΔRDS is drawn. A, B and C are points on RD, RS and DS respectively such that AB || DS and RB : BS = 5 : 3. DC = 20 units and CS = 12 units.



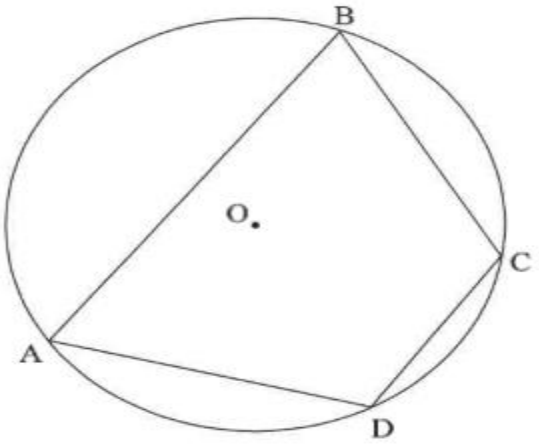
8.2.1 Prove, giving reasons, that BC || AD. (3)

8.2.2 If it is further given that RD = 48 units, calculate, giving reasons, the value of the ratio AD : AB. (3)
[15]

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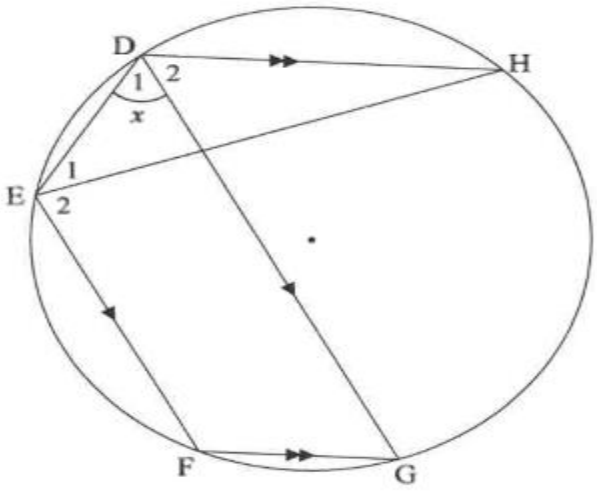
QUESTION 9

9.1 In the diagram, O is the centre of the circle. ABCD is a cyclic quadrilateral.



Use the diagram in the ANSWER BOOK to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is prove that $\hat{B} + \hat{D} = 180°$. (5)

9.2 In the diagram, DEFG is a cyclic quadrilateral such that EF || DG. H is another point on the circle such that DH || FG. Chord EH is drawn. Let $\hat{D}_1 = x$.

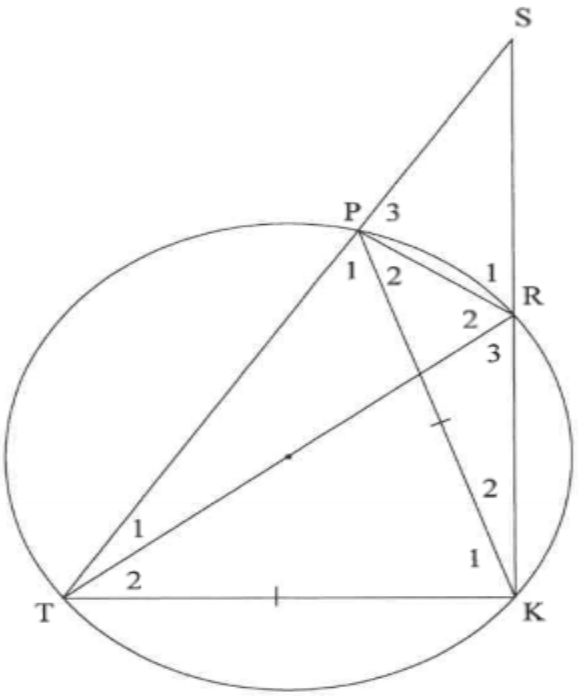


Prove, giving reasons, that $\hat{D}_1 = \hat{D}_2$. (4)
[9]

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QUESTION 10

In the diagram, TR is a diameter of the circle. PRKT is a cyclic quadrilateral. Chords TP and KR are produced to intersect at S. Chord PK is drawn such that PK = TK.



10.1 Prove, giving reasons, that:

10.1.1 SR is a diameter of a circle passing through points S, P and R (4)

10.1.2 $\hat{S} = \hat{P}_2$ (5)

10.1.3 $\Delta SPK \;|||\; \Delta PRK$ (3)

10.2 If it is further given that SR = RK, prove that $ST = \sqrt{6}RK$. (5)
[17]

TOTAL: 150

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MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

MAY/JUNE/*MEI/JUNIE* 2023

MARKS: 150
PUNTE: 150

These marking guidelines consist of 21 pages./
Hierdie nasienriglyne bestaan uit 21 bladsye.

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QUESTION/*VRAAG 1*

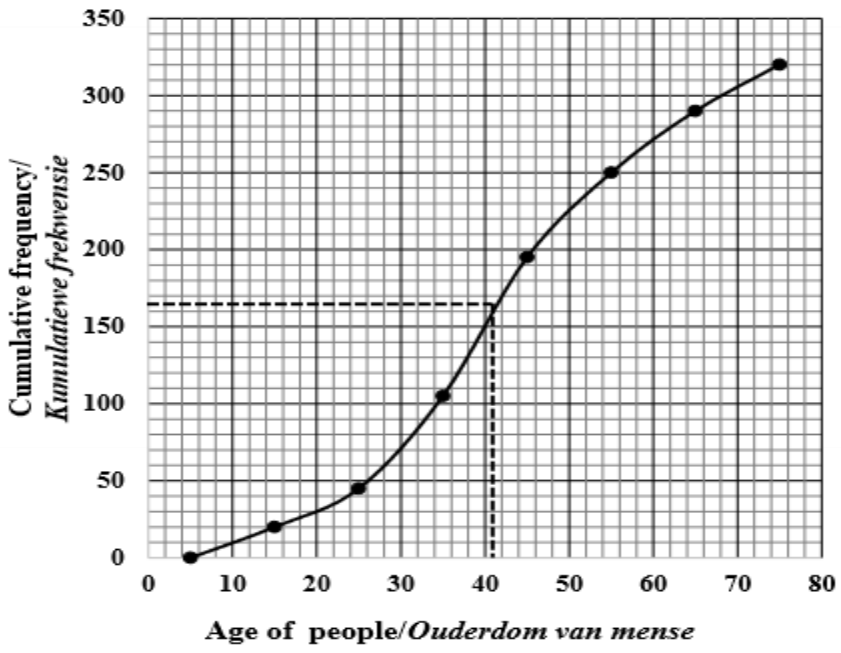
1.1.1	$a = 1730{,}22$ $b = 13{,}96$ $\hat{y} = 1730{,}22 + 13{,}96x$	✓ $a = 1730{,}22$ ✓ $b = 13{,}96$ ✓ equation (3)
1.1.2	$\hat{y} = 1730{,}22 + 13{,}96x$ $\hat{y} = 1730{,}22 + 13{,}96(28\,500)$ $\hat{y} = \text{R}399\,590{,}22$ **OR/*OF*** $\hat{y} = \text{R}399\,599{,}64$ (calc)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.1.3	$r = 0{,}98002\ldots$ $r = 0{,}98$	✓ answer (1)
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. / *Daar is 'n baie sterk positiewe korrelasie tussen die bedrag spandeer op advertensie en die verkope.*	✓ strong positive (1)
1.2.1	$\bar{x} = \frac{1\,552\,195}{9}$ $\bar{x} = 172\,466{,}11$	✓ $\bar{x} = \frac{1\,552\,195}{9}$ ✓ answer (2)
1.2.2	$\sigma = 56\,950{,}09$	✓ answer (1)
1.2.3	$\bar{x} + \sigma$ $= 172\,466{,}11 + 56\,950{,}09$ $= 229\,416{,}20$ 2 years/*jaar*	✓ $\bar{x} + \sigma$ ✓ answer (2)
		[12]

QUESTION/*VRAAG 2*

2.1	$35 < x \leq 45$	✓ answer (1)
2.2	320 people/*mense*	✓ answer (1)
2.3		

AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY
$5 < x \leq 15$	20	20
$15 < x \leq 25$	25	45
$25 < x \leq 35$	60	105
$35 < x \leq 45$	90	195
$45 < x \leq 55$	55	250
$55 < x \leq 65$	40	290
$65 < x \leq 75$	30	320

OGIVE/*OGIEF*

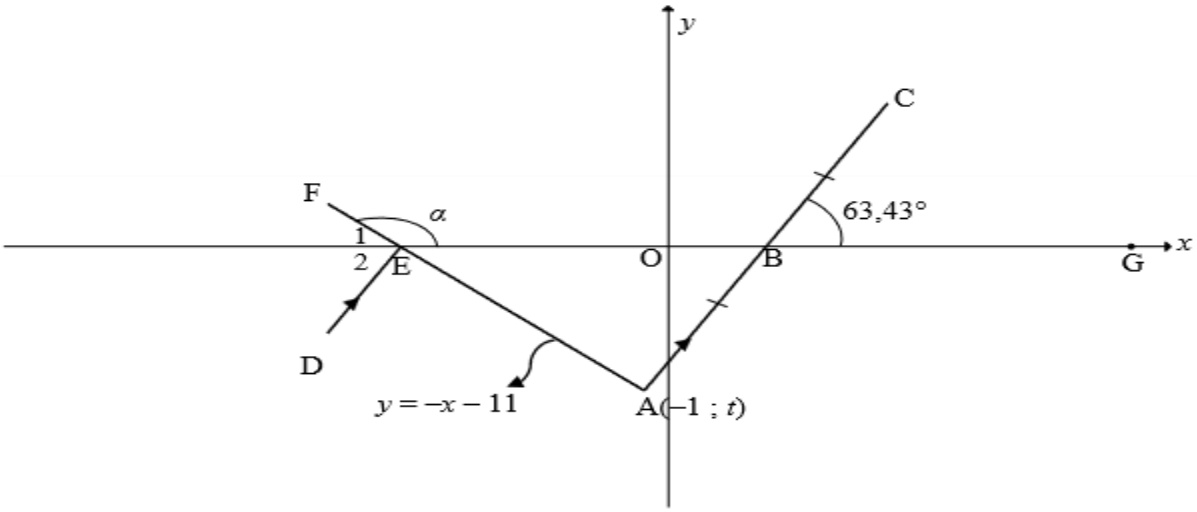


✓ cumulative frequency
✓ grounding
✓ plotting at upper limit
✓ shape (4)

2.4	Median = 41	✓✓ answer (2)
		[8]

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QUESTION/*VRAAG 3*



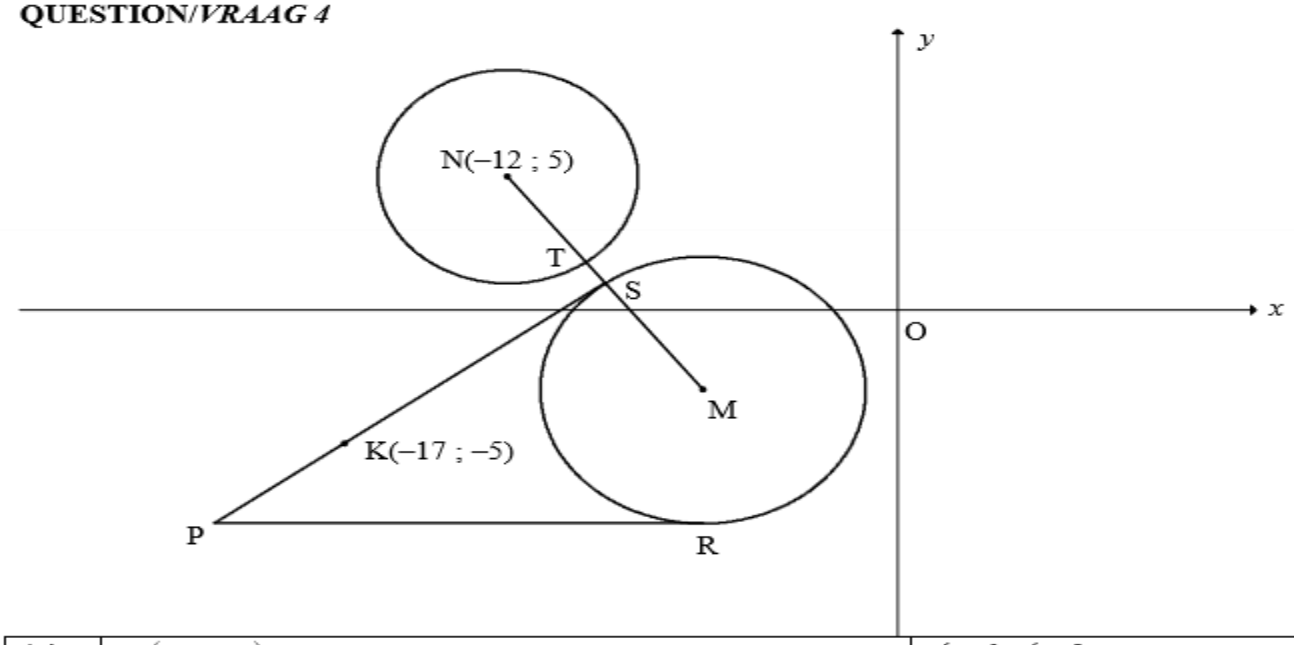
3.1.1	$y=-x-11$ A$(-1\ ;\ t)$ $t=-(-1)-11$ $t=-10$	✓ substitution ✓ value of t (2)
3.1.2	$\tan\alpha=-1$ $ref.\angle=45°$ $\therefore\alpha=135°$	✓ $\tan\alpha=-1$ ✓ $135°$ (2)
3.1.3	$\tan 63{,}43°=m_{AC}$ $m_{AC}=2$	✓ $\tan 63{,}43°=m_{AC}$ ✓ answer (2)
3.2	$m_{AC}=2$ A$(-1\ ;-10)$ $y=2x+k$ **OR/*OF*** $y-y_1=2(x-x_1)$ $-10=2(-1)+k$ $y-(-10)=2(x-(-1))$ $k=-8$ $y=2x-8$ $y=2x-8$	✓ substitution of m and A ✓ equation (2)

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3.3.1	$y=2x-8$ $0=2x-8$ $x_B=4$ $\frac{x_C+(-1)}{2}=4$ $\qquad$ $\frac{y_C+(-10)}{2}=0$ $x_C=9$ $\qquad$ $y_C=10$ **OR/*OF*** by translation / *met translasie* $A\rightarrow B\ (x;y)\rightarrow(x+5;y+10)$ $B\rightarrow C\ (4;0)\rightarrow(4+5;\ 0+10)=(9\ ;\ 10)$	✓ $x_B=4$ ✓ $x_C=9$ ✓ $y_C=10$ (3) ✓ $(x+5;y+10)$ ✓ $x_C=9$ ✓ $y_C=10$ (3)
3.3.2	$A\hat{B}E=63,43°$ [vert. opp ∠'s =] $\hat{E}_2=63,43°$ [corres. ∠'s, DE ‖ AB] $\hat{E}_1=45°$ [∠s on a str line] $F\hat{E}D=108,43°$ **OR/*OF*** $E\hat{A}B=135°-63,43°$ $E\hat{A}B=71,57°$ $D\hat{E}A=E\hat{A}B=71,57°$ $F\hat{E}D=108,43°$ **OR/*OF*** $A\hat{B}E=63,43°$ [vert. opp ∠'s] $D\hat{E}O=116,57°$ [co-int. ∠'s, DE ‖ AB] $F\hat{E}D=360°-(116,57°+135°)$ $=108,43°$	✓ $A\hat{B}E=63,43°$ ✓ $\hat{E}_1=45°$ ✓ $F\hat{E}D=108,43°$ (3) ✓ $E\hat{A}B=71,57°$ ✓ $D\hat{E}A=E\hat{A}B=71,57°$ ✓ $F\hat{E}D=108,43°$ (3) ✓ $A\hat{B}E=63,43°$ ✓ $D\hat{E}O=116,57°$ ✓ $F\hat{E}D=108,43°$ (3)
3.4	$y=0$ $x_E=-11$ $\frac{x_G+(-11)}{2}=4$ $x_G=19$ $(x-19)^2+y^2=15^2$ $(x-19)^2+y^2=225$	✓ $x_E=-11$ ✓ $x_G=19$ ✓ $(x-19)^2+y^2$ ✓ 225 (4)
		[18]

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QUESTION/*VRAAG 4*

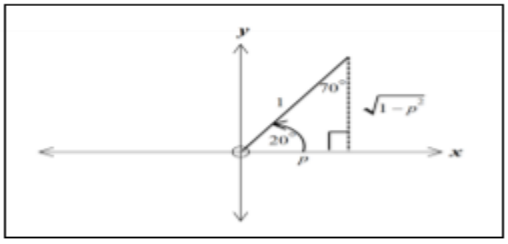


4.1	$M(-6;-3)$	✓ -6 ✓ -3 (2)
4.2.1	$x^2+y^2+24x-10y+153=0$ $(x+12)^2+(y-5)^2=-153+144+25$ $(x+12)^2+(y-5)^2=16$ $r^2=16$ $r=4$ units	✓ $r^2=-153+144+25$ ✓ length of radius (2)
4.2.2	$NM=\sqrt{(-12-(-6))^2+(5-(-3))^2}$ NM = 10 units SM = 5 units $\therefore$ TS = 10 – 5 – 4 = 1 unit	✓ substitution into distance formula ✓ NM = 10 units ✓ SM = 5 units ✓ answer (4)
4.3.1	R(–6 ; –8) $y=-8$	✓ $y_R=-8$ ✓ answer (2)

4.3.2	$m_{NM} = \frac{5-(-3)}{-12-(-6)}$ $m_{NM} = -\frac{4}{3}$ $m_{tangent} = \frac{3}{4}$ $-5 = \frac{3}{4}(-17)+c$ **OR/*OF*** $y-y_1 = \frac{3}{4}(x-x_1)$ $c = \frac{31}{4}$ $y-(-5) = \frac{3}{4}(x-(-17))$ $y = \frac{3}{4}x+\frac{31}{4}$ $y = \frac{3}{4}x+\frac{31}{4}$ **OR/OF** NS = SM = 5 $S\left(\frac{-12-6}{2}; \frac{5-3}{2}\right)$ S (−9 ; 1) $m_{SK} = \frac{1-(-5)}{-9+17}$ $= \frac{6}{8} = \frac{3}{4}$ $y+5 = \frac{3}{4}(x+17)$ $y = \frac{3}{4}x+\frac{31}{4}$ *or* $y = \frac{3}{4}x+7\frac{3}{4}$	✓ substitution ✓ $m_{NM} = -\frac{4}{3}$ ✓ $m_{tangent} = \frac{3}{4}$ ✓ substitution of m and N ✓ equation (5) ✓ S midpoint ✓ coordinates of S ✓ $m_{tangent} = \frac{3}{4}$ ✓ substitution of m and K(−17 ; −5) or S ✓ equation (5)
4.4.1	$-8 = \frac{3}{4}x+\frac{31}{4}$ $-32 = 3x+31$ $3x = -63$ $x = -21$ P(−21 ; −8) R(−6 ; −8) PR = PS = 15 units [tangents from same point] MS = MR = 5 units Perimeter PSMR = 15 + 15 + 5 + 5 = 40 units	✓ $-8 = \frac{3}{4}x+\frac{31}{4}$ ✓ $x = -21$ ✓ PR = PS = 15 units ✓ MS = MR = 5 units ✓ answer (5)

4.4.2	$\frac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ $\frac{\frac{1}{2}NS.SP}{\frac{1}{2}SP.MS+\frac{1}{2}MR.PR}$ $=\frac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $=\frac{1}{2}$ **OR** $\Delta NPS \equiv \Delta SPM \equiv \Delta MPR$ $\frac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ $=\frac{1}{2}$	✓ substitution ✓ answer (2) ✓ congruent ✓ answer (2)
		[22]

QUESTION/*VRAAG 5*

5.1	$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$ $=\frac{1-(-\sin\theta)(-\sin\theta)}{\cos\theta}$ $=\frac{1-\sin^2\theta}{\cos\theta}$ $=\frac{\cos^2\theta}{\cos\theta}$ $=\cos\theta$	✓ $-\sin\theta$ ✓ $-\sin\theta$ ✓ $\cos\theta$ ✓ $\cos^2\theta$ ✓ answer (5)
5.2.1	$\cos 200°$ $=-\cos 20°$ $=-p$	✓ reduction ✓ answer (2)
5.2.2	$\sin(-70°)$ $=-\sin 70°$ $=-\cos 20°$ $=-p$ **OR/*OF*** $\sin(-70°)$ $=-\sin 70°$ $=-p$ 	✓ reduction ✓ answer (2) ✓ reduction ✓ answer (2)
5.2.3	$\sin 10°$ $\cos(2(10°))=1-2\sin^2 10°$ $2\sin^2 10°=1-\cos 20°$ $\sin 10°=\sqrt{\frac{1-\cos 20°}{2}}$ $\sin 10°=\sqrt{\frac{1-p}{2}}$ **OR/*OF*** $\sin 10°$ $\sin(30°-20°)$ $=\sin 30°\cos 20°-\cos 30°\sin 20°$ $=\frac{1}{2}p-\frac{\sqrt{3}}{2}\sqrt{1-p^2}=\frac{p-\sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF***	✓ double angle ✓ $\sin 10°$ as subject ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)

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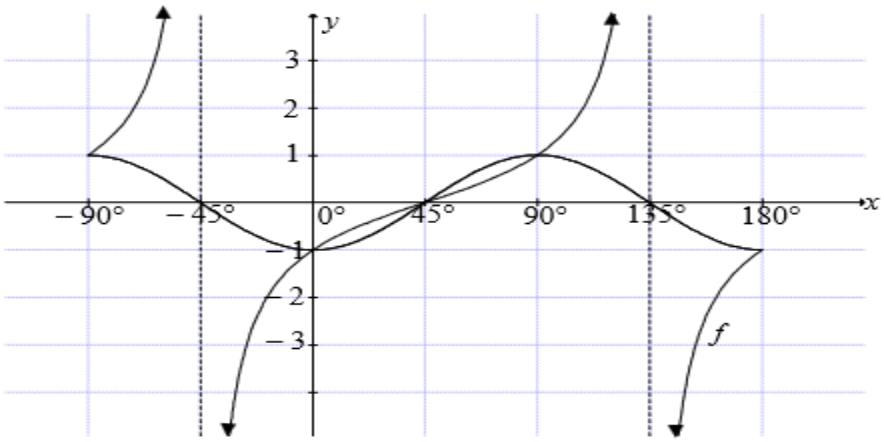
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	$\sin 10°$ $\sin(70°-60°)$ $=\sin 70°\cos 60°-\cos 70°\sin 60°$ $=p.\frac{1}{2}-\sqrt{1-p^2}\times\frac{\sqrt{3}}{2}=\frac{p-\sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF*** $\sin 10°$ $=\cos 80°$ $\cos(60°+20°)$ $=\cos 60°\cos 20°-\sin 60°\sin 20°$ $=\frac{1}{2}p-\frac{\sqrt{3}}{2}.\sqrt{1-p^2}$	✓ using special angle ✓ expanding ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)
5.3	$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$ $=\cos[A+55°-(A+10°)]$ $=\cos 45°$ $=\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	✓✓ compound identity ✓ answer (3)
5.4.1	LHS $=\frac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}$ RHS $=-\sin x$ $=\frac{\cos^2 x-\sin^2 x+2\sin x\cos x-\cos^2 x}{\sin x-2\cos x}$ $=\frac{-\sin^2 x+2\sin x\cos x}{\sin x-2\cos x}$ $=\frac{-\sin x(\sin x-2\cos x)}{\sin x-2\cos x}$ $=-\sin x$ $\therefore$ LHS = RHS	✓ $\cos^2 x-\sin^2 x$ ✓ $2\sin x\cos x$ ✓ common factor of $-\sin x$ (3)
5.4.2	$\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$ $=\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin x(\sin x-2\cos x)}$ $=\frac{\cos 2x+\sin 2x-\cos^2 x}{(\sin x-2\cos x)}\times\frac{1}{-3\sin x}$ $=(-\sin x)\times\frac{1}{-3\sin x}$ $=\frac{1}{3}$	✓ common factor of $-3\sin x$ ✓ substitution ✓ answer (3)

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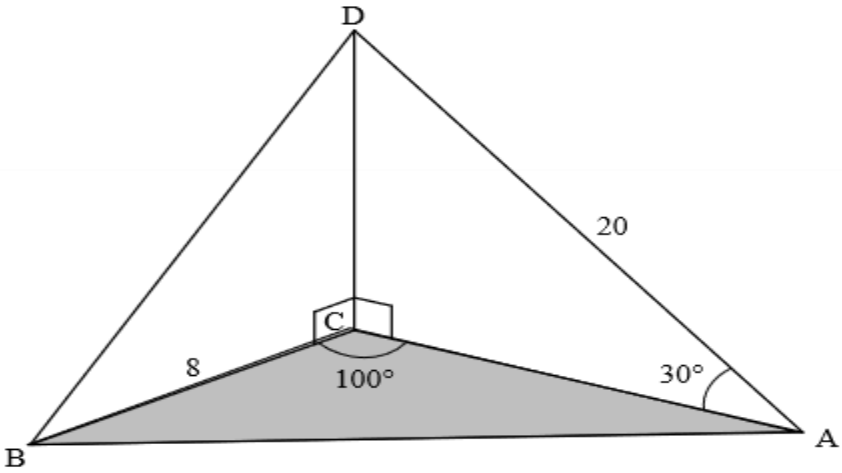
5.5.1	$3\tan 4x = -2\cos 4x$ $3\left(\frac{\sin 4x}{\cos 4x}\right) = -2\cos 4x$ $3\sin 4x + 2\cos^2 4x = 0$ $3\sin 4x + 2(1-\sin^2 4x) = 0$ $-2\sin^2 4x + 3\sin 4x + 2 = 0$ $2\sin^2 4x - 3\sin 4x - 2 = 0$ $(2\sin 4x + 1)(\sin 4x - 2) = 0$ $\sin 4x = -\frac{1}{2}$ or $\sin 4x \neq 2$	✓ identity ✓ $1-\sin^2 4x$ ✓ standard form ✓ factors (4)
5.5.2	$\sin 4x = -\frac{1}{2}$ *ref.* $\angle = 30°$ $4x = 210° + k.360°$ or $4x = 330° + k.360°$ $x = 52{,}5° + k.90°$; $k \in Z$ $x = 82{,}5° + k.90°$; $k \in Z$	✓ 210° ; 330° ✓ 52,5° ; 82,5° ✓ $k.90°$; $k \in Z$ (3)
		[28]

QUESTION/VRAAG 6

6.1	Period = 180°	✓ answer (1)
6.2		✓ x-intercepts ✓ turning points ✓ end points (3)
6.3	$y \in [-1;1]$ **OR/*OF*** $-1 \le y \le 1$	✓ answer (1)
6.4	$g(x) = -\cos 2x$ $g(x+45°) = -\cos 2(x+45°)$ $= -\cos(2x+90°)$ $= \sin 2x$	✓ $-\cos 2(x+45°)$ ✓ answer (2)
6.5.1	$x \in (-90°;-45°)$ **OR/*OF*** $-90° < x < -45°$	✓✓ $x \in (-90°;-45°)$ (2)
6.5.2	$2\cos 2x - 1 > 0$ $\cos 2x > \frac{1}{2}$ $-\cos 2x < -\frac{1}{2}$ $x \in (-30°;30°)$ **OR/*OF*** $-30° < x < 30°$	✓ $\cos 2x > \frac{1}{2}$ ✓ $-\cos 2x < -\frac{1}{2}$ ✓ $x = \pm 30°$ ✓ interval (4)
		[13]

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QUESTION/*VRAAG 7*

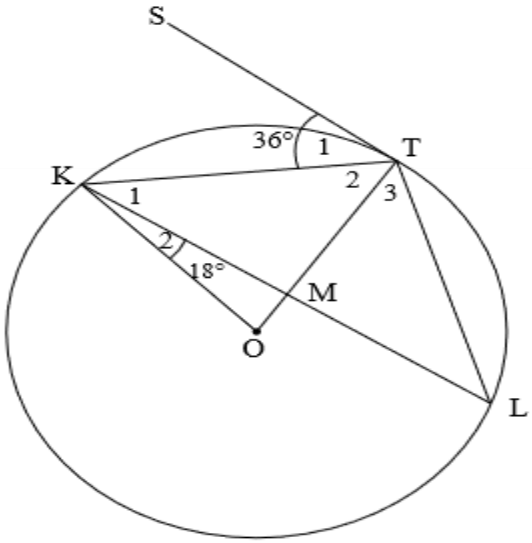


7.1.1	$\frac{AC}{20} = \cos 30°$ $AC = 20\cos 30°$ $AC = 10\sqrt{3} = 17,32$ units **OR/*OF*** $\frac{AC}{\sin 60°} = \frac{20}{\sin 90°}$ $\therefore AC = 20\sin 60 = 17,32$	✓ trig ratio ✓ answer (2) ✓ trig ratio ✓ answer (2)
7.1.2	$AB^2 = AC^2 + BC^2 - 2AC.BC\cos A\hat{C}B$ $AB^2 = \left(10\sqrt{3}\right)^2 + 8^2 - 2\left(10\sqrt{3}\right)(8)\cos 100°$ $AB = 20,30$ units	✓ cosine formula ✓ substitution into cosine formula ✓ answer (3)
7.2	$\frac{\sin A\hat{D}B}{AB} = \frac{\sin A\hat{B}D}{AD}$ $\frac{\sin A\hat{D}B}{20,3} = \frac{\sin 73,4°}{20}$ $\sin A\hat{D}B = \frac{20,3\sin 73,4°}{20}$ $A\hat{D}B = 76,58°$	✓ sine formula in ΔABD ✓ substitution into sine formula ✓ answer (3)
		[8]

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QUESTION/*VRAAG 8*

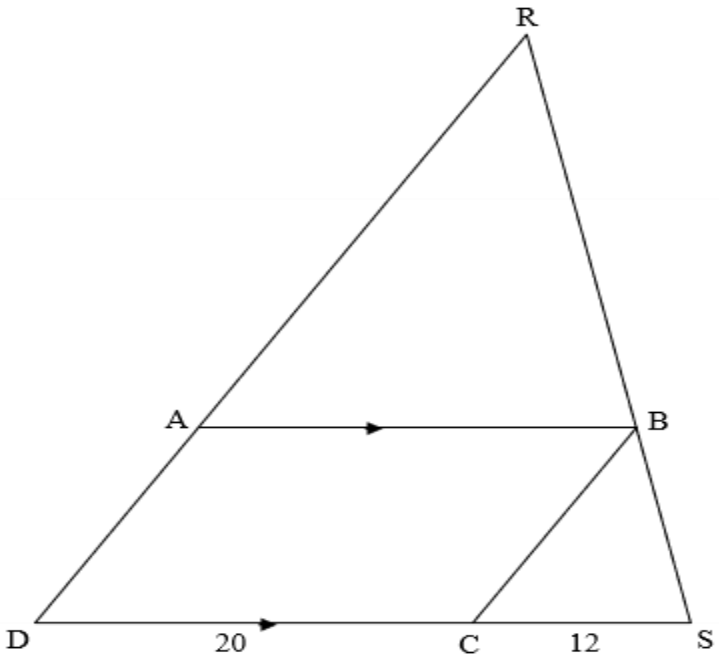
8.1



8.1.1(a)	$\hat{T}_2 = 54°$	[tan ⊥rad]	✓ S ✓R (2)
8.1.1(b)	$\hat{L}=36°$	[tan - chord theorem]	✓ S ✓R (2)
8.1.1(c)	$K\hat{O}T=72°$	[∠ at centre = 2×∠ at circumference]	✓ S ✓R (2)
	OR/*OF*		
	$O\hat{K}T=\hat{T}_2=54°$	[∠s opposite = radii]	✓ S/R
	$K\hat{O}T=180°-(54°+54°)$ $=72°$	[sum of int ∠'s of Δ]	✓ S (2)
8.1.2	$K\hat{M}O=180°-(18°+72°)$ $=90°$	[sum of int ∠'s of Δ]	✓ S ✓ S
	∴ KM = ML	[line from centre ⊥ to chord]	✓ R (3)
	OR/*OF*		
	$O\hat{K}T=54°$	[∠s opposite = radii]	
	$\hat{K}_1=54°-18°=36°$		✓ S
	$T\hat{M}K=90°$	[sum of int ∠'s of Δ]	✓ S
	∴ KM = ML	[line from centre ⊥ to chord]	✓ R (3)

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8.2



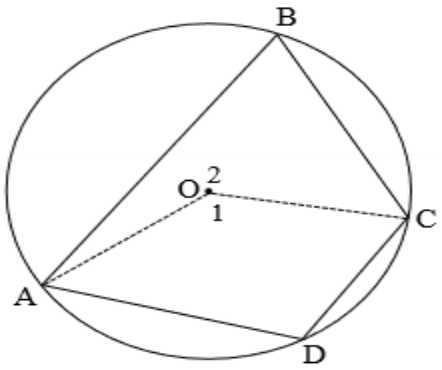
8.2.1	$\frac{DC}{CS}=\frac{20}{12}=\frac{5}{3}$ $\therefore \frac{DC}{CS}=\frac{RB}{BS}$ $\therefore$ BC ‖ DR [converse line ‖ one side of Δ OR sides in the same proportion] $\therefore$ BC ‖ AD	✓ S ✓ S ✓ R (3)
8.2.2	$\frac{AR}{AD}=\frac{RB}{BS}$ [line ‖ one side of Δ] **OR**[Prop Theorem AB ‖ DS] $\frac{AR}{AD}=\frac{5}{3}$ $\frac{48-AD}{AD}=\frac{5}{3}$ $\therefore 5AD = 144-3AD$ $AD = 18$ AB = 20 [opp sides of parm] $\therefore$ AD : AB = 18 : 20 = 9 : 10	✓ $\frac{AR}{AD}=\frac{5}{3}$ ✓ AD = 18 ✓ ratio (3)

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	OR/OF $\frac{AR}{RD}=\frac{5}{8}$.....prop thm AB || DS $\frac{AR}{48}=\frac{5}{8}$ $\therefore AR=30$ and $AD=18$ $\therefore \frac{AR}{RD}=\frac{AB}{DS}$.....$\|\|\|\ \Delta's$ $\therefore AB=20$ $\therefore AB:AD=18:20=9:10$	✓ $\frac{AR}{RD}=\frac{5}{8}$ ✓ $AD=18$ ✓ ratio (3)
		[15]

QUESTION/*VRAAG 9*

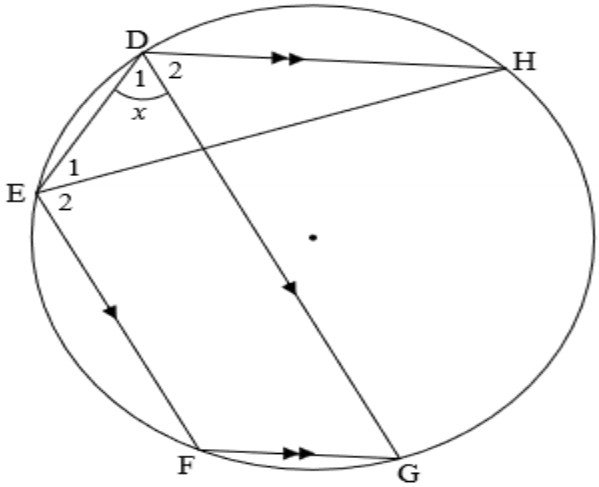
9.1



9.1	Constr: Draw radii OA and OC. Proof: $\hat{O}_1 = 2\hat{B}$ [$\angle$ at centre = $2\times\angle$ at circumference] $\hat{O}_2 = 2\hat{D}$ [$\angle$ at centre = $2\times\angle$ at circumference] $\hat{O}_1 + \hat{O}_2 = 360°$ [revolution] $2\hat{B} + 2\hat{D} = 360°$ [revolution] $\therefore \hat{B} + \hat{D} = 180°$	✓ Construction ✓ S ✓ R ✓ S/R ✓ S (5)

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9.2



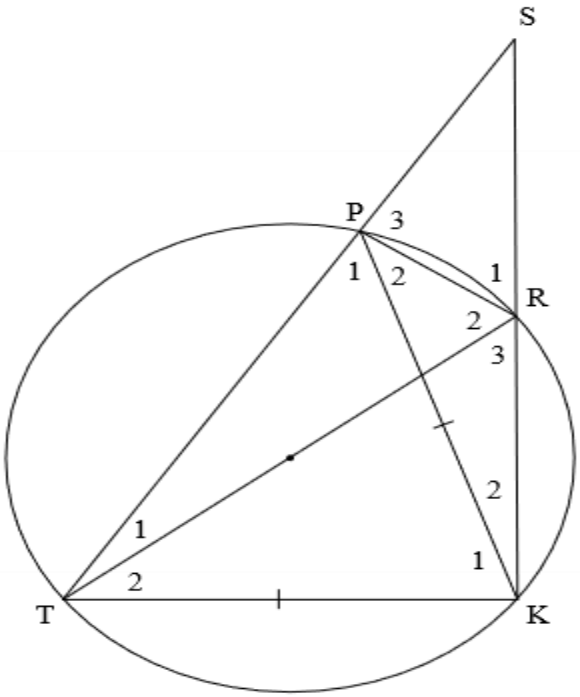
9.2	$E\hat{F}G = 180° - \hat{D}_1$ [opp ∠'s of cyclic quad] $\therefore E\hat{F}G = 180° - x$ $E\hat{F}G = 180° - \hat{G}$ [co-int ∠'s; EF ‖ DG] $\hat{G} = x$ But $\hat{G} = \hat{D}_2$ [alt ∠'s; DH ‖ FG] $\therefore \hat{D}_1 = \hat{D}_2 = x$	✓S ✓R ✓S / R ✓ S / R (4)
		[9]

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QUESTION/VRAAG 10

10.1



10.1.1	$T\hat{P}R = 90°$	[∠ in semi-circle]	✓S ✓R
	$S\hat{P}R = 90°$	[∠'s on a straight line]	✓S
	∴ SR is a diameter	[converse ∠ in semi-circle]	✓R
			(4)
	OR		
	$T\hat{K}R = 90°$	[∠ in semi-circle]	✓S ✓R
	$S\hat{P}R = 90°$	[ext ∠ of cyclic quad]	✓S
	∴ SR is a diameter	[converse ∠ in semi-circle]	✓R
		OR [chord subtends a right angle]	(4)

10.1.2	$\hat{R}_1 = P\hat{T}K$ [ext ∠ of cyclic quad] $\hat{P}_1 = P\hat{T}K = \hat{R}_1$ [∠s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext ∠ of Δ] $\therefore \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]	✓S ✓R ✓S /R ✓S ✓R (5)
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] ΔSPK ||| ΔPRK [∠, ∠, ∠] **OR/*OF*** In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of ∠s in Δ] ΔSPK ||| ΔPRK	✓S ✓S ✓S/R (3) ✓S ✓S ✓S/R (3)
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [ΔSPK ||| ΔPRK] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] $TK = PK$ [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$	✓S ✓S ✓ $PK^2 = SK.RK$ ✓SK = 2RK ✓ $ST^2 = 6RK^2$ (5)
		[17]

TOTAL/TOTAAL: 150

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2015

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and a 25-page answer book.

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QUESTION 2

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Calculate the mean of the data. (2)

2.2 Determine the median of the data. (2)

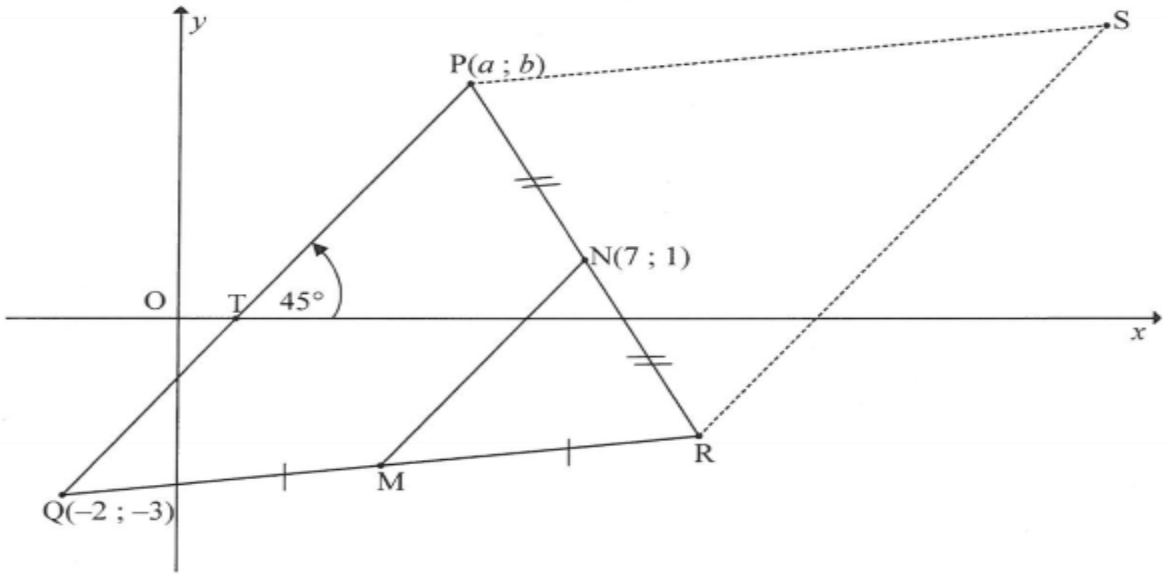
2.3 Determine the standard deviation of the data. (2)

2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)
[9]

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QUESTION 3

In the diagram below, the line joining Q(–2 ; –3) and P(a ; b), a and $b > 0$, makes an angle of 45° with the positive x-axis. QP = $7\sqrt{2}$ units. N(7 ; 1) is the midpoint of PR and M is the midpoint of QR.



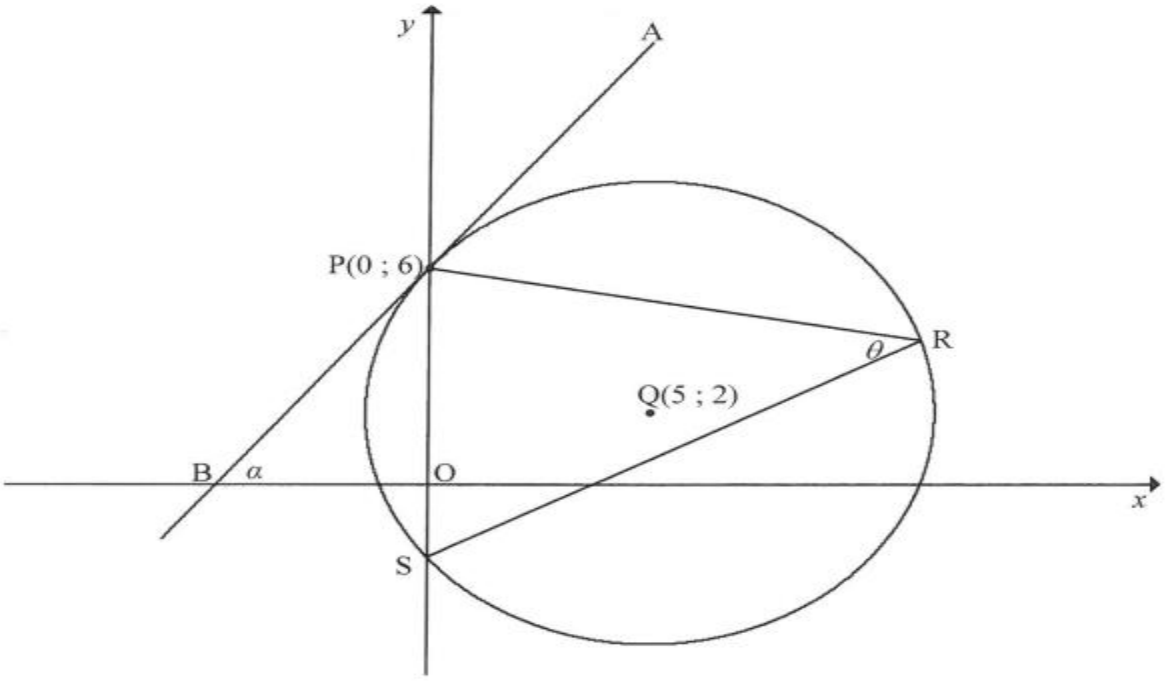
Determine:

3.1	The gradient of PQ	(2)
3.2	The equation of MN in the form $y = mx + c$ and give reasons	(4)
3.3	The length of MN	(2)
3.4	The length of RS	(1)
3.5	The coordinates of S such that PQRS, in this order, is a parallelogram	(3)
3.6	The coordinates of P	(6)
		[18]

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QUESTION 4

In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y-axis at P(0 ; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1 Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

4.4 Calculate the size of α. (2)

4.5 Calculate, with reasons, the size of θ. (4)

4.6 Calculate the area of ΔPQS. (4)
[20]

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QUESTION 5

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90° + x)}{\sin(30° - x).\cos x + \cos(30° - x).\sin x}$$ (6)

5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

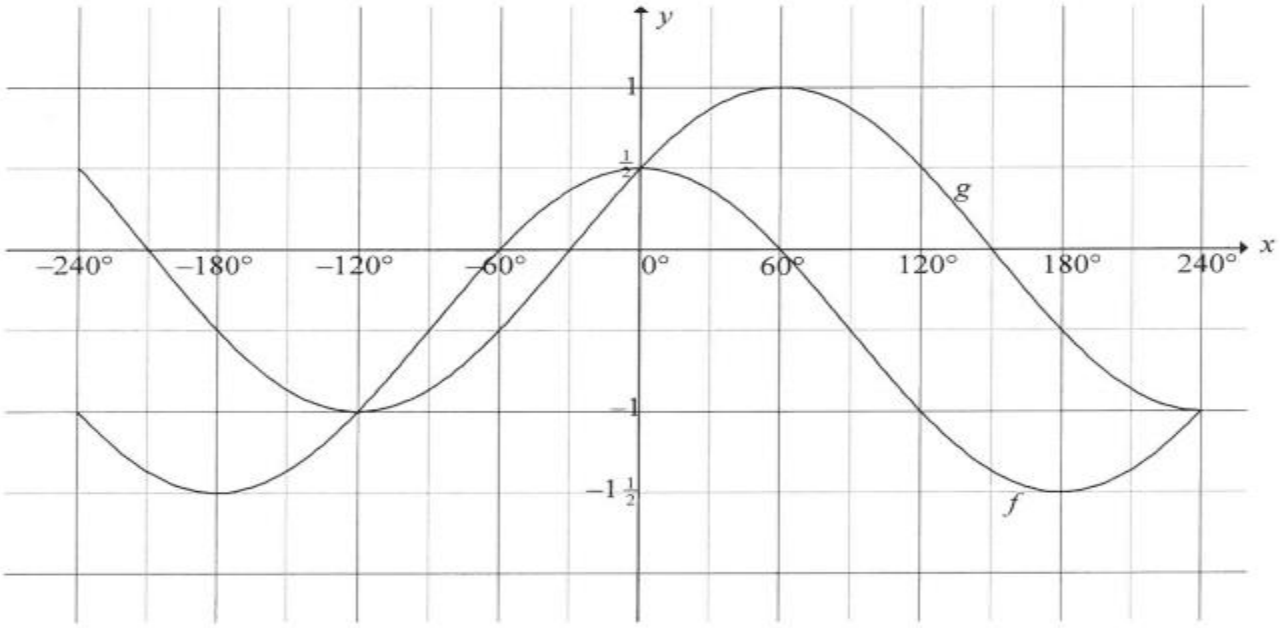
5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

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QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240° \le x \le 240°$. The graphs intersect at $\left(0° ; \frac{1}{2}\right)$, $(-120° ; -1)$ and $(240° ; -1)$.



6.1 Determine the values of p and q. (4)

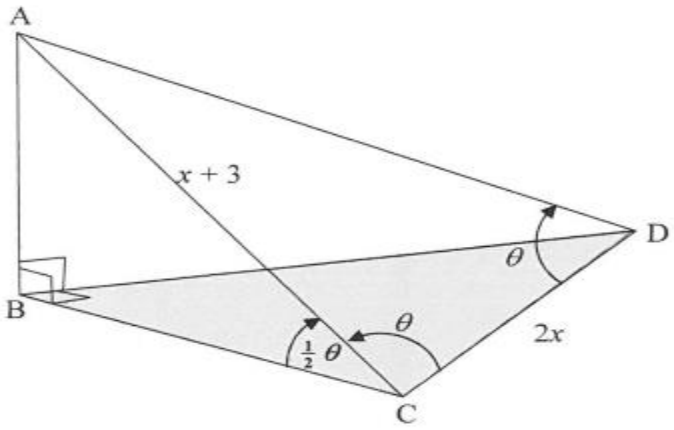
6.2 Determine the values of x in the interval $-240° \le x \le 240°$ for which $f(x) > g(x)$. (2)

6.3 Describe a transformation that the graph of g has to undergo to form the graph of h, where $h(x) = -\cos x$. (2)
[8]

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QUESTION 7

A corner of a rectangular block of wood is cut off and shown in the diagram below.
The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$.
Also $A\hat{C}B = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.



7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

7.2 Prove that $\cos\theta = \frac{x}{x+3}$. (4)

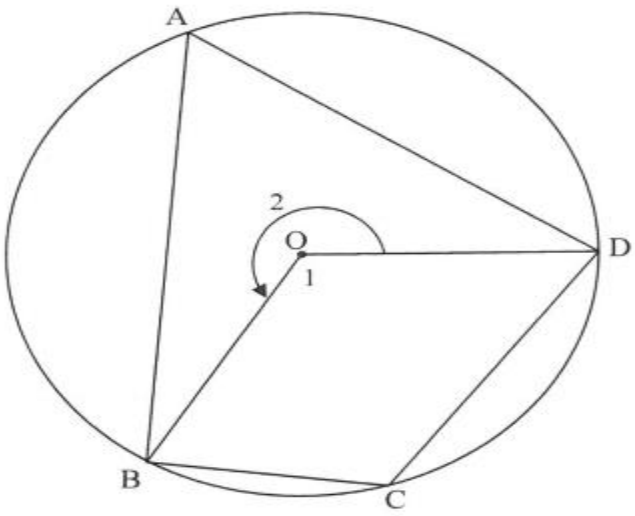
7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5)
[10]

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Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 In the diagram below, cyclic quadrilateral ABCD is drawn in the circle with centre O.



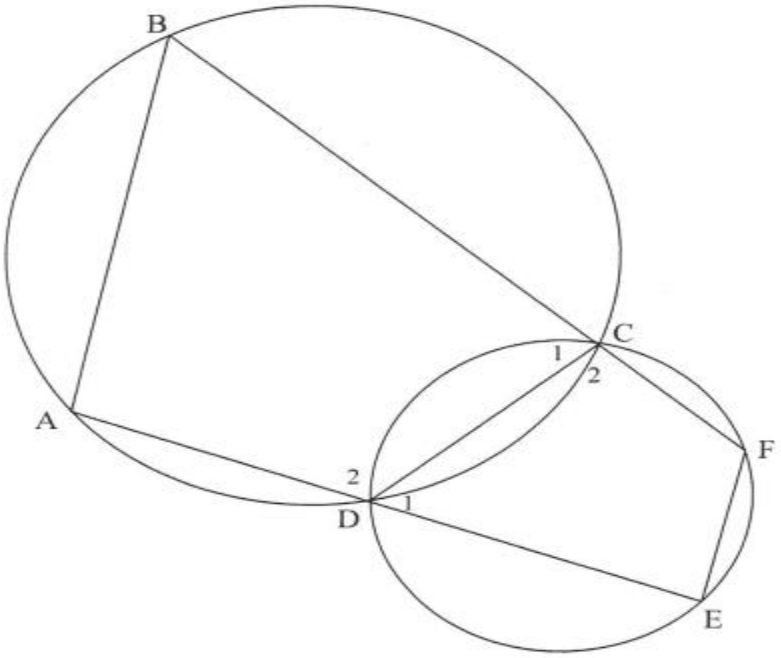
8.1.1 Complete the following statement:

The angle subtended by a chord at the centre of a circle is … the angle subtended by the same chord at the circumference of the circle. (1)

8.1.2 Use QUESTION 8.1.1 to prove that $\hat{A}+\hat{C}=180°$. (3)

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8.2 In the diagram below, CD is a common chord of the two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.

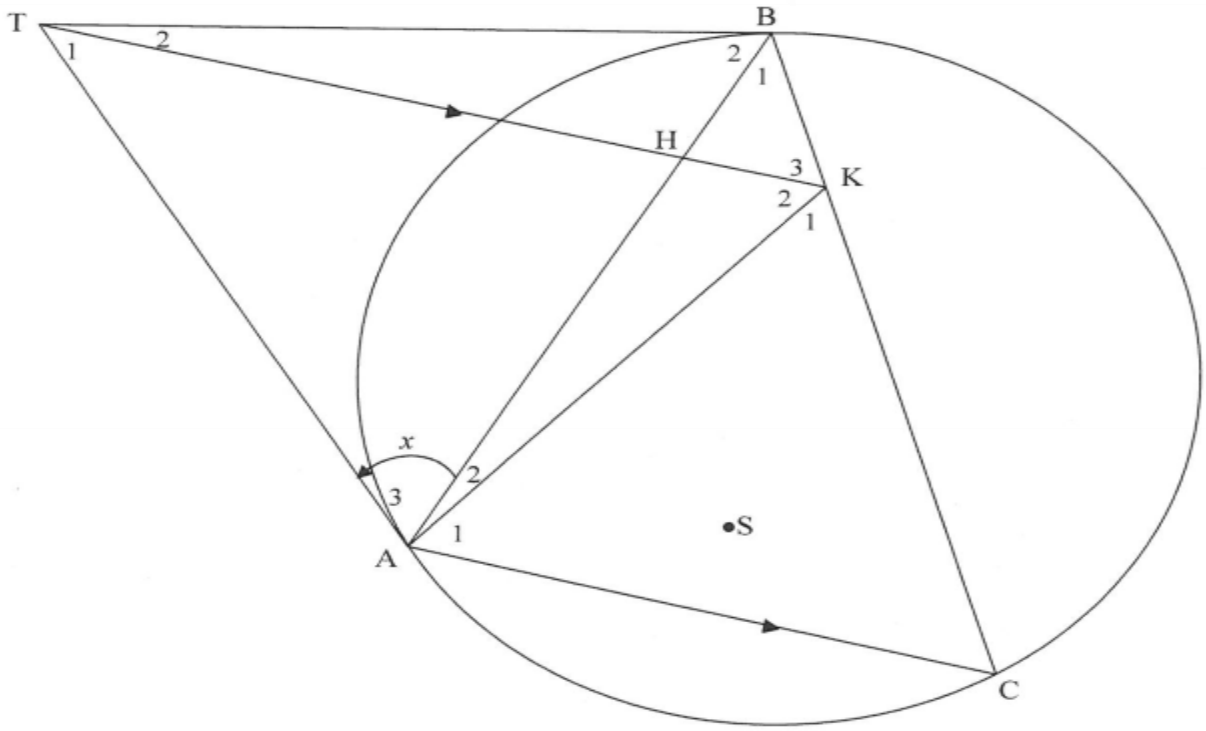


Prove that EF | | AB. (5)
[9]

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QUESTION 9

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC. AK is drawn. Let $\hat{A}_3 = x$.



9.1 Prove that $\hat{K}_3 = x$. (4)

9.2 Prove that AKBT is a cyclic quadrilateral. (2)

9.3 Prove that TK bisects $A\hat{K}B$. (4)

9.4 Prove that TA is a tangent to the circle passing through the points A, K and H. (2)

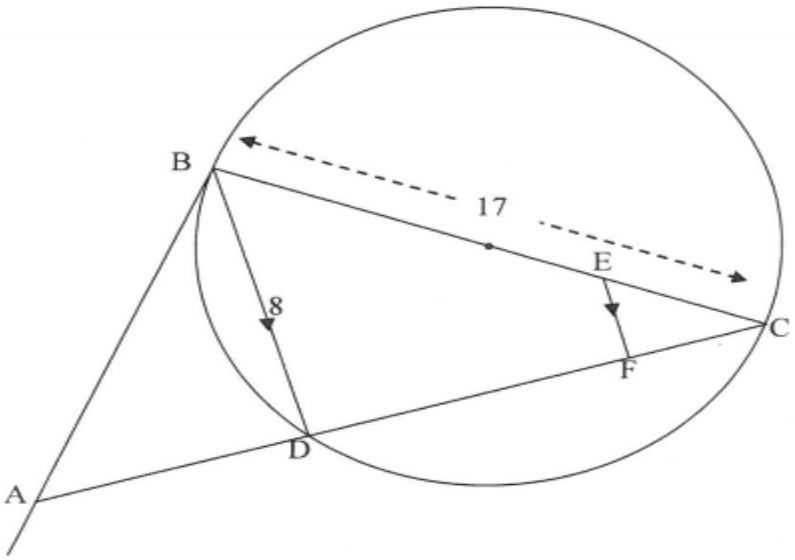
9.5 S is a point in the circle such that the points A, S, K and B are concyclic. Explain why A, S, B and T are also concyclic. (2)

[14]

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QUESTION 10

In the diagram below, BC = 17 units, where BC is a diameter of the circle. The length of chord BD is 8 units. The tangent at B meets CD produced at A.



10.1 Calculate, with reasons, the length of DC. (3)

10.2 E is a point on BC such that BE : EC = 3 : 1. EF is parallel to BD with F on DC.

10.2.1 Calculate, with reasons, the length of CF. (3)

10.2.2 Prove that $\Delta BAC \,|||\, \Delta FEC$. (5)

10.2.3 Calculate the length of AC. (4)

10.2.4 Write down, giving reasons, the radius of the circle passing through points A, B and C. (2)
[17]

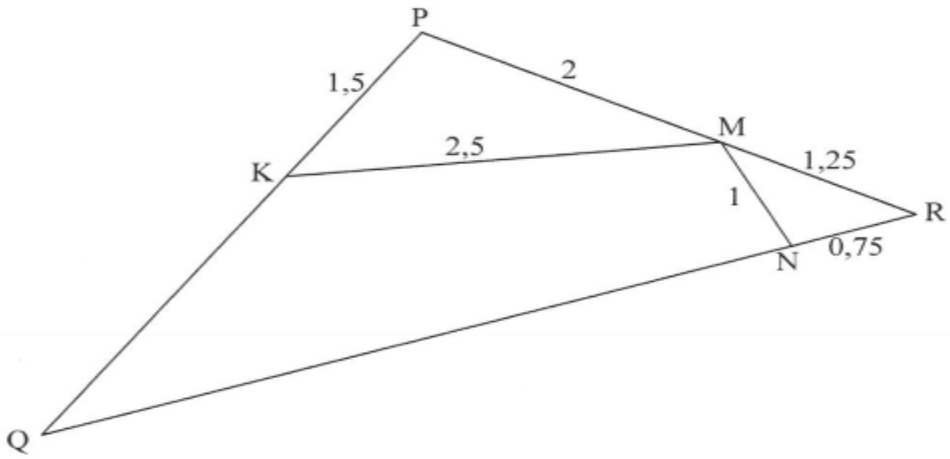
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QUESTION 11

11.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are ... (1)

11.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ΔPQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1,25 and NR = 0,75.



11.2.1 Prove that ΔKPM ||| ΔRNM. (3)

11.2.2 Determine the length of NQ. (6)
[10]

TOTAL: 150

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GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2015

MEMORANDUM

MARKS: 150
PUNTE: 150

This memorandum consists of 27 pages./
Hierdie memorandum bestaan uit 27 bladsye.

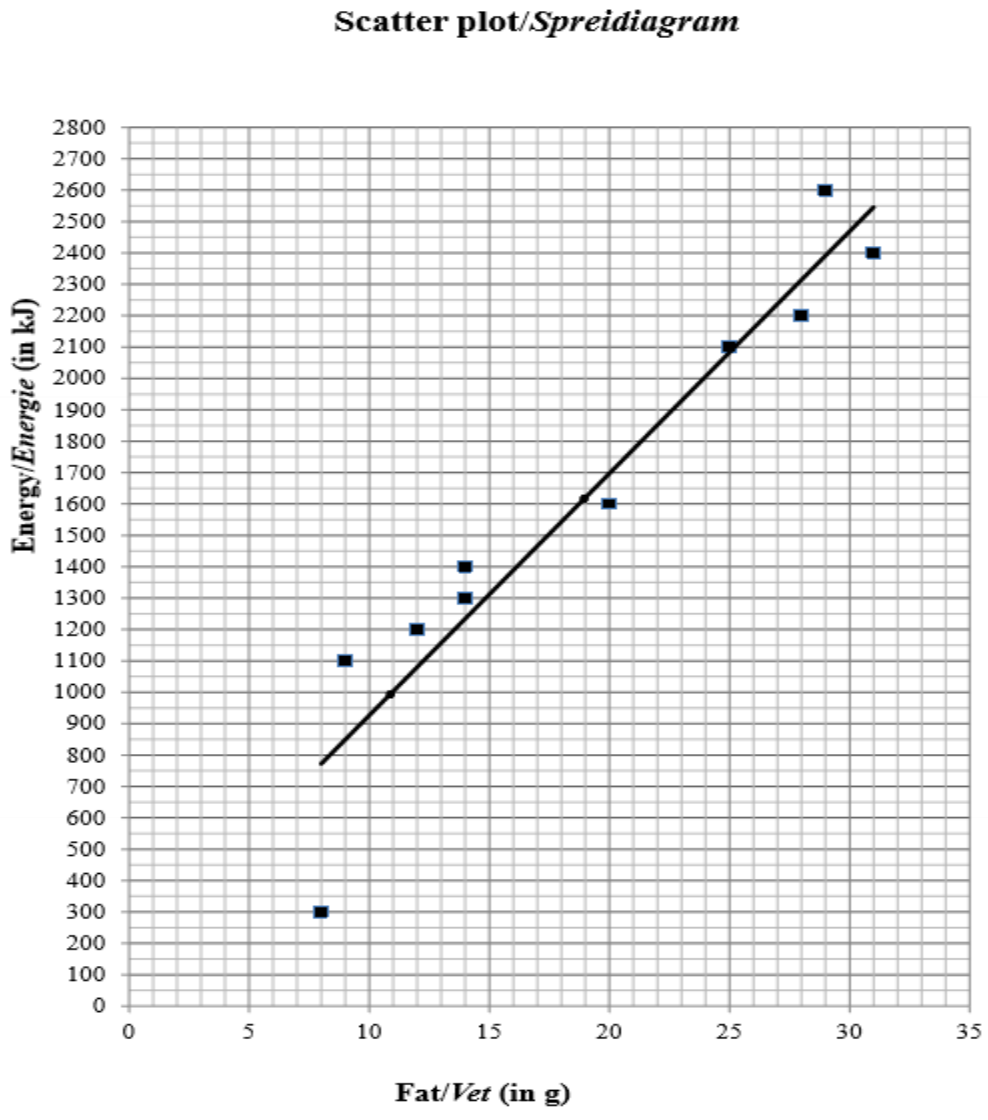
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Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 1

Fat/*Vet* (in g)	9	14	25	8	12	31	28	14	29	20
Energy/*Energie* (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1



1.1
no marks:
0 – 2 points correctly

✓plotting
3 – 5 points correctly

✓✓plotting
6 – 9points correctly

✓✓✓plotting all 10 points correctly

geen punte:
0 – 2 punte korrek

✓ *stip 3 – 5 pte korrek*

✓✓*stip 6 –9 pte korrek*

✓✓✓ *stip al 10 pte korrek*

(3)

1.2.2

1.2.2
✓ *y* – int close to (0 ; 150)
✓ one pt close to (25 ; 2100) or (20 ; 1700)
(2)

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1.2.1	$\hat{y} = 154{,}60 + 77{,}13(18)$ $= 1\,542{,}94 \approx 1\,500$ kJ	✓ subst ✓ answ rounded off correctly/ *antw korrek afgerond* (2)
1.3	(8 ; 300)	✓ answ/*antw* (1)
1.4	$r = 0{,}9520... \approx 0{,}95$	✓✓ answ/*antw* (2)
1.5	very strong positive relationship/ *baie sterk positiewe verband*	✓ strong/ *sterk* (1) **[11]**

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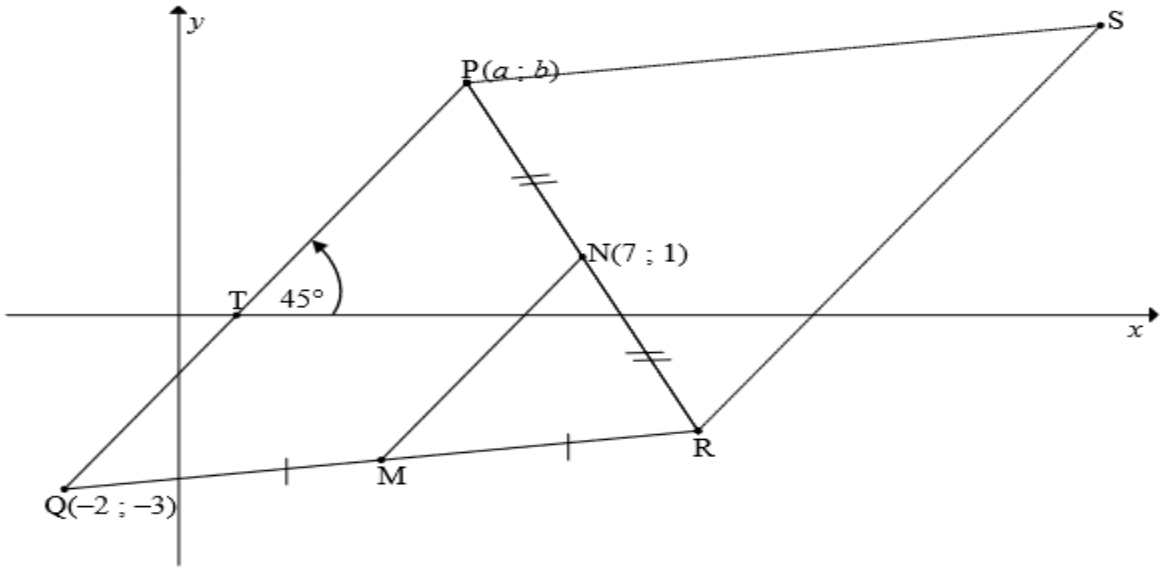
QUESTION/*VRAAG* 2

Sum of the values on uppermost faces/ ***Som van die waardes op boonste vlakke***	**Frequency/** ***Frekwensie***
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1	mean/*gemiddelde* $= \frac{2(0)+3(3)+4(2)+...12(1)}{30} = \frac{202}{30}$ $= 6,73$	✓202 ✓ answ/*antw* (2)
2.2	median/*mediaan* $= \frac{T_{15}+T_{16}}{2} = \frac{7+7}{2} = 7$	✓✓ answ/*antw* (2)
2.3	SD/*SA* $= 2,264... \approx 2,26$	✓✓ answ/*antw* (2)
2.4	$(6,73 - 2,26 ; 6,73 + 2,26)$ $= (4,47 ; 8,99)$ $\therefore 4 + 4 + 8 + 3 = 19$ times/*keer*	✓lower boundary ✓upper boundary ✓ answ/*antw* (3) **[9]**

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QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \tan 45°$ $= 1$	✓ $m = \tan 45°$ ✓ answ/*antw* (2)
3.2	MN || QP [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore y = x - 6$ **OR/*OF*** MN || PQ [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore y = x - 6$	✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4) ✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4)
3.3	$MN = \frac{1}{2}PQ$ [midpoint theorem/midp stelling] $\therefore MN = \frac{7\sqrt{2}}{2} \approx 4{,}95$	✓ S ✓ answ/antw (2)

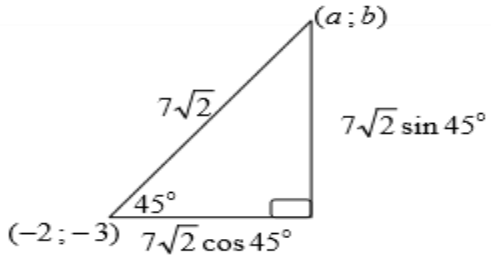
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3.5	QN = NS [diag of ||m/hoekl van ||m] $\frac{-2+x_S}{2}=7$ and/en $\frac{-3+y_S}{2}=1$ $\therefore x_S=16$ $\therefore y_S=5$ **OR/*OF*** QN = NS [diag of ||m/hoekl van ||m] $\therefore$ by inspection/deur inspeksie: S(16 ; 5)	✓ method/metode ✓ *x*-value/waarde ✓ *y*-value/waarde (3) ✓ method/metode ✓ *x*-value/waarde ✓ *y*-value/waarde (3)
3.6	Equation of/*Vgl van PQ*: $y=x+c$ $-3=-2+c$ $y=x-1$ $\therefore a=b+1$ .....(1) From distance formula/*Van afstandsformule*: $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $7\sqrt{2}=\sqrt{(a-(-2))^2+(b-(-3))^2}$ $\therefore 98=(a+2)^2+(b+3)^2$ .....(2) Subst (1) into (2): $98=(b+1+2)^2+(b+3)^2$ $98=b^2+6b+9+b^2+6b+9$ $0=2b^2+12b-80$ $0=b^2+6b-40$ $\therefore 0=(b+10)(b-4)$ $\therefore b=4$ (since $b>0$) Subst $b=4$ into (1): $\therefore a=4+1=5$ $\therefore$ P(5 ; 4) **OR/*OF*** Equation of/*Vgl van PQ*: $y=x+c$ $-3=-2+c$ $y=x-1$ $\therefore a=b+1$ .....(1) From distance formula/*Van afstandsformule*: $7\sqrt{2}=\sqrt{(a-(-2))^2+(b-(-3))^2}$ $\therefore 98=(a+2)^2+(b+3)^2$ .....(2) Subst (1) into (2): $98=(b+1+2)^2+(b+3)^2$ $98=2(b+3)^2$ $49=(b+3)^2$ $\pm 7=b+3$ $\pm 7-3=b$ $\therefore b=4$ (since $b>0$) Subst $b=4$ into (1): $\therefore a=4+1=5$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ st form/*st vorm* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6) ✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6)

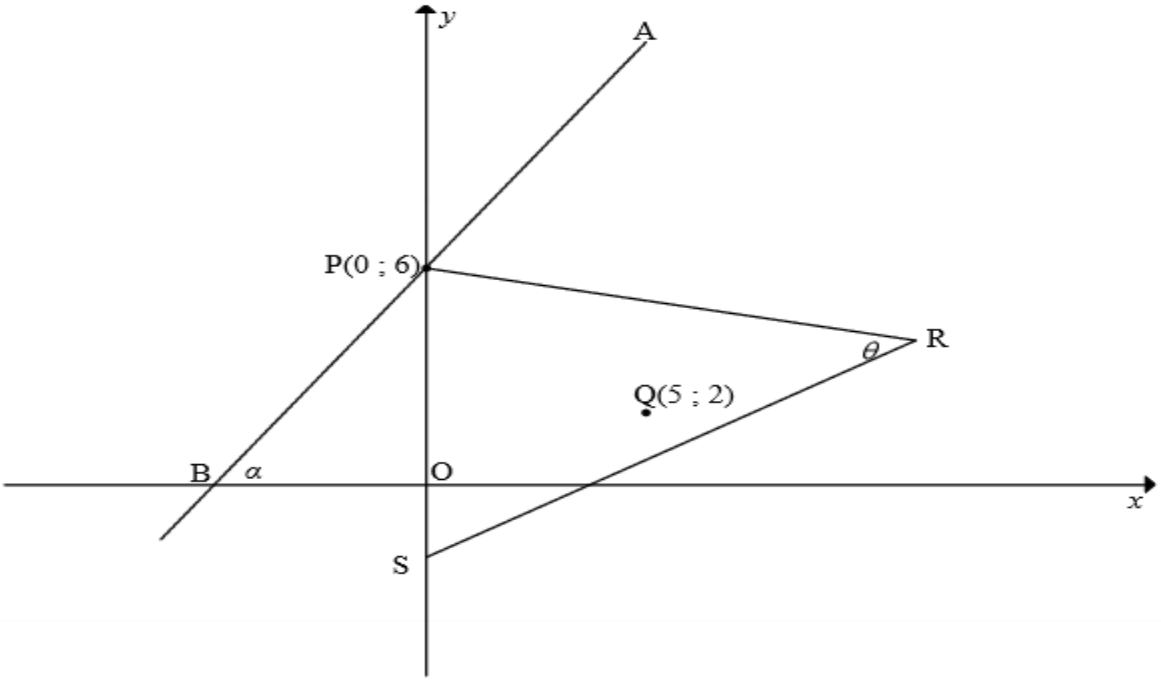
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	OR/*OF*	
	Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1)	✓ eq of/*vgl van* PQ
	From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$	✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule*
	$98 = (a+2)^2 + (a-1+3)^2$ $= 2(a+2)^2$	✓ subst eq of/*vgl v.* PQ
	$\therefore a + 2 = 7$ (since/*aangesien* $a > 0$)	✓ simplification/ *vereenvoudig*
	$\therefore a = 5$ Subst $a = 4$ into (1): $\therefore b = 5 - 1 = 4$ $\therefore$ P(5 ; 4)	✓ value of/*waarde van a* ✓ value of/*waarde van b* (6)
	OR/*OF*	
		✓✓✓✓
	$a = -2 + 7\sqrt{2}\cos 45° = 5$ $b = -3 + 7\sqrt{2}\sin 45° = 4$	✓ ✓ (6) **[17]**

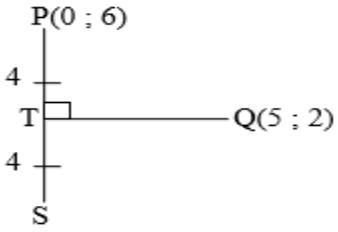
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QUESTION/*VRAAG* 4



4.1	$(x-5)^2+(y-2)^2=r^2$ $(0-5)^2+(6-2)^2=r^2$ $25+16=r^2$ $41=r^2$ $\therefore\ (x-5)^2+(y-2)^2=41$ **OR/*OF*** $PQ=\sqrt{(0-5)^2+(6-2)^2}$ $=\sqrt{25+16}$ $r=\sqrt{41}$ $\therefore\ (x-5)^2+(y-2)^2=41$	✓ subst (5 ; 2) into circle eq/*in sirkelvgl* ✓ value of/*waarde van* r^2 ✓ equation/*vgl* (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/*in afst. form* ✓ value of/*waarde van r* ✓ equation/*vgl* (3)
4.2	$(0-5)^2+(y-2)^2=41$ $25+(y-2)^2=41$ $25+y^2-4y+4=41$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y\neq 6$ *or / of* $y=-2$ $\therefore$ S(0 ; –2) or $y=-2$	✓ $x=0$ ✓ st form/*st. vorm* ✓ answ/*antw* (neg value) (3)

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	OR/*OF* $(0-5)^2+(y-2)^2 = 41$ $25+(y-2)^2 = 41$ $(y-2)^2 = 16$ $y-2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6$ or / *of* $y = -2$ $\therefore$ S(0 ; –2)	✓ $x = 0$ ✓ square form/ *kwadraatvorm* ✓ answ/*antw* (neg value) (3)
	OR/*OF* Draw/*Trek* QT ⊥ PS PT = TS [line from centre ⊥ to chord/ *lyn van midpt ⊥ koord*] PT = $y_P - y_Q = 6-2 = 4$ $y_Q - y_S = 4$ $y_S = 2-4 = -2$ $\therefore$ S(0 ; –2)  	✓ $x = 0$ ✓✓ $y = -2$ (3)
4.3	$m_{PQ} = \frac{6-2}{0-5}$ $= -\frac{4}{5}$ $m_{PQ} \times m_{APB} = -1$ [tan/*raakl* ⊥ radius] $\therefore\ m_{APB} = \frac{5}{4}$ $\therefore y = \frac{5}{4}x + 6$	✓ subst (0 ; 6) & (5 ; 2) into grad form/*in grad. formule* ✓ m_{PQ} ✓ m_{APB} ✓ equation/*vgl* (4)
4.4	$\tan\alpha = \frac{5}{4}$ $\therefore\ \alpha = 51{,}34°$ **OR/*OF*** B(4,8 ; 0) $\therefore \tan\alpha = \frac{6}{4{,}8}$ $\therefore\ \alpha = 51{,}34°$	✓ $\tan\alpha = m_{APB}$ ✓ answ/*antw* (2) ✓ $\tan\alpha = \frac{6}{4{,}8}$ ✓ answ/*antw* (2)

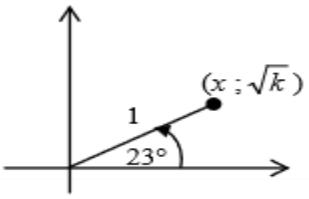
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4.5	$\theta = B\hat{P}S$ [tan-chord th/*raakl-koordst.*] $= 90° - \alpha$ [$\angle$ sum in Δ/$\angle$ *som van* Δ] $= 90° - 51{,}34°$ $= 38{,}66°$ **OR/*OF*** $PS = 8$ $PQ = SQ = \sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos P\hat{Q}S$ $64 = 41 + 41 - 2.41.\cos P\hat{Q}S$ $\cos P\hat{Q}S = \frac{18}{82}$ $P\hat{Q}S = 77{,}32°$ $\theta = \frac{1}{2}P\hat{Q}S$ [$\angle$ at centre $= 2 \times \angle$ circumf] $= 38{,}66°$	✓ S ✓ R ✓ $90° - \alpha$ ✓ answ/*antw* (4) ✓ correct subst into cosine rule ✓ $P\hat{Q}S = 77{,}32°$ ✓ R ✓ answ/*antw* (4)
4.6	Area $\Delta PQS = \frac{1}{2}$ PS ×height/*hoogte* $= \frac{1}{2}(8)(5)$ $= 20$ sq units/*vk eenh* **OR/*OF*** $P\hat{Q}S = 2 \times 38{,}66°$ [$\angle$at centre $= 2\times\angle$ at circum/ *midpts$\angle$=2omtreks$\angle$*] $= 77{,}32°$ Area $\Delta PQS = \frac{1}{2}PQ.QS.\sin P\hat{Q}S$ $= \frac{1}{2}.\sqrt{41}.\sqrt{41}.\sin 77{,}32°$ $= 20$ sq units/*vk eenh*	✓ area formula/*e*: ΔPQS ✓ PS = 8 ✓ $\perp h = 5$ ✓ answ/*antw* (4) ✓ size of/*grootte v* $P\hat{Q}S$ ✓ area rule/*reël*: ΔPQS ✓ subst correctly/ *subst korrek* ✓ answ/*antw* (4) **[20]**

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QUESTION/*VRAAG* 5

5.1.1	$\sin 203°$ $= -\sin 23°$ $= -\sqrt{k}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.1.2	$\cos^2 23° = 1 - \sin^2 23°$ $= 1 - k$ $\cos 23° = \sqrt{1-k}$ **OR/*OF*** $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1-k}$ $\cos 23° = \frac{\sqrt{1-k}}{1} = \sqrt{1-k}$  	✓identity/*identiteit* ✓ $\cos^2 23°$ ito/*itv k* ✓ answ/*antw* (3) ✓ $x^2 = 1 - k$ ✓ x ito/*itv k* ✓ answ/*antw* (3)
5.1.3	$\tan(-23°) = -\tan 23°$ $= -\frac{\sin 23°}{\cos 23°}$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$ **OR/*OF*** $\tan(-23°) = -\tan 23°$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2) ✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.2	$\frac{4\cos x.(-\sin x)}{\sin(30° - x + x)}$ $= \frac{-4\sin x.\cos x}{\sin 30°}$ $= \frac{-4\sin x.\cos x}{\frac{1}{2}}$ $= -8\sin x.\cos x$ $= -4(2\sin x.\cos x)$ $= -4\sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\sin(\alpha + \beta)$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

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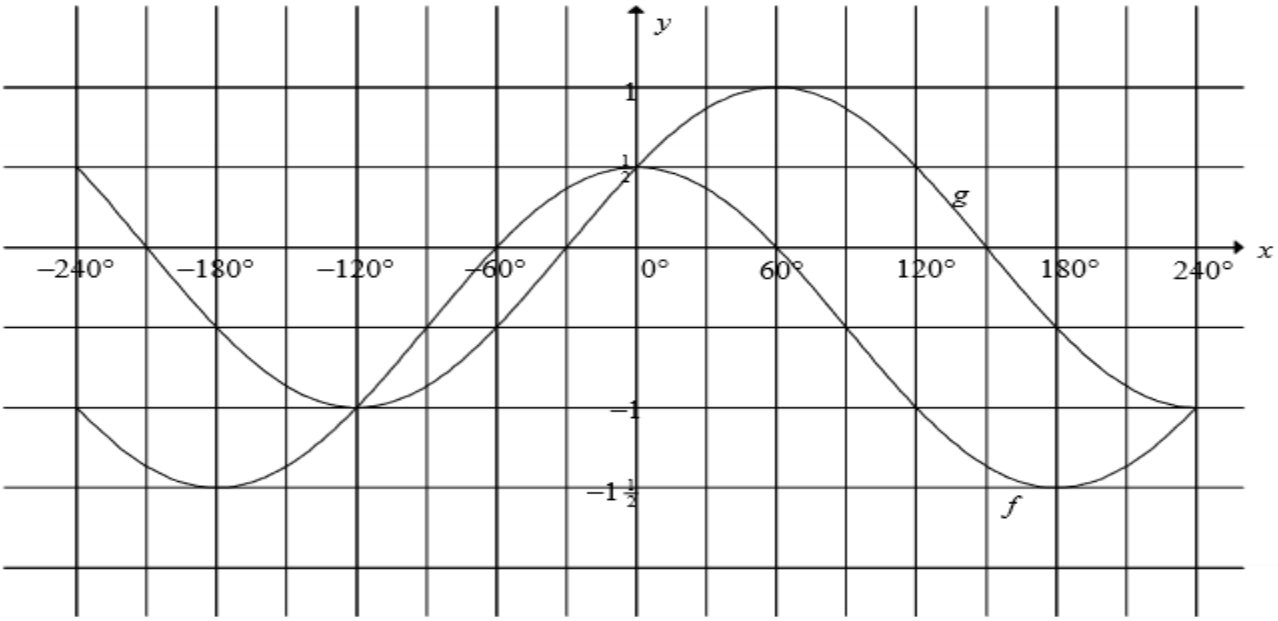
	OR/OF $\frac{4\cos x.(-\sin x)}{(\sin 30°\cos x-\cos 30°\sin x)\cos x+(\cos 30°\cos x+\sin 30°\sin x)\sin x}$ $=\frac{-4\sin x.\cos x}{(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x)\cos x+(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x)\sin x}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}(\cos^2 x+\sin^2 x)}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}(1)}$ $=-8\cos x\sin x$ $=-4(2\sin x\cos x)$ $=-4\sin 2x$	✓ $\cos x$ ✓$-\sin x$ ✓ $\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

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5.3	$\cos 2x - 7\cos x - 3 = 0$ $2\cos^2 x - 1 - 7\cos x - 3 = 0$ $2\cos^2 x - 7\cos x - 4 = 0$ $(2\cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2}$ or/*of* $\cos x = 4$ (no solution) $\therefore x = 120° + n.360°$ or/*of* $x = 240° + n.360°$; $n \in Z$ **OR/OF** $\therefore x = \pm 120° + n.360°$; $n \in Z$	✓ expansion/ *uitbreiding* ✓ $2\cos^2 x - 7\cos x - 4 = 0$ ✓ factors/*faktore* ✓ $\cos x = -\frac{1}{2}$ ✓ 120° & 240° ✓ $+ n.360°$ **OR/OF** ✓ $\pm 120°$ ✓ $+ n.360°$ (6)
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ $= 3(\frac{1}{3}) - 4(\frac{1}{3})^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	✓ expansion of/ *uitbreiding van* $\sin(2\theta + \theta)$ ✓ expansions of $\sin 2\theta$ AND $\cos 2\theta$ ✓ $1 - \sin^2\theta$ ✓ subst ✓ answ/*antw* (5) **[24]**

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QUESTION/*VRAAG* 6



6.1	$f(x)=\cos x-\frac{1}{2}$ and/*en* $g(x)=\sin(x+30°)$ $\therefore p=30°$ and/*en* $q=-\frac{1}{2}$ **OR/*OF*** $\sin(60°+p)=1$ and/*en* $\cos 0°+q=\frac{1}{2}$ $\therefore p=30°$ $\therefore q=-\frac{1}{2}$	✓ $f(x)=\cos x-\frac{1}{2}$ ✓ $g(x)=\sin(x+30°)$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4) ✓ $\sin(60°+p)=1$ ✓ $\cos 0°+q=\frac{1}{2}$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4)
6.2	$x\in(-120°;0°)$ **OR/*OF*** $-120°<x<0°$	✓ critical values/ *kritiese waardes* ✓ correct interval/ *korrekte interval* (2)

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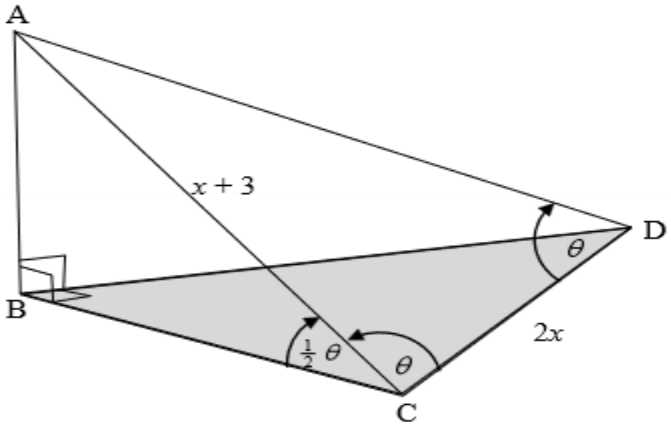
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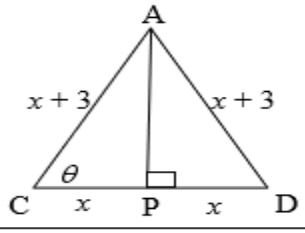
6.3	The graph of g has to shift 60° to the left and then be reflected about the x-axis./*Die grafiek van g moet 60° na links skuif en dan om die x-as gereflekteer word.*	✓ 60° left/*links* ✓ reflection about x-axis/*refleksie om x-as* (2)
	OR/*OF* The graph of g must be reflected about the x-axis and then be shifted 60° to the left./*Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuif word.*	✓ reflection about x-axis/*refleksie om x-as* ✓ 60° left/*links* (2)
	OR/*OF* The graph of g has to shift 120° to the right./*Die grafiek van g moet 120° na regs geskuif word.*	✓ ✓ 120° right/*regs* (2)
	OR/*OF* The graph of g has to shift 240° to the left./*Die grafiek van g moet met 240° na links geskuif word*	✓ ✓ 240° left/*links* (2) **[8]**

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QUESTION/*VRAAG* 7



7.1	$C\hat{A}D = 180° - 2\theta$ [∠s sum of Δ/*∠e som van Δ*]	✓ answ/*antw* (1)
7.2	$\frac{\sin\theta}{x+3} = \frac{\sin(180° - 2\theta)}{2x}$ $\frac{\sin\theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin\theta}{x+3} = \frac{2\sin\theta.\cos\theta}{2x}$ $\cos\theta = \frac{2x\sin\theta}{2(x+3)\sin\theta}$ $\cos\theta = \frac{x}{x+3}$	✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ $\sin 2\theta$ ✓ $2\sin\theta.\cos\theta$ ✓ $\cos\theta$ as subject/ *as onderwerp* (4)
	OR/*OF* $AD = x + 3$ [sides opp = ∠s/*sye to = ∠e*] $AC^2 = AD^2 + CD^2 - 2AD.CD.\cos\theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3).\cos\theta$ $0 = 4x^2 - 4x(x+3)\cos\theta$ $\cos\theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$	✓ $AD = x + 3$ ✓ correct subst into cosine rule/*korrekte subst in cos-reël* ✓ simplification/ *vereenvoudiging* ✓ $\cos\theta$ as subject/ *as onderwerp* (4)
	OR/*OF* Draw/*Trek* AP ⊥ CD $\cos\theta = \frac{x}{x+3}$ 	✓✓ constr/*konstr* ✓✓ sketch shown/ *toon skets* (4)

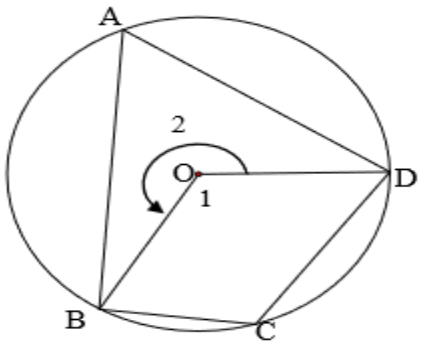
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7.3	$\cos\theta = \frac{2}{5}$ $\therefore\ \theta = 66{,}42°$ In ΔABC: $\sin\frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33{,}21° = \frac{AB}{5}$ $\therefore\ AB = 5\sin 33{,}21°$ $= 2{,}74$ **OR/*OF*** $\sin\frac{\theta}{2} = \frac{AB}{5}$ $\therefore AB = 5\sin\frac{\theta}{2}$ but/*maar*: $\cos\theta = \frac{2}{5}$ $1 - 2\sin^2\frac{\theta}{2} = \frac{2}{5}$ $\sin^2\frac{\theta}{2} = \frac{3}{10}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore AB = 5\sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2{,}74$	✓ $\cos\theta = \frac{2}{5}$ ✓ size of/*grootte v* θ ✓ correct ratio/ *korrekte verh* ✓ subst correctly/ *korrek* ✓ answ/*antw* (5) ✓ $AB = 5\sin\frac{\theta}{2}$ ✓ equation/*vgl* ✓ simplification/ *vereenvoudiging* ✓ value of/*waarde v* $\sin\frac{\theta}{2}$ ✓ answ/*antw* (5) **[10]**

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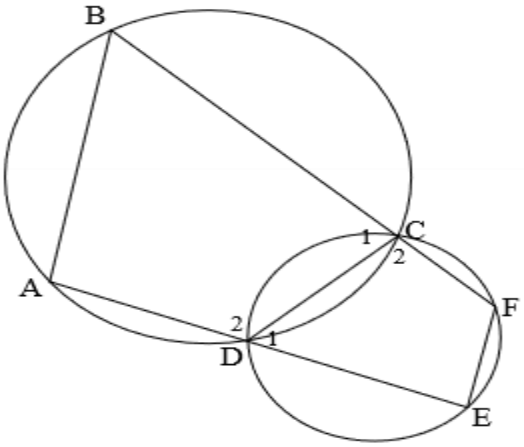
QUESTION/*VRAAG* 8



8.1.1	twice or double */twee keer of dubbel*	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2 ×omtreks∠*] $\hat{O}_2 = 2\hat{C}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2 ×omtreks∠*] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s in a rev/*∠e in omw of om 'n pt*] $2\hat{A} + 2\hat{C} = 360°$ $\therefore \hat{A} + \hat{C} = 180°$ **OR/OF** Let/*Gestel* $\hat{O}_1 = 2x$ $\hat{A} = x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2 ×omtreks∠*] $\hat{O}_2 = 360° - 2x$ [∠s in a rev/*∠e in omw of om 'n pt*] $\hat{C} = 180° - x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2 ×omtreks∠*] $\therefore \hat{A} + \hat{C} = 180°$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

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8.2

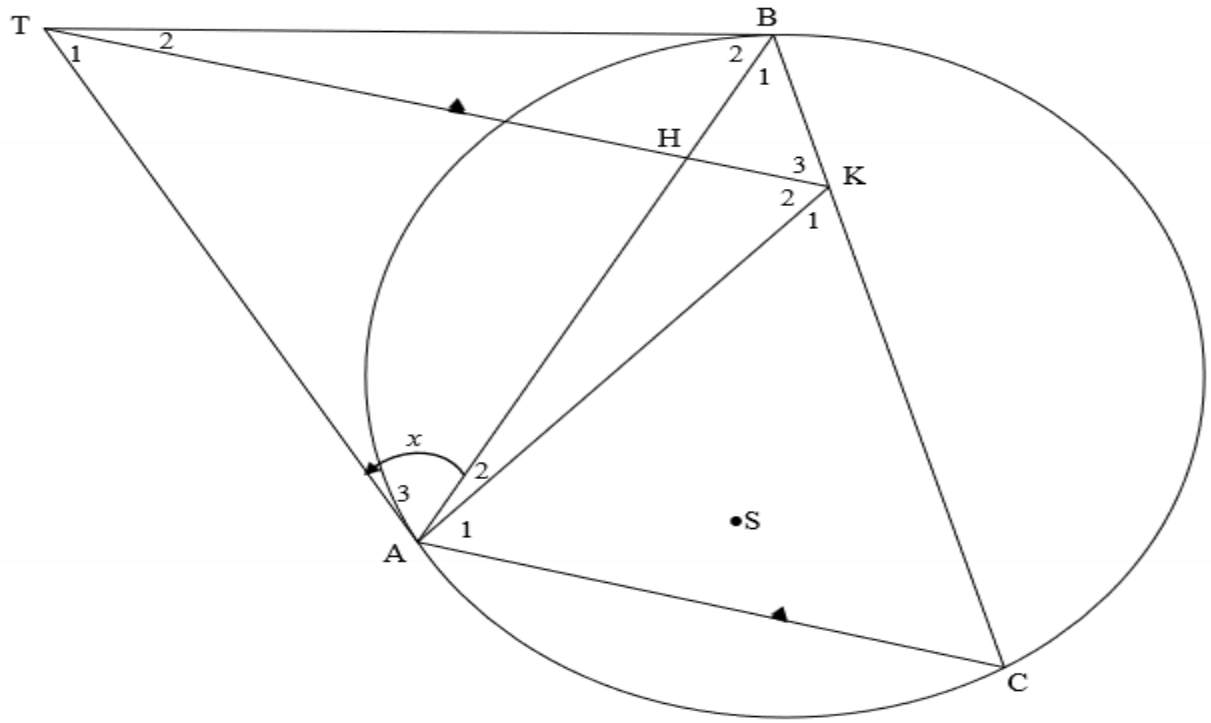


8.2	$\hat{A} = \hat{C}_2$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{E} = 180° - \hat{C}_2$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore \hat{E} = 180° - \hat{A}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R (5)
	OR/*OF*		
	$\hat{B} = \hat{D}_1$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{F} = 180° - \hat{D}_1$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore \hat{F} = 180° - \hat{B}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R (5) **[9]**

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QUESTION/*VRAAG 9*



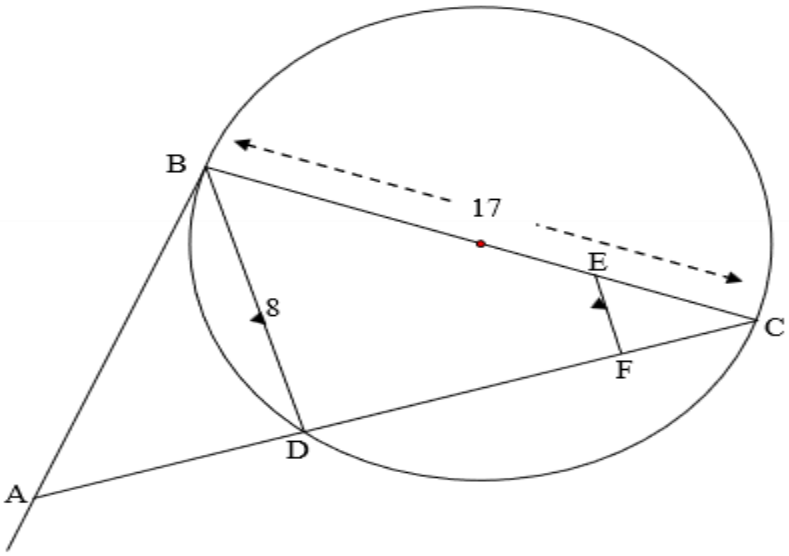
9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp ∠s/*ooreenk ∠e* ; CA| |KT] [tan-chord th/*raakl-koordst*]	✓ S ✓ R ✓ S ✓ R (4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ ∴ AKBT is cyc quad	[proved/*bewys* in 9.1] [line (BT) subtends equal ∠s/ *lyn (BT) onderspan gelyke ∠e*] **OR/OF** [converse ∠s in same segment/ *omgek ∠e in dies segment*]	✓ S ✓ R (2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ ∴ TK bisects/*halveer* $A\hat{K}B$ **OR/*OF*** $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/*raakl-koordst*] [∠s in the same segm/*∠e in dies segm*] [∠s in the same seg/*∠e in dies segm*] [tans from same pt; ∠s opp equal sides/ *rkle v dies pt; ∠e to gelyke sye*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R

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	$\therefore \quad = \hat{K}_3$ [proven in 9.1] $\therefore$ TK bisects/*halveer* $A\hat{K}B$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/*bewys*] $\therefore$ TA tangent [converse tan chord theorem OR ∠ between line and chord/ *omgekeerde raakl-kdst* ***OF*** *∠ tussen lyn en koord*]	✓ S ✓ R (2)
9.5	$B\hat{S}A = B\hat{K}A = 2x$ [A,S,K & B concyclic/*konsiklies*] $A\hat{T}B = 180° - 2x$ [A,T,B & K concyclic/*konsiklies*] $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is ook konsiklies* [opp ∠s of quad = 180°/*tos ∠e van vierhoek=180°*] **OR/*OF*** A, S K and B are concyclic. A, K, B and T are concyclic. $\therefore$ A, S, B and T are concyclic. **OR/*OF*** The circle passing through points A, K and B contains the point S on the circumference (A, ,S, K and B concyclic)./*Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).* The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./*Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).* $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is konsiklies*	✓ S (both/*beide* statements/ *bewerings*) ✓ R (2) ✓ S ✓ S (2) ✓ S ✓ S (2) **[14]**

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QUESTION/*VRAAG* 10



10.1	$B\hat{D}C = 90°$ $DC^2 = 17^2 - 8^2$ $= 225$ $\therefore DC = 15$	[∠ in semi circle/∠ *in halfsirkel*] [Th of/*stelling v* Pythagoras]	✓ S ✓ using/*gebruik* Pyth *korrek*/ correctly ✓ answ/*antw* (3)
10.2.1	$\frac{CF}{CD} = \frac{CE}{CB}$ $\therefore \frac{CF}{15} = \frac{1}{4}$ $\therefore CF = 3,75$	[line || one side of Δ/*lyn* || *een sy van* Δ] **OR/OF** ΔCEF ||| ΔCBD	✓ S/R ✓ subst correctly/ *korrek* ✓ answ/*antw* (3)
10.2.2	$B\hat{D}C = 90°$ $E\hat{F}C = B\hat{D}C$ $A\hat{B}C = 90°$ In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ $\hat{C} = \hat{C}$ $\therefore$ ΔBAC ||| Δ FEC **OR/*OF*** $B\hat{D}C = 90°$ $E\hat{F}C = B\hat{D}C$ $A\hat{B}C = 90°$ In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ $\hat{C} = \hat{C}$	[∠ in semi circle/∠ *in halfsirkel*] [corresp ∠s/*ooreenk ∠e*; EF| |BD] [tan ⊥ diameter/*raakl ⊥ middellyn*] [proven/bewys] [common/*gemeen*] [∠∠∠] [∠ in semi circle/∠ *in halfsirkel*] [corresp ∠s/*ooreenk ∠e*; EF| |BD] [tan ⊥ diameter/*raakl ⊥ middellyn*] [proven/bewys] [common/*gemeen*]	✓ S/R ✓ S ✓ R ✓ S ✓ R (5) ✓ S/R ✓ S ✓ R ✓ S

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	$B\hat{A}C = F\hat{E}C$ [∠ sum in Δ/∠ *som van* Δ] $\therefore \Delta BAC \mid\mid\mid \Delta FEC$	✓ S (5)
10.2.3	$EC = \frac{1}{4} \times 17 = 4,25$ $\frac{AC}{EC} = \frac{BC}{FC}$ [ΔBAC ∣∣∣ Δ FEC] $\frac{AC}{4,25} = \frac{17}{3,75}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3,75}{4,25} = \frac{17}{AC}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\Delta BCA \mid\mid\mid \Delta DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19,27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\hat{C} = A\hat{B}D$ [tan-chord theorem/*rkl-kdstelling*] $\frac{AD}{8} = \tan A\hat{B}D$ $= \tan\hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$	✓ length of/*lengte v* EC ✓ S ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓✓ correct ratios/ *korrekte verh's* ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓ S **OR** Pyth th ✓ correct ratio ✓ subst ✓ answ/*antw* (4) ✓ S ✓ correct ratio ✓ subst ✓ answ/*antw* (4)

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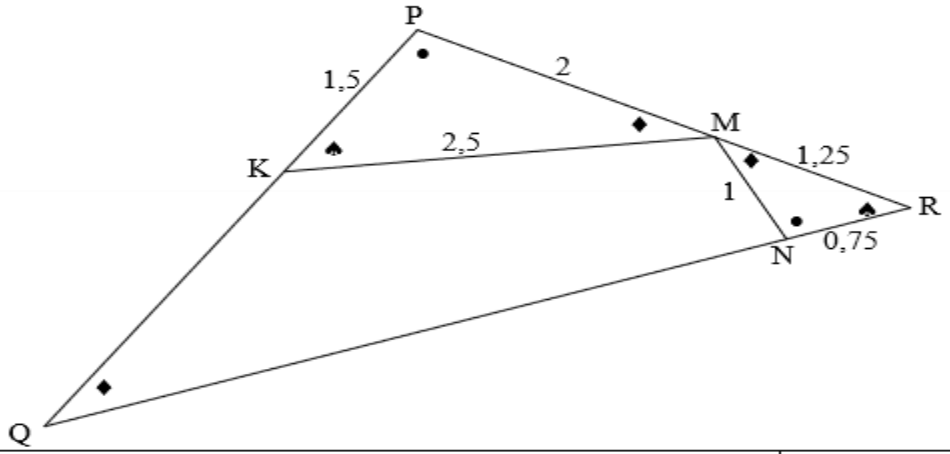
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10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends 90° **OR** converse ∠ in semi circle] *AC is middellyn van die sirkel wat deur die punte A, B en C gaan* [*koord onderspan 90° **OF** omgek ∠ in halfsirkel*] $\therefore \text{radius} = \frac{1}{2} \times 19{,}27 = 9{,}63$ or/*of* $9\frac{19}{30}$ or/*of* $\frac{1}{2}$AC	✓ S/R ✓ answ/*antw* (2) **[17]**

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QUESTION/*VRAAG* 11

11.1	equiangular or similar/*gelykhoekig of gelykvormig*	✓ answ/*antw* (1)



11.2.1	$\frac{KP}{RN}=\frac{1,5}{0,75}=2$; $\frac{PM}{NM}=\frac{2}{1}=2$; $\frac{KM}{RM}=\frac{2,5}{1.25}=2$ $\therefore \frac{KP}{RN}=\frac{PM}{NM}=\frac{KM}{RM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** $\frac{RN}{KP}=\frac{0,75}{1,5}=\frac{1}{2}$; $\frac{NM}{PM}=\frac{1}{2}$; $\frac{RM}{KM}=\frac{1,25}{2,5}=\frac{1}{2}$ $\therefore \frac{RN}{KP}=\frac{NM}{PM}=\frac{RM}{KM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** In ΔMNR: $1,25^2=1^2+0,75^2=1,5625$ $\therefore M\hat{N}R=90°$ [converse Pyth theorem] In ΔPKM: $2,5^2=1,5^2+2^2=6,25$ $\therefore \hat{P}=90°$ [converse Pyth theorem] $\cos P\hat{K}M=\frac{1,5}{2,5}=\frac{3}{5}$ and $\cos \hat{R}=\frac{0,75}{1,25}=\frac{3}{5}$ $\therefore P\hat{K}M=\hat{R}$ In ΔKPM and ΔRNM $P\hat{K}M=\hat{R}$ [proved] $\hat{P}=M\hat{N}R$ [proved] $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [∠;∠;∠ OR 3rd ∠]	✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓ $\hat{P}=M\hat{N}R$ ✓ $P\hat{K}M=\hat{R}$ ✓[∠;∠;∠ OR 3rd ∠] (3)

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11.2.2	$\hat{PKM} = \hat{R}$ [ΔKPM ||| ΔRNM] $\therefore$ $\hat{P}$ is common/*gemeen* $\therefore$ ΔRPQ ||| ΔKPM [∠∠∠] $\frac{RP}{KP} = \frac{RQ}{KM}$ [ΔRPQ ||| ΔKPM] $\therefore \frac{3,25}{1,5} = \frac{RQ}{2,5}$ $\therefore RQ = \frac{2,5 \times 3,25}{1,5} = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$ **OR/OF** $R\hat{N}M = \hat{P}$ [ΔKPM ||| ΔRNM] $\therefore$ $\hat{R}$ is common/*gemeen* $\therefore$ ΔRNM ||| ΔRPQ [∠∠∠] $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ||| ΔRPQ] $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$ **OR/*OF*** In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90°$ [converse Pyth theorem] In ΔMNR and ΔQPR ∠R is common $M\hat{N}R = \hat{P} = 90°$ $\therefore$ ΔMNR ||| ΔQPR [∠∠∠] $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ||| ΔRPQ] $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$	✓ S ✓ ΔRPQ|||ΔKPM ✓ S ✓ subst correctly/ *korrek* ✓ RQ = $5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) ✓ S ✓ ΔRNM ||| ΔRPQ ✓ S ✓ subst correctly/ *korrek* ✓ RQ = $5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) ✓ S ✓ ΔMNR ||| ΔQPR ✓ S ✓ subst correctly/ *korrek* ✓ RQ = $5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) **[10]**

TOTAL/*TOTAAL*: 149

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basic education

Department:
Basic Education
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GRADE 12

MATHEMATICS P2

NOVEMBER 2016

MARKS: 150

TIME: 3 hours

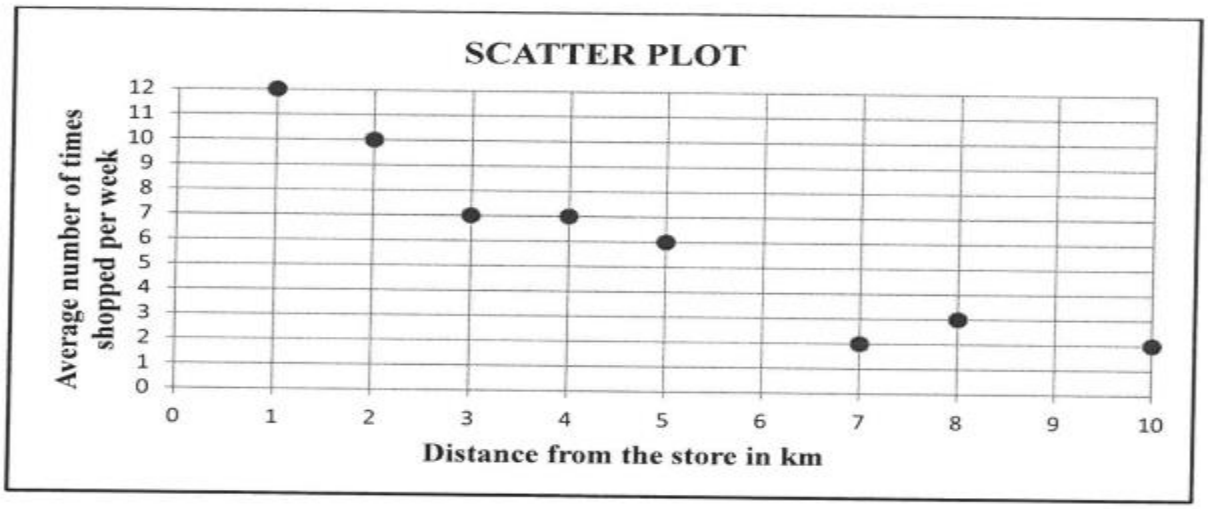
This question paper consists of 14 pages, 1 information sheet and an answer book of 28 pages.

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QUESTION 1

A survey was conducted at a local supermarket relating the distance that shoppers lived from the store to the average number of times they shopped at the store in a week. The results are shown in the table below.

Distance from the store in km	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2



1.1 Use the scatter plot to comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week. (1)

1.2 Calculate the correlation coefficient of the data. (1)

1.3 Calculate the equation of the least squares regression line of the data. (3)

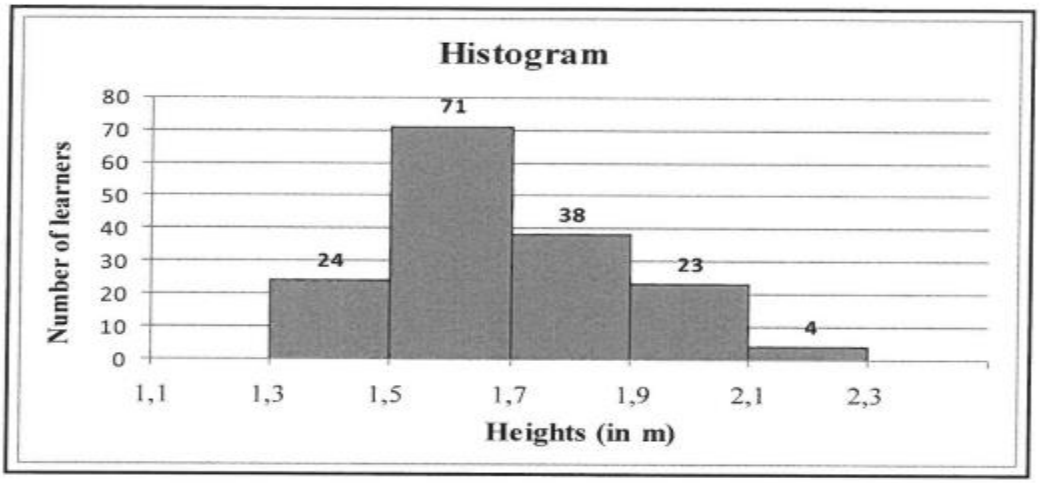
1.4 Use your answer at QUESTION 1.3 to estimate the average number of times that a shopper living 6 km from the supermarket will visit the store in a week. (2)

1.5 Sketch the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)
[9]

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QUESTION 2

The heights of 160 learners in a school are measured. The height of the shortest learner is 1,39 m and the height of the tallest learner is 2,21 m. The heights are represented in the histogram below.



2.1 Describe the skewness of the data. (1)

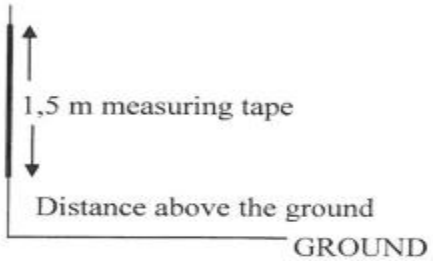
2.2 Calculate the range of the heights. (2)

2.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.4 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

2.5 Eighty learners are less than x metres in height. Estimate x. (2)

2.6 The person taking the measurements only had a 1,5 m measuring tape available. In order to compensate for the short measuring tape, he decided to mount the tape on a wall at a height of 1 m above the ground. After recording the measurements he discovered that the tape was mounted at 1,1 m above the ground instead of 1 m.



How does this error influence the following:

2.6.1 Mean of the data set (1)

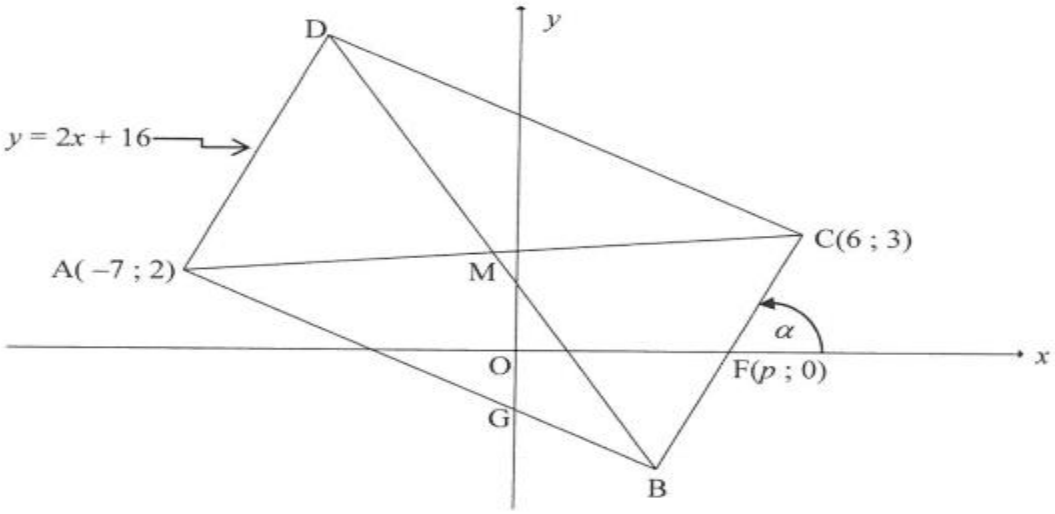
2.6.2 Standard deviation of the data set (1)
[13]

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QUESTION 3

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.



3.1 Calculate the coordinates of M. (2)

3.2 Write down the gradient of BC in terms of p. (1)

3.3 Hence, calculate the value of p. (3)

3.4 Calculate the length of DB. (3)

3.5 Calculate the size of α. (2)

3.6 Calculate the size of $O\hat{G}B$. (3)

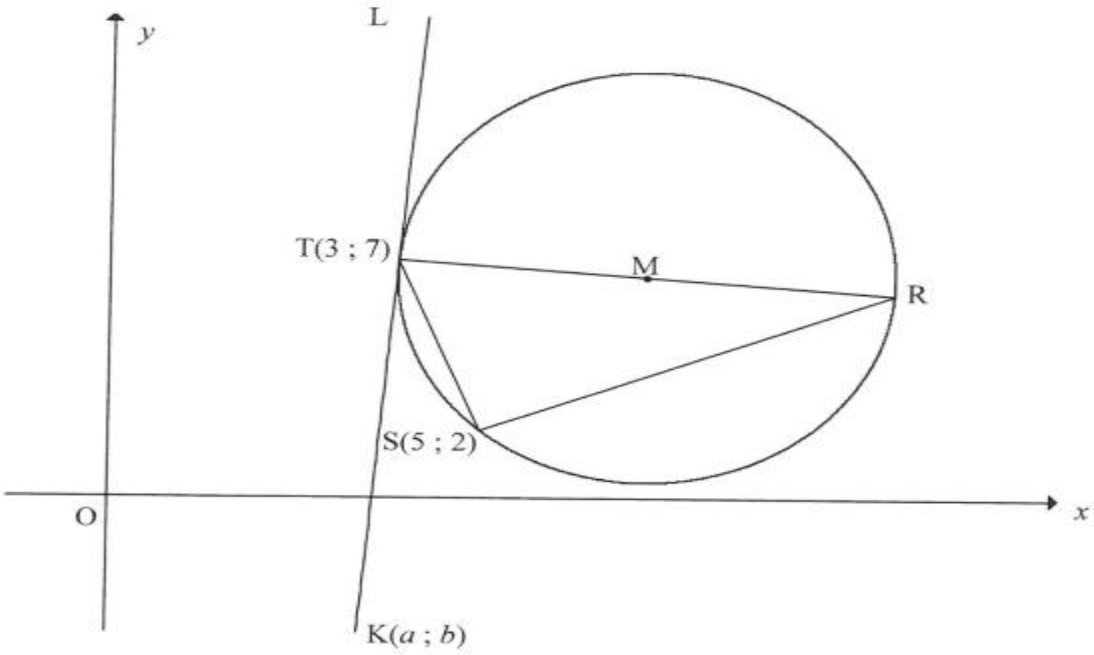
3.7 Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

3.8 If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer. (2)
[19]

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QUESTION 4

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

4.4.4 Show that $b = 12a - 29$. (3)

4.4.5 If TK = TR, calculate the coordinates of K. (6)
[23]

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QUESTION 5

5.1 Given: $\sin 16° = p$
Determine the following in terms of p, **without using a calculator**.

5.1.1 $\sin 196°$ (2)

5.1.2 $\cos 16°$ (2)

5.2 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$ (3)

5.3 Simplify $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90° + A)}$ completely, given that $0° < A < 90°$. (5)

5.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0° \leq B \leq 90°$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

5.4.1 $\cos B$ (3)

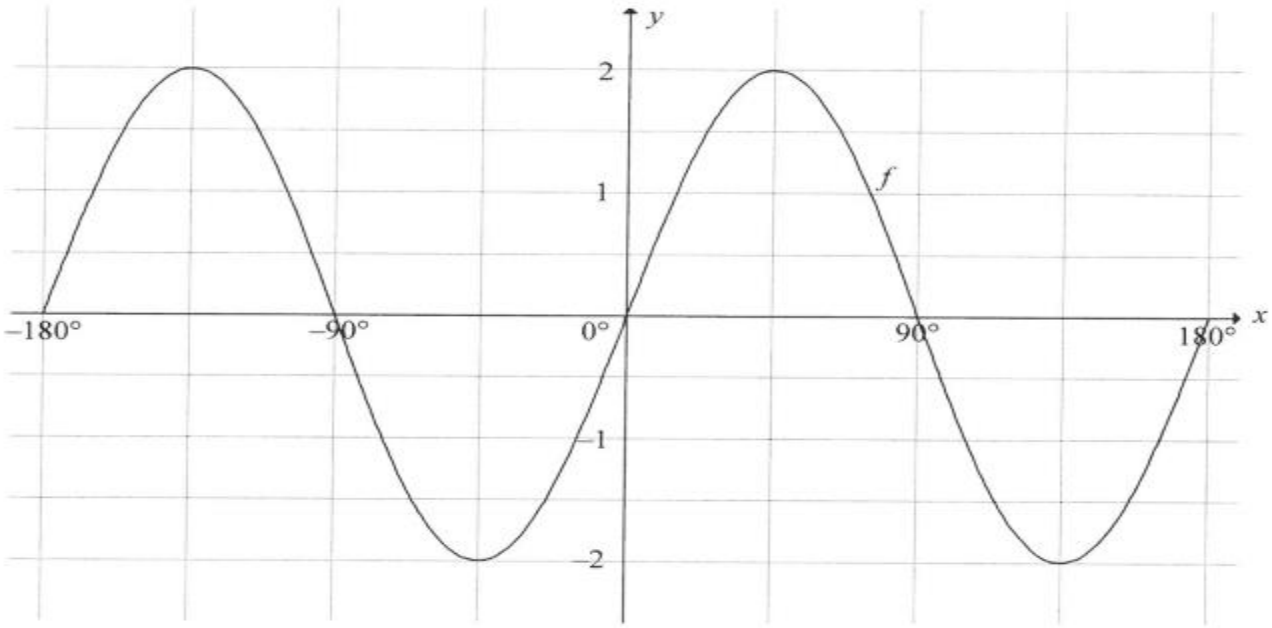
5.4.2 $\sin B$ (2)

5.4.3 $\cos(B + 45°)$ (4)
[21]

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QUESTION 6

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

6.3 Determine the general solution of $f(x) = g(x)$. (4)

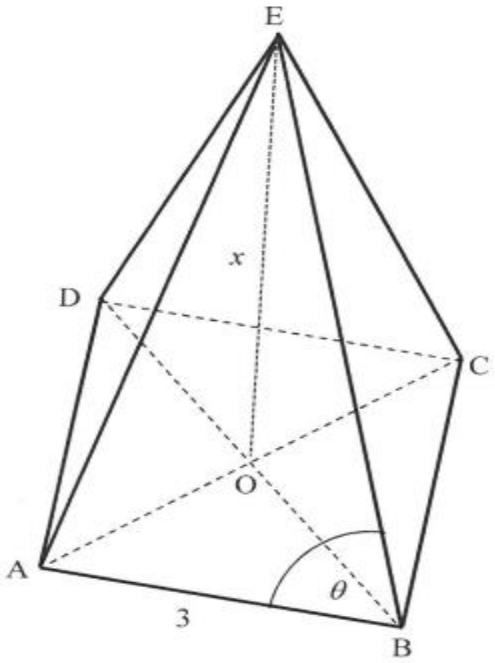
6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180° ; 0°]$. (3)
[12]

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QUESTION 7

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $E\hat{B}A = \theta$, AB = 3 m and EO, the perpendicular height of the pyramid, is x.

Volume of pyramid $= \frac{1}{3}$(area of base) $\times$ ($\perp$ height)



7.1 Calculate the length of OB. (3)

7.2 Show that $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ (5)

7.3 If the volume of the pyramid is 15 m^3, calculate the value of θ. (4)

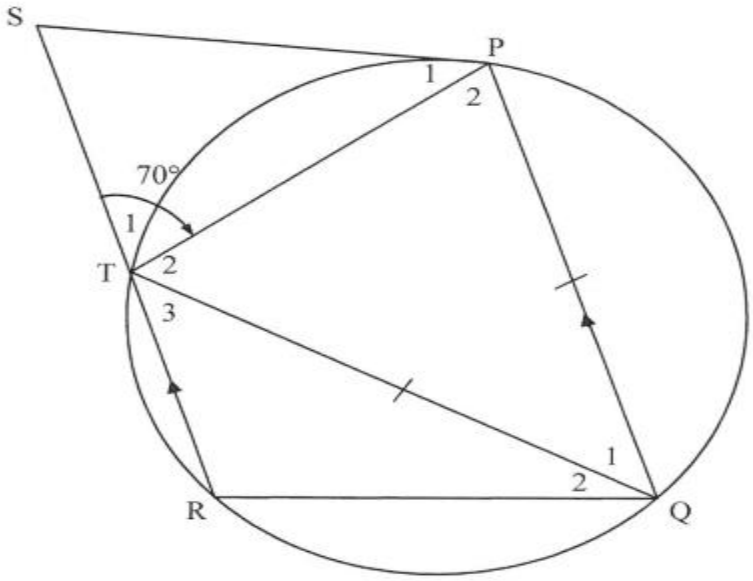
[12]

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Give reasons for ALL statements and calculations in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below PQRT is a cyclic quadrilateral having RT ∥ QP. The tangent at P meets RT produced at S. QP = QT and $P\hat{T}S = 70°$.



8.1.1 Give a reason why $\hat{P}_2 = 70°$. (1)

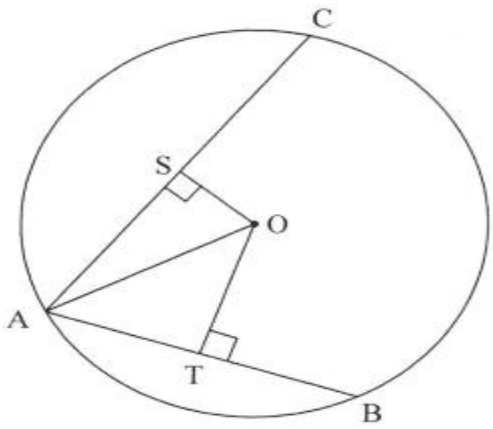
8.1.2 Calculate, with reasons, the size of:

(a) $\hat{Q}_1$ (3)

(b) $\hat{P}_1$ (2)

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8.2 A, B and C are points on the circle having centre O. S and T are points on AC and AB respectively such that OS ⊥ AC and OT ⊥ AB. AB = 40 and AC = 48.



8.2.1 Calculate AT. (1)

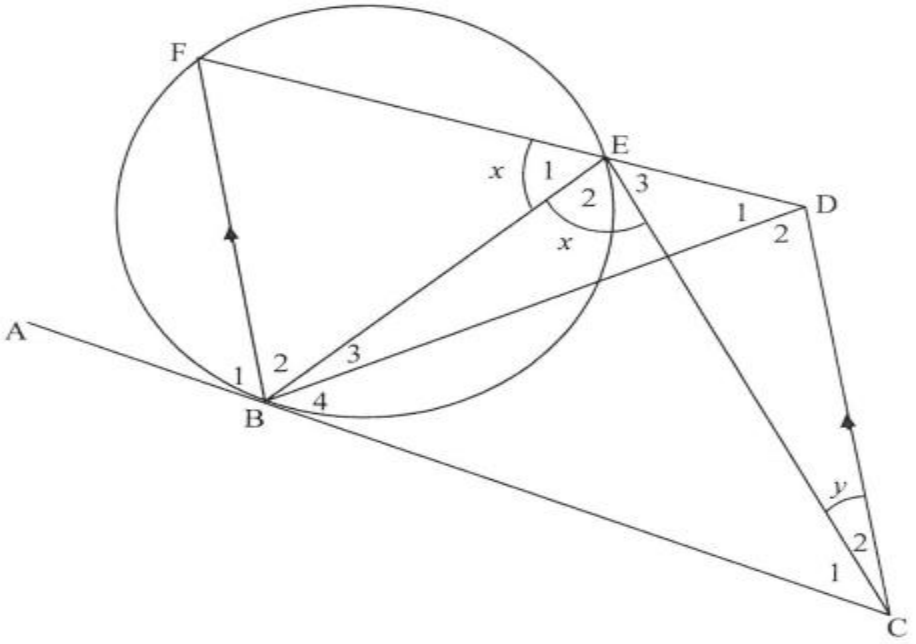
8.2.2 If $OS = \frac{7}{15}OT$, calculate the radius OA of the circle. (5)

[12]

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QUESTION 9

ABC is a tangent to the circle BFE at B. From C a straight line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$.



9.1 Give a reason why EACH of the following is TRUE:

9.1.1 $\hat{B}_1 = x$ (1)

9.1.2 $B\hat{C}D = \hat{B}_1$ (1)

9.2 Prove that BCDE is a cyclic quadrilateral. (2)

9.3 Which TWO other angles are each equal to x? (2)

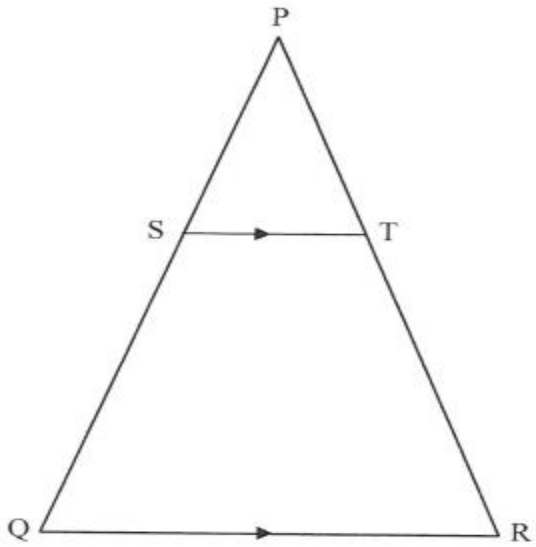
9.4 Prove that $\hat{B}_2 = \hat{C}_1$. (3)

[9]

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QUESTION 10

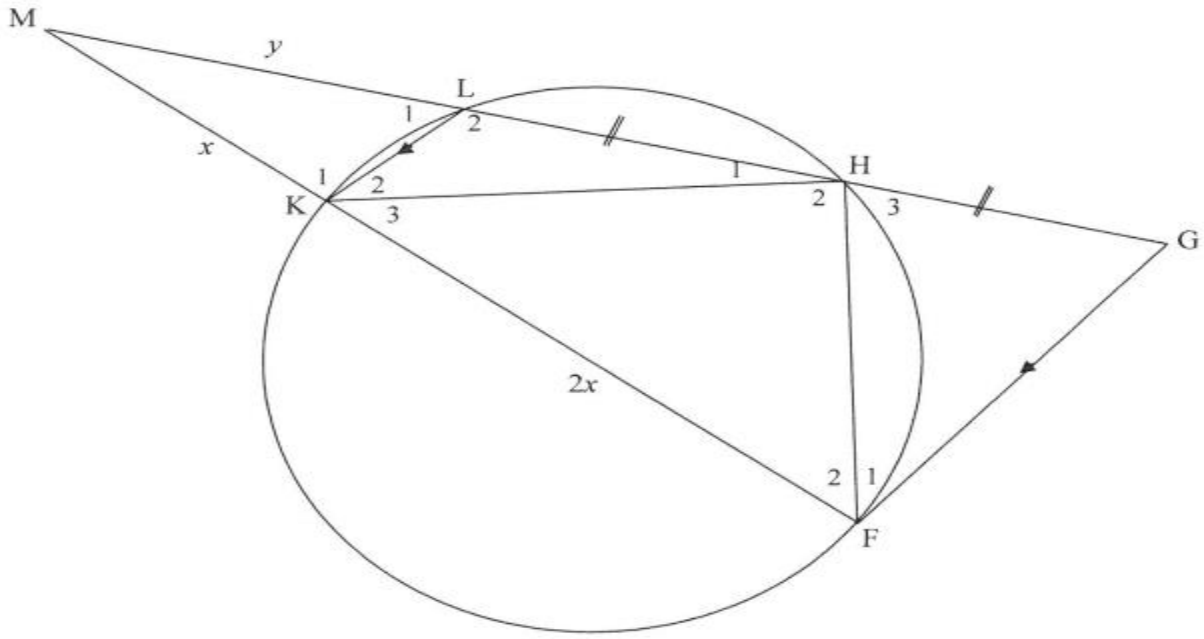
10.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that ST $\parallel$ QR.



Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

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10.2 In the diagram HLKF is a cyclic quadrilateral. The chords HL and FK are produced to meet at M. The line through F parallel to KL meets MH produced at G. MK = x, KF = $2x$, ML = y and LH = HG.



10.2.1 Give a reason why $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Prove that:

(a) $GH = y$ (3)

(b) $\Delta MFH \,|||\, \Delta MGF$ (5)

(c) $\dfrac{GF}{FH} = \dfrac{3x}{2y}$ (2)

10.2.3 Show that $\dfrac{y}{x} = \sqrt{\dfrac{3}{2}}$ (3)

[20]

TOTAL: 150

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MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2016

MEMORANDUM

MARKS/*PUNTE*: 150

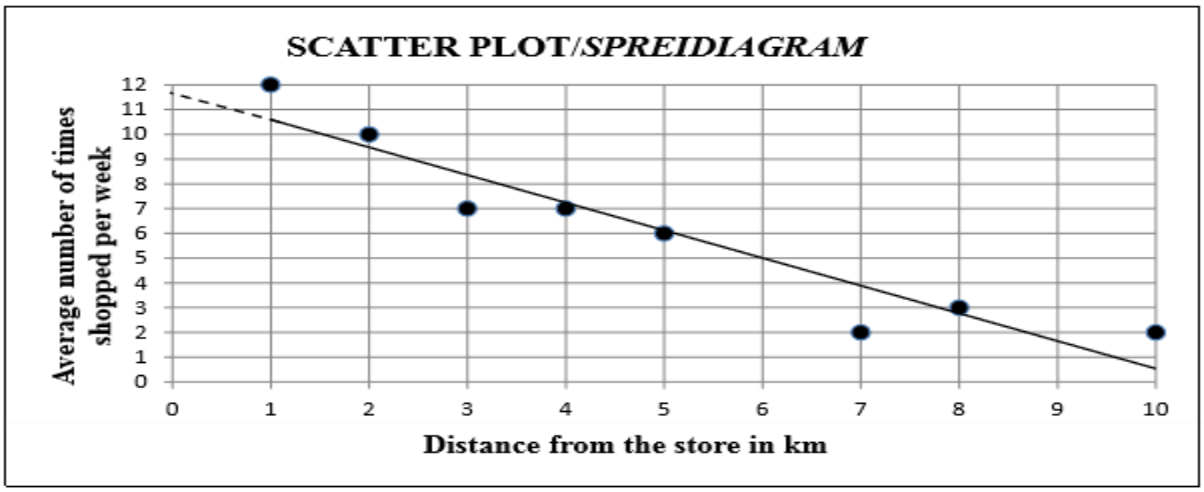
This memorandum consists of 26 pages.
Hierdie memorandum bestaan uit 26 bladsye.

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Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 1

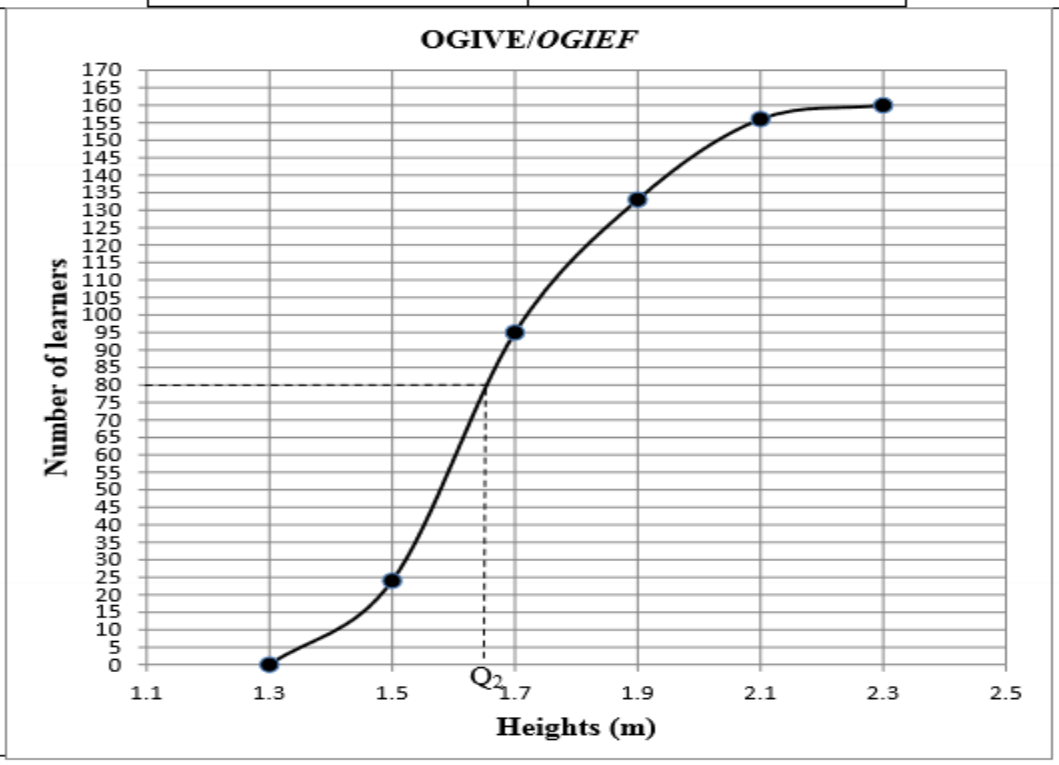
Distance from the store in km *Afstand vanaf die winkel in* **km**	1	2	3	4	5	7	8	10
Average number of times shopped per week *Gemiddelde aantal keer wat kopers die winkel per week besoek*	12	10	7	7	6	2	3	2



1.1	Strong/*Sterk*	✓ (1)
1.2	–0,95 (–0,9462..)	✓ (1)
1.3	$a = 11{,}71$ (11,7132…) $b = -1{,}12$ (–1,1176…) $\hat{y} = -1{,}12x + 11{,}71$	✓ value of a ✓ value of b ✓ equation/*vgl* (3)
1.4	$\hat{y} = -1{,}12(6) + 11{,}71$ $= 5$ times	✓ substitition ✓ answer (2)
1.5	On scatter plot/*Op spreidiagram*	✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) (2) **[9]**

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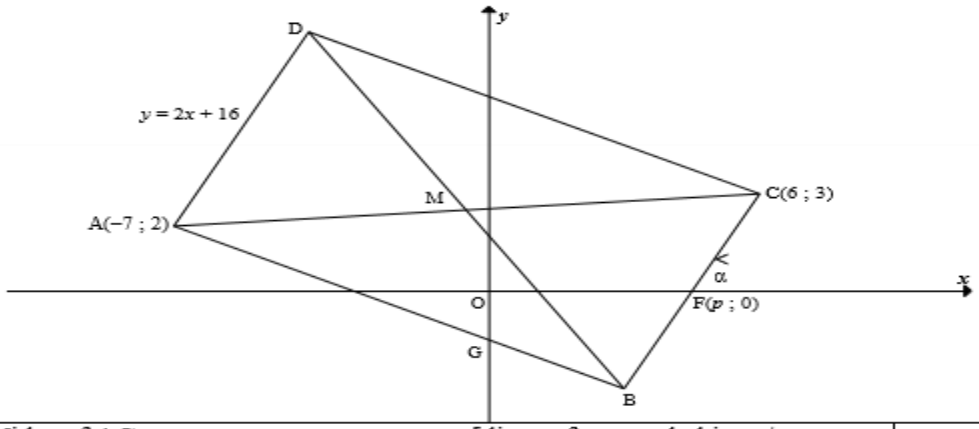
QUESTION/*VRAAG* 2

2.1	Positively skewed **OR** skewed to the right/*positief skeef* ***OF*** *skeef na regs*	✓ answer (1)
2.2	Range/*Omvang* = 2,21 – 1,39 = 0,82 m	✓ subtract values ✓answer (2)
2.3	**Intervals** / ***Klasse*** | **Cumulative frequency** / ***Kumulatiewe frekwensie*** $1,3 \le x < 1,5$ | 24 $1,5 \le x < 1,7$ | 95 $1,7 \le x < 1,9$ | 133 $1,9 \le x < 2,1$ | 156 $2,1 \le x < 2,3$ | 160	✓95 , 133, 156 ✓160 (2)
2.4		✓ upper limits / *boonste limiete* ✓ cum *f*/ *kum f* ✓ shape/ *vorm* ✓ grounded *geanker* (4)
2.5	method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69)	✓ method ✓ answer (2)
2.6.1	The mean would change by 0,1 *m* *Die gemiddelde sal met 0,1 m verander*	✓ answer (1)
2.6.2	No influence/change as there is no difference in variation of data./*Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.*	✓ answer (1) **[13]**

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QUESTION/*VRAAG 3*

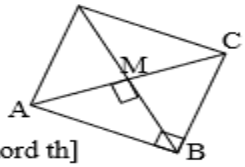


3.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $= M\left(\frac{-7+6}{2} ; \frac{2+3}{2}\right)$ $= M\left(-\frac{1}{2} ; \frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
3.2	$m_{BC} = \frac{3-0}{6-p} = \frac{3}{6-p}$ **OR/*OF*** $m_{BC} = \frac{0-3}{p-6} = \frac{-3}{p-6}$	✓answer (1) ✓answer (1)
3.3	$m_{AD} = m_{BC}$ [AD || BC] $m_{BC} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ **OR/OF** $y - y_1 = 2(x - x_1)$ $C(6;3)$ $y - 3 = 2(x - 6)$ $\therefore y = 2x - 9$ *but* $y = 0$ $\therefore x = 4\frac{1}{2} = p$ **OR/OF**	✓ $m_{BC} = 2$ ✓ equating ✓answer (3) ✓ $m_{BC} = 2$ ✓ substituting (6 ; 3) ✓answer (3)

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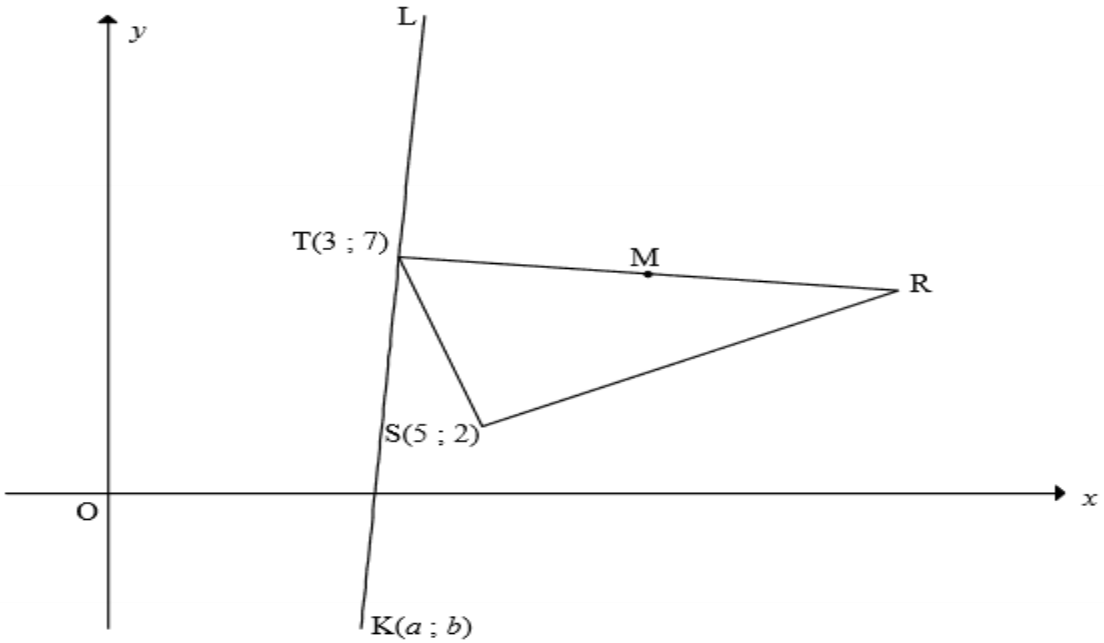
	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2}$ $\quad \therefore p = \frac{9}{2}$	✓ $m_{BC} = 2$ ✓ substituting ✓answer (3)
3.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore$ DB = $\sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
3.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
3.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp ∠s/*regoorst ∠e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum ∠s quad/*som ∠e vierh* = 360°] $\therefore O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ $90° + O\hat{G}A = 153{,}43°$ $\therefore O\hat{G}A = 63{,}43°$ $O\hat{G}B = 180° - 63{,}43°$ $= 116{,}57°$ **OR/*OF*** $F\hat{O}G = G\hat{B}F = 90°$ $\therefore$GOFB is cyc quad $O\hat{G}B = 180° - 63{,}43°$ [∠s of cyc quad = 180°] $= 116{,}57°$ **OR/OF** $O\hat{F}B = 63{,}43°$ $X\hat{O}G = F\hat{B}G = 90°$ $\therefore$ OGBF is a cyclic quad $\therefore O\hat{G}B = 180° - 63{,}43°$ $O\hat{G}B = 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3)

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3.7	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r=\frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\left(\frac{\sqrt{170}}{2}\right)^2=\frac{85}{2}=42,5$	✓ M is centre ✓ $r=\frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$C\hat{B}M = B\hat{A}M = 45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] ∴ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF*** $A\hat{M}B = 90°$ [diag of square bisect ⊥] ∴ AB is diameter BC ⊥ AB ∴ BC is tangent [line ⊥ radius *or* converse tan-chord th] 	✓S ✓ R (2) ✓S ✓ R (2) **[19]**

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QUESTION/*VRAAG 4*



4.1	∠ in semi circle/ ∠ at centre = 2∠ on circle ∠ *in halfsirkel* /∠ by middelpt = 2∠ op sirkel	✓ R (1)
4.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)

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	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓ substitution m and (5 ; 2) ✓ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR/OF** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2) ✓ substitution ✓ answer (2)
4.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore$ R(15 ; 6) Answer only: full marks Answer only: only 1 coordinate correct (1 mark) **OR/*OF*** $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9+6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ **OR/OF**	✓ M ✓ x coordinate ✓ y coordinate (3) ✓ M ✓ x coordinate ✓ y coordinate (3)

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	$m_{TM} = \frac{9-3}{6\frac{1}{2}-7} = -\frac{1}{12}$ $TM: \quad 7 = -\frac{1}{12}(3) + c \quad y = -\frac{1}{12}x + \frac{29}{4}$ .....(1) $SR: \quad y = \frac{2}{5}x$ .....(2) $\frac{2}{5}x = -\frac{1}{12}x + \frac{29}{4}$ $\frac{29}{60}x = \frac{29}{4}$ $\therefore x = 15$ $\therefore y = \frac{2}{5}(15) = 6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
4.4.3	$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $ST = \sqrt{(5-3)^2 + (2-7)^2}$ $ST = \sqrt{4+25} = \sqrt{29}$ $\sin R = \frac{TS}{TR} = \frac{\sqrt{29}}{\sqrt{145}} or \frac{\sqrt{5}}{5} or \frac{1}{\sqrt{5}} or \ 0{,}45$ **OR/OF** $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ area of $\Delta TSR = \frac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right) = 29$ $29 = \frac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R = \frac{\sqrt{5}}{5} or \frac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
4.4.4	$m_{TR} = \frac{7-6\frac{1}{2}}{3-9} = -\frac{1}{12}$ **OR/*OF*** $m_{TR} = \frac{7-6}{3-15} = -\frac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ [$r \perp$ tangent] $m_{KTL} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x - 3)$ $y = 12x - 29$ substitute $K(a;b)$: $b = 12a - 29$ **OR/*OF***	✓ $m_{TR} = -\frac{1}{12}$ ✓ $m_{KTL} = 12$ ✓ $y = 12x - 29$ (3)

	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ [$r \perp$ tangent] $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a-29$ **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓ substitution (3 ; 7) & (a ; b) (3) ✓ subst into Pyth ✓ multiplication ✓ simplification (3)
4.4.5	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a-29-7)^2 = 145$ $(a-3)^2 + (12a-36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$ $\therefore$ K(2 ; –5) **OR/*OF***	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓ standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

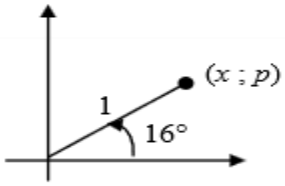
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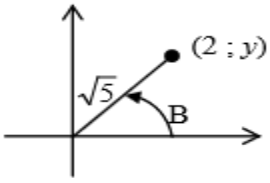
	$TK = TR$ $\sqrt{(a-3)^2+(b-7)^2} = \sqrt{145}$ $(a-3)^2+(b-7)^2 = 145$ Substitute $b = 12a-29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2 = 145$ $(a-3)^2+(12a-36)^2 = 145$ $(a-3)^2+144(a-3)^2 = 145$ $(a-3)^2 = 1$ $a-3 = \pm 1$ $a = 2$ or 4 $\therefore b = 12(2)-29$ or $b = 12(4)-29$ $= -5$ $= 19$ $\therefore$ K(2 ; –5) **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2+(b-6)^2 = 145+145$ $(a-15)^2+(12a-29-6)^2 = 290$ $(a-15)^2+(12a-35)^2 = 290$ $a^2-30a+225+144a^2-840a+1225 = 290$ $145a^2-870a+1160 = 0$ $a^2-6a+8 = 0$ $\therefore (a-2)(a-4) = 0$ $a = 2$ or $a = 4$ $\therefore b = 12(2)-29$ or $b = 12(4)-29$ $= -5$ $= 19$ K(2 ; –5)	✓ substitution into distance formula ✓ substitution of $b = 12a-29$ ✓ $(a-3)^2 = 1$ ✓ ± 1 ✓ values of a ✓ value of b (6) ✓ substitution ✓ substitution of $b = 12a-29$ ✓ standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

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QUESTION/*VRAAG 5*

5.1.1	$\sin 196° = -\sin 16°$ $= -p$	✓reduction ✓answer (2)
5.1.2	$\cos 16° = \sqrt{1-\sin^2 16°}$ $= \sqrt{1-p^2}$ **OR/*OF*** $x^2 + p^2 = 1$ $x = \sqrt{1-p^2}$ $\therefore \cos 16° = \frac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$ 	✓statement ✓answer (2) ✓x in terms of p ✓answer (2)
5.2	$\sin(A+B) = \cos[90° - (A+B)]$ $= \cos[(90° - A) - B]$ $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ $= \sin A\cos B + \cos A\sin B$	✓co-ratio ✓correct form ✓expansion (3)
5.3	$\frac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90° + A)}$ $= \frac{\sqrt{\sin^2 2A}}{\cos A.(-\sin A)}$ $= \frac{\sin 2A}{\cos A.(-\sin A)}$ $= \frac{2\sin A\cos A}{\cos A.(-\sin A)}$ $= -2$ **OR/OF** $\frac{\sqrt{1-\cos^2 2A}}{\cos(-A)\cos(90° + A)} = \frac{\sqrt{1-(2\cos^2 A - 1)^2}}{\cos A.-\sin A}$ $= \frac{\sqrt{1-(4\cos^4 A - 4\cos^2 A + 1)}}{\cos A.-\sin A} = \frac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A.-\sin A}$ $= \frac{\sqrt{4\cos^2 A(1-\cos^2 A)}}{\cos A.-\sin A} = \frac{\sqrt{4\cos^2 A\sin^2 A}}{\cos A.-\sin A}$ $= \frac{2\cos A.\sin A}{\cos A.-\sin A}$ $= -2$ **OR/OF**	✓$\sqrt{\sin^2 2A}$ ✓cosA ✓$-\sin A$ ✓$2\sin A\cos A$ ✓ answer (5) ✓$2\cos^2 A - 1$ ✓cosA ✓$-\sin A$ ✓identity ✓ answer (5)

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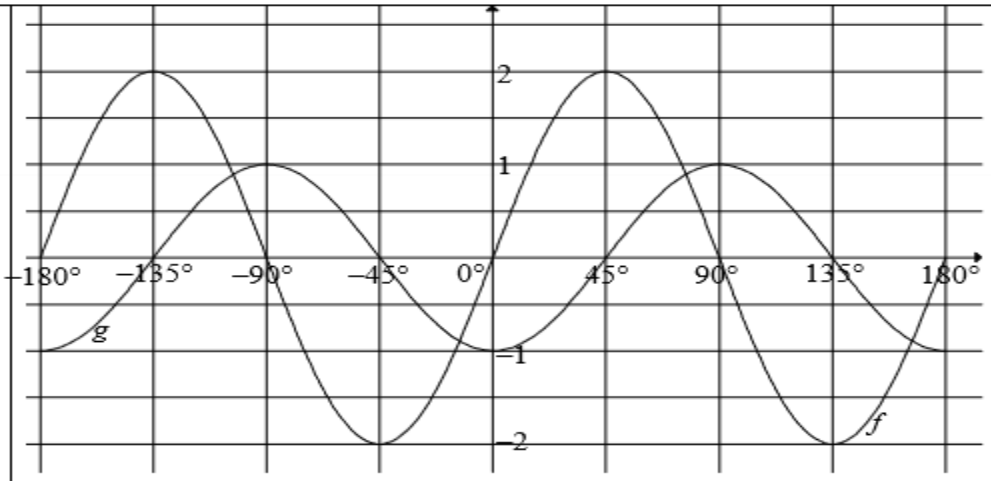
	$\frac{\sqrt{1-(1-2\sin^2 A)^2}}{\cos A. - \sin A}$ $= \frac{\sqrt{1-(1-4\sin^2 A + 4\sin^2 A)}}{\cos A. - \sin A}$ $= \frac{\sqrt{4\sin^2 A(1-\sin^2 A)}}{\cos A. - \sin A}$ $= \frac{2\sin A\sqrt{\cos^2 A}}{\cos A. - \sin A}$ $= -2$	✓$1-2\sin^2 A$ ✓cosA ✓$-\sin A$ ✓identity ✓ answer (5)
5.4.1	$\cos 2B = \frac{3}{5}$ $2\cos^2 B - 1 = \frac{3}{5}$ $\cos^2 B = \frac{4}{5}$ $\therefore \cos B = \sqrt{\frac{4}{5}}$ *or* $\frac{2}{\sqrt{5}}$ *or* $\frac{2\sqrt{5}}{5}$ $[0° \le B \le 90°]$ **OR/*OF*** $\cos B = \frac{\sqrt{\cos 2B + 1}}{2}$ $= \frac{\sqrt{\frac{3}{5}+1}}{2}$ $= \frac{2\sqrt{5}}{5}$	✓ identity ✓ value of $\cos^2 B$ ✓ answer (3) ✓ $= \frac{\sqrt{\cos 2B + 1}}{2}$ ✓ value of $\cos^2 B$ ✓ answer (3)
5.4.2	$\sin^2 B = 1 - \cos^2 B$ $= 1 - \left(\frac{2}{\sqrt{5}}\right)^2$ $= \frac{1}{5}$ $\therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$ **OR/*OF*** $(2)^2 + y^2 = (\sqrt{5})^2$ $4 + y^2 = 5$ $y^2 = 1$ $y = 1$ $\therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$  	✓ $\sin^2 B = \frac{1}{5}$ ✓ answer (2) ✓ $y = 1$ ✓ answer (2)

	OR/*OF* $\cos 2B = \frac{3}{5}$ $1 - 2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$	✓ $\sin^2 B = \frac{1}{5}$ ✓ answer (2)
5.4.3	$\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}$ *or* $\frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
	OR/*OF* $\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}$ *or* $\frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
		[21]

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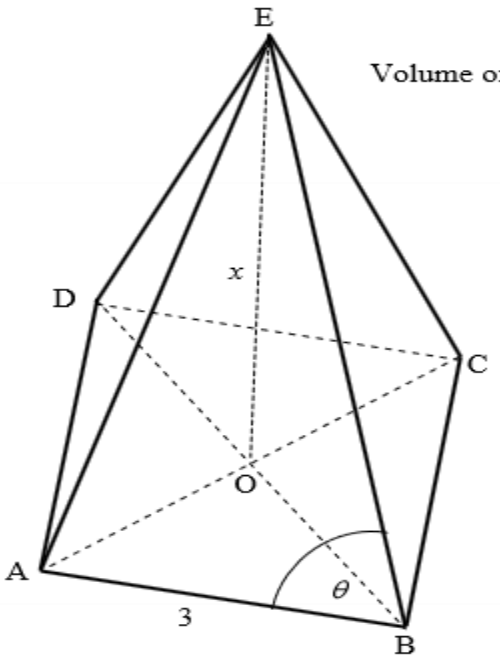
QUESTION/*VRAAG 6*

6.1		✓ *x*- intercepts/ *afsnitte* ✓ *y*- intercept/ *afsnit* ✓ turning pts/ *draaipte* (3)
6.2	$f(x)-3=2\sin 2x-3$ $\therefore$ maximum value $=2-3=-1$	✓✓ answer (2)
6.3	$2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.180°;k\in Z$ $x=76{,}72°+k.90°;k\in Z$ *or* $x=-13.28°+k.90°,k\in Z$ **OR/*OF*** $2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.360°$ *or* $333{,}43°+k.360°,k\in Z$ $x=76{,}72°+k.180°$ *or* $166{,}72°+k.180°;k\in Z$	✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ or $-26{,}56°$ ✓ $76{,}72°$ or $-13{,}28°$ ✓ $k.90°;k\in Z$ (4) ✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ & $333{,}43°$ ✓ $76{,}72°$ & $166{,}72°$ ✓ $k.180°;k\in Z$ (4)
6.4	$x\in(-103{,}28°;-13{,}28°)$ **OR/*OF*** $-103{,}28°<x<-13{,}28°$	✓✓ values ✓notation (3) ✓✓ values ✓notation (3) **[12]**

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QUESTION/*VRAAG* 7



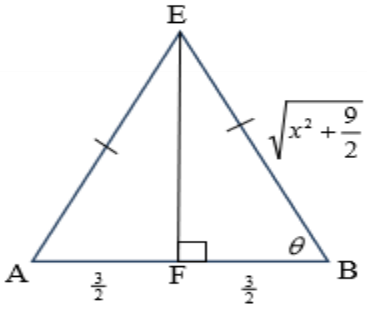
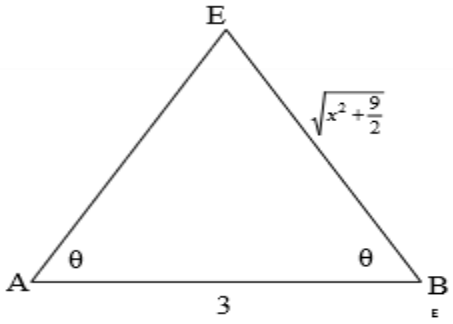
$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$

7.1	$DB^2 = 3^2 + 3^2$ [Theorem of Pyth] $= 18$ $DB = \sqrt{18}$ $OB = \frac{1}{2}DB = \frac{\sqrt{18}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12 **OR/*OF*** $\sin 45° = \frac{OB}{3}$ $OB = 3\sin 45°$ $OB = \frac{3\sqrt{2}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* 2,12 **OF/OR** $\cos 45° = \frac{OB}{3}$ $\frac{1}{\sqrt{2}} = \frac{OB}{3}$ $OB = \frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12	✓ substitution into Pyth ✓ value of DB ✓ answer (3) ✓ correct ratio ✓ OB as subject ✓ answer (3) ✓ correct ratio ✓ special angle ✓ answer (3)

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	OR/OF $A\hat{O}B = 90°$ (diagonals bisect ⊥) $OB = OA$ $AB^2 = AO^2 + BO^2$ [pyth] $\therefore AB^2 = 2OB^2$ $2OB^2 = 3^2$ $\therefore OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12	✓ OB = OA ✓ Pyth ✓ answer (3)
7.2	$BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\cos\theta = \frac{AB^2 + EB^2 - AE^2}{2AB.EB} = \frac{AB^2}{2AB.EB}$ [EB = AE] $\cos\theta = \frac{AB}{2EB}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ cos θ as subject ✓ simplification (5)
	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\left(\sqrt{x^2 + \frac{9}{2}}\right)^2 = 9 + \left(\sqrt{x^2 + \frac{9}{2}}\right)^2 - 2(3)\left(\sqrt{x^2 + \frac{9}{2}}\right)\cos\theta$ $\cos\theta = \frac{9}{6\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ substituting ✓ cos θ as subject (5)

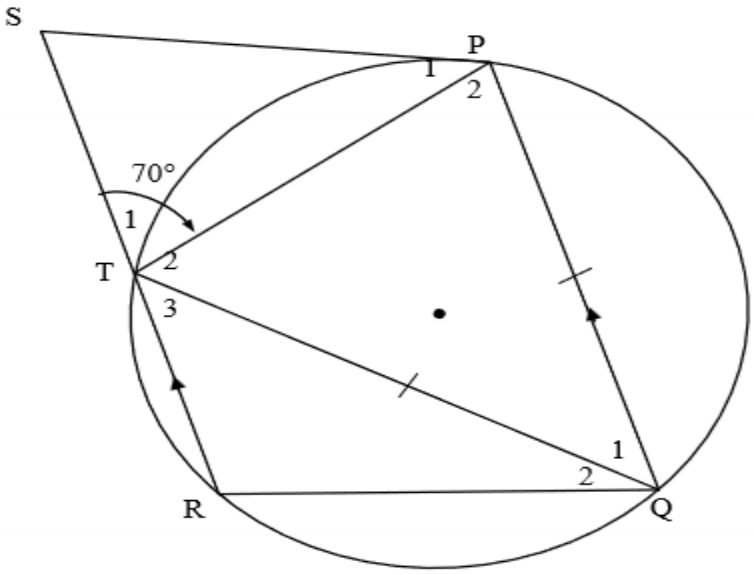
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	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{\frac{3}{2}}{\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  	✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution (5)
	OR/OF $\hat{E} = 180° - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \frac{3}{\sin 2\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\sin\theta\cos\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\cos\theta} = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  	✓ $\hat{E} = 180° - 2\theta$ ✓ $\sin E = \sin 2\theta$ ✓ subst into sine rule ✓ diagram ✓ $2\sin\theta\cos\theta$ (5)
7.3	Volume $= \frac{1}{3}$(area of base) $\times$ ($\perp$ height) $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos\theta = \frac{3}{2\sqrt{25 + \frac{9}{2}}}$ $\therefore \theta = 73{,}97°$	✓ substitution ✓ x-value ✓ substitution ✓ answer (4) **[12]**

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QUESTION/*VRAAG 8*

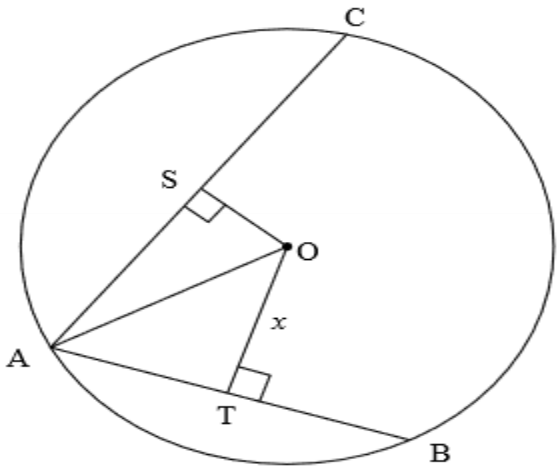
8.1



8.1.1	Alternate angles / *verwiss hoeke*, PQ ∥ SR		✓ R (1)
8.1.2(a)	$\hat{T}_2 = 70°$ $\therefore \hat{Q}_1 = 180° - 2(70°)$ $= 40°$	[∠s opp = sides/*∠e teenoor = sye*] [∠s/*e* Δ = 180°]	✓ S ✓R ✓ answer (3)
8.1.2(b)	$\hat{P}_1 = 40°$	[tangent chord th/*raakl-koordst*]	✓ S ✓R (2)

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8.2



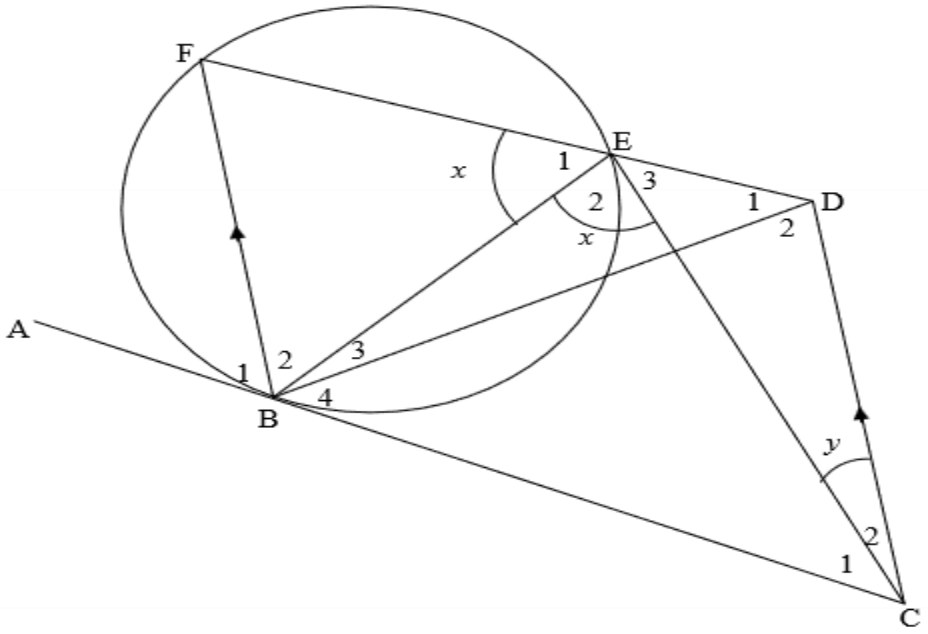
8.2.1	AT = 20 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	✓S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ **OR/OF** Let OS = 7, then OT = 15 In ΔAOT: $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS: $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore OA = 25$ **OR/OF**	✓ equating ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ OT ✓ radius (5) ✓✓ testing in ΔAOT ✓✓ testing in ΔAOS ✓conclusion (5)

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	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS]	✓ equating
	$OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT]	
	Let $OT = 15x$. Then $OS = 7x$	✓ AS = 24
	But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	✓ substitution
	$(15x)^2 + 400 = (7x)^2 + 576$	
	$225x^2 + 400 = 49x^2 + 576$	
	$176x^2 = 176$	✓ $x = 1$
	$x = 1$	✓ radius
	$\therefore AO = \sqrt{225 + 400}$	(5)
	$= 25$	
	OR/OF	✓ AS = 24
	AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	
	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS]	✓ substitution
	$= \left(\frac{7}{15}OT\right)^2 + AS^2$	$OS = \frac{7}{15}OT$
		✓ equating
	$AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2$ [Pyth : ΔAOT]	✓ subst Pyth
	$\frac{176}{225}AO^2 = \frac{4400}{9}$	
	$AO^2 = 625$	✓ radius
	$AO = 25$	(5) **[12]**

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QUESTION/*VRAAG 9*

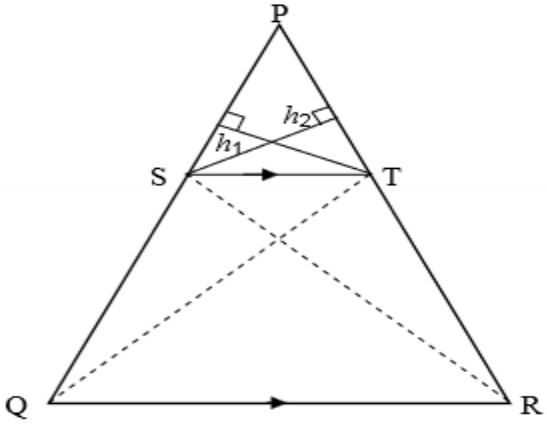


9.1.1	tangent chord theorem/*raaklyn-koordstelling*	✓ R (1)
9.1.2	corresponding/*ooreenkomstige* ∠s/*e*; FB || DC	✓ R (1)
9.2	$\hat{E}_1 = B\hat{C}D$ ∴ BCDE = cyclic quad [converse ext ∠ cyc quad/*omgek: buite∠kdvh*]	✓ S ✓ R (2)
9.3	$\hat{D}_2 = \hat{E}_2$ [∠*s* in the same segment/∠*e in dies segment*] $\hat{D}_2 = F\hat{B}D$ [alt ∠*s*, BF||CD/*verwiss* ∠*e*,*BF*||*CD*]	✓ S ✓ S (2)
9.4	$\hat{B}_3 = y$ *OR* $\hat{B}_3 = \hat{C}_2$ [∠s in the same segment/∠*e in dies segment*] $\hat{B}_2 = x - y$ *OR* $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] ∴ $\hat{B}_2 = \hat{C}_1$ **OR/*OF*** In ΔBFE and ΔBEC $\hat{E}_1 = \hat{E}_2$ [= *x*] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] ∴ ΔBFE///ΔCBE [∠,∠,∠] ∴ $\hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S (3) ✓ identifying Δ's ✓ S ✓ S (3) **[9]**

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QUESTION/*VRAAG* 10

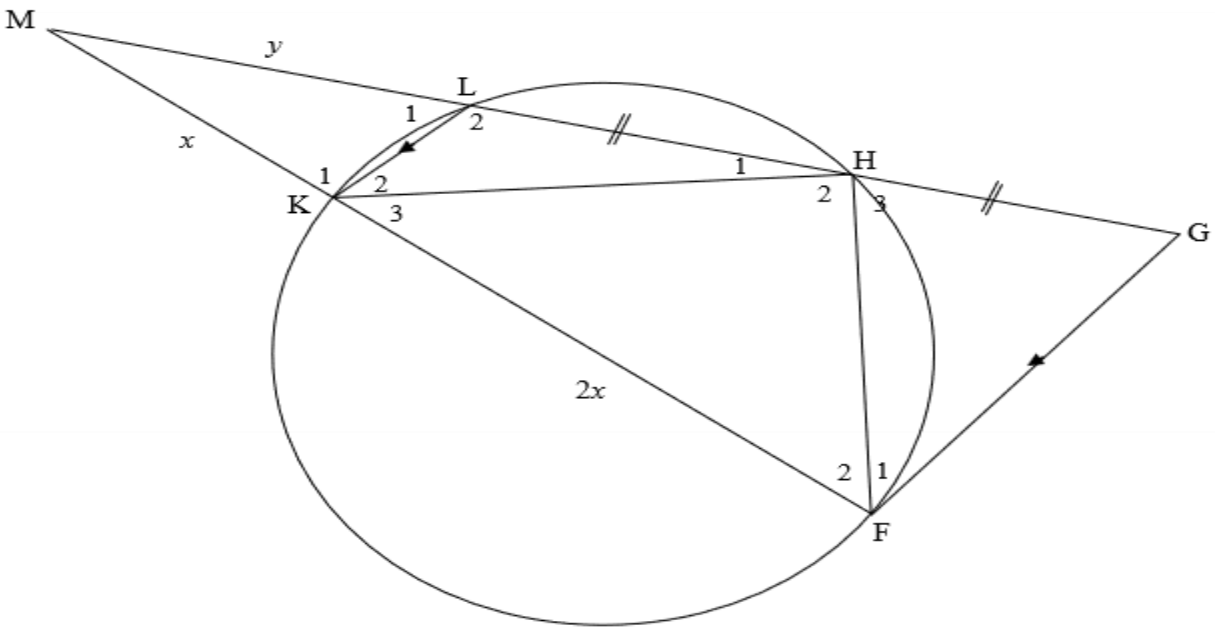
10.1



10.1	Constr : Join S to R and T to Q and draw h_1 from S ⊥ PT and h_2 from T ⊥ PS/ *Verbind SR en TQ en trek* h_1 *van S ⊥ PT en* h_2 *van T ⊥ PS*]	✓ constr/*konstruksie*
	Proof :	
	$\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2} = \frac{PS}{SQ}$ equal altitudes	✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta QST}$ $= \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2}$
	$\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{\frac{1}{2}PT \times h_1}{\frac{1}{2}TR \times h_1} = \frac{PT}{TR}$ equal altitudes	✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{PT}{TR}$
	area ΔPST = area ΔPST [common]	
	But area ΔQST = area ΔSTR [same base, height; ST ‖ QR]	
	$\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR}$	✓ S ✓ R
	$\therefore \frac{PS}{SQ} = \frac{PT}{TR}$	✓ S
		(6)

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

10.2



10.2.1	Corresponding/*Ooreenkomstige* ∠s/*e*; GF ∥ LK	✓ R (1)
10.2.2(a)	$\frac{GL}{LM}=\frac{FK}{KM}$ OR $\frac{GL}{y}=\frac{2x}{x}$ [prop theorem/*eweredighst*; GF ∥ LK] $\frac{2GH}{y}=\frac{2x}{x}$ [LH = HG] $\therefore GH = y$	✓S ✓R ✓ GL = 2GH (3)

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10.2.2(b)	$\hat{K}_1 = G\hat{F}M$	[corresponding/*ooreenkomst* ∠ s; GF || LK]	✓S ✓ R
	$L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$	[ext ∠ cyclic quad/*buite*∠*koordevh*]	✓ S
	$M\hat{H}F = G\hat{F}M$		
	In ΔMFH and ΔMGF:		✓ S
	$\hat{M} = \hat{M}$	[common/*gemeen*]	
	$M\hat{H}F = G\hat{F}M$	[proven/*bewys*]	
	∴ ΔMFH ||| ΔMGF	[∠∠∠]	✓ R (5)
	OR/*OR*		
	$\hat{K}_1 = G\hat{F}M$	[corresponding/*ooreenkomst* ∠ s; GF || LK]	
	$L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$	[ext ∠ cyclic quad/*buite*∠*koordevh*]	✓S ✓ R
	$M\hat{H}F = G\hat{F}M$		✓ S
	In ΔMFH and ΔMGF:		
	$\hat{M} = \hat{M}$	[common/*gemeen*]	✓ S
	$M\hat{H}F = G\hat{F}M$	[proven/*bewys*]	
	$\hat{F}_2 = \hat{G}$	[∠s of Δ = 180°]	✓ S (5)
	∴ ΔMFH ||| ΔMGF		
10.2.2(c)	$\therefore \frac{GF}{FH} = \frac{MF}{MH}$	[||| Δs]	✓S ✓R
	$= \frac{3x}{2y}$		(2)
10.2.3	$\frac{MF}{MH} = \frac{MG}{MF}$	[||| Δs]	✓ S
	$\frac{3x}{2y} = \frac{3y}{3x}$	[from 10.2.2(*c*)]	✓substitution
	$\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$		✓ simplification
	$\frac{y}{x} = \sqrt{\frac{3}{2}}$		(3) **[20]**
		TOTAL MARKS	**150**

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basic education

Department:
Basic Education
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GRADE 12

MATHEMATICS P2

NOVEMBER 2017

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 28 pages.

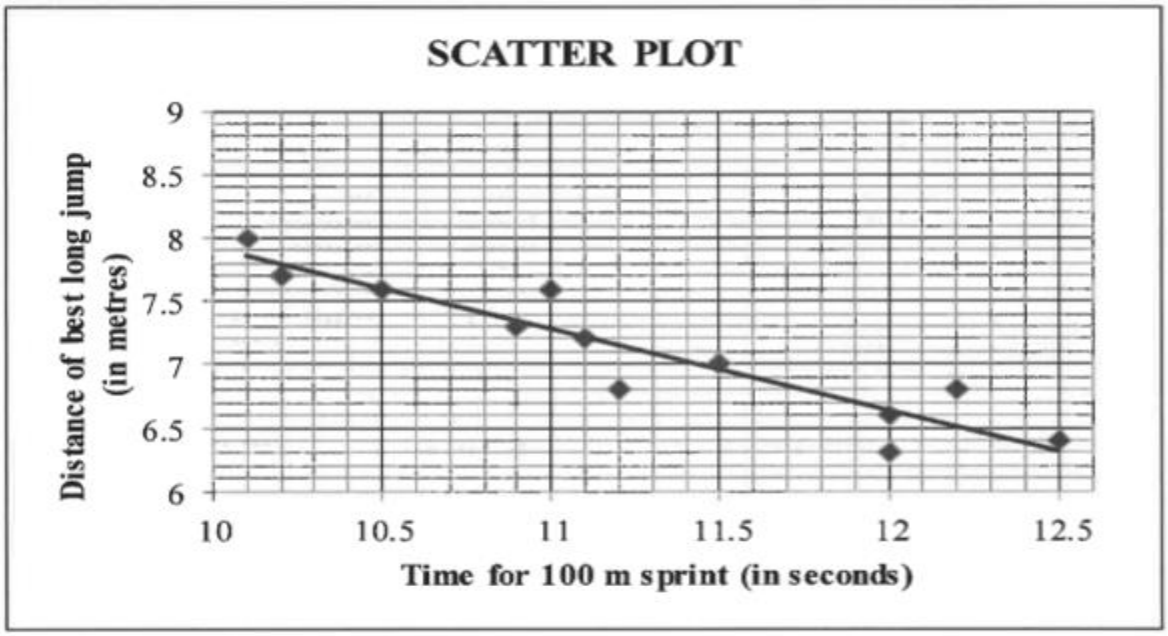
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QUESTION 1

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100 metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

1.1 Determine the values of a and b. (3)

1.2 An athlete runs the 100 metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete. (2)

1.3 Another athlete completes the 100 metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2)
[7]

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QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

2.2 The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

2.3 Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

2.4 The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

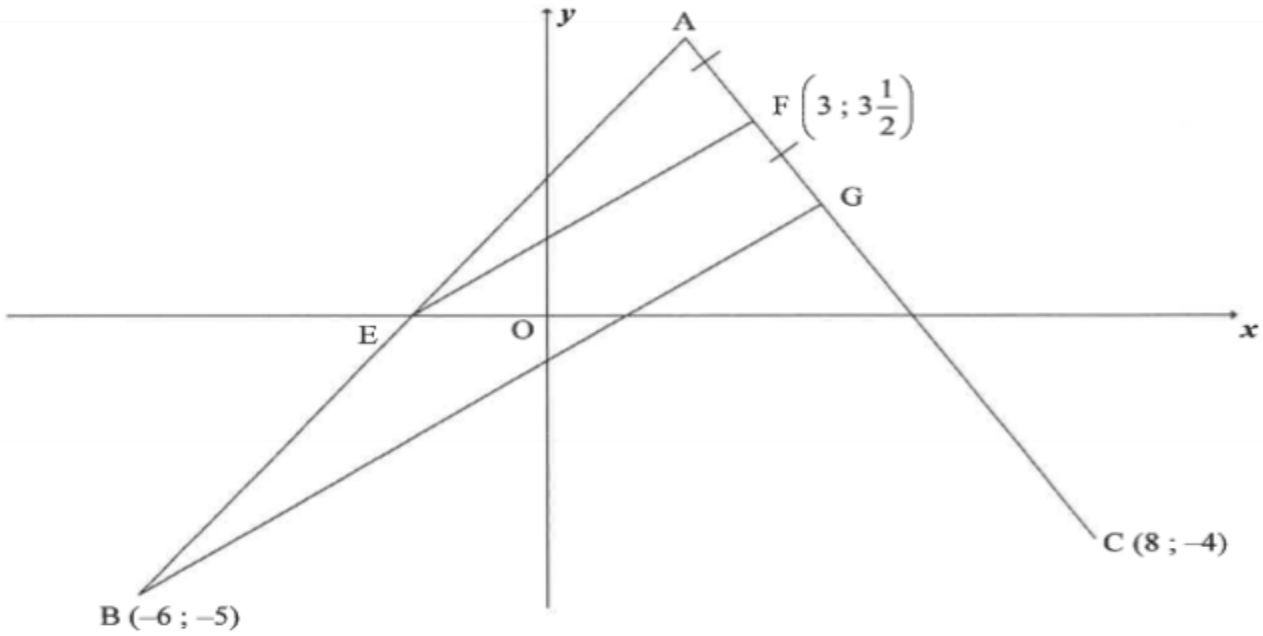
2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2)
[13]

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QUESTION 3

In the diagram, A, B(–6 ; –5) and C(8 ; –4) are points in the Cartesian plane. $F\left(3; 3\frac{1}{2}\right)$ and G are points on line AC such that AF = FG. E is the *x*-intercept of AB.



3.1 Calculate:

3.1.1 The equation of AC in the form $y = mx + c$ (4)

3.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$ (3)

3.2 Show by calculation that the coordinates of A is (2 ; 5). (2)

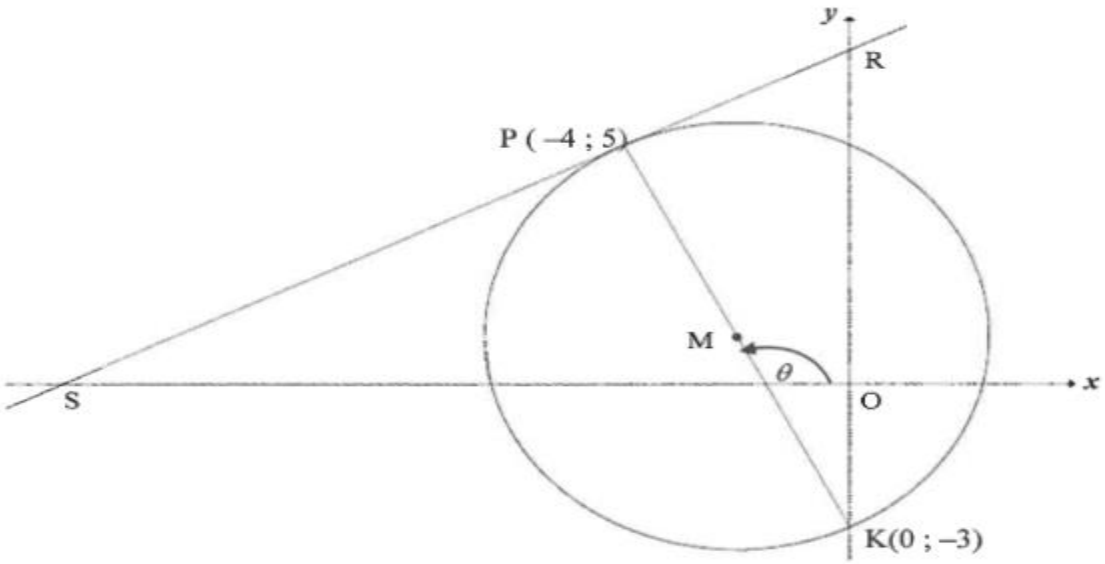
3.3 Prove that EF || BG. (4)

3.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)
[17]

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QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1 The gradient of SR (4)

4.1.2 The equation of SR in the form $y = mx + c$ (2)

4.1.3 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

4.1.4 The size of $P\hat{K}R$ (3)

4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)

4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

4.3 Calculate the area of ΔSMK. (5)
[23]

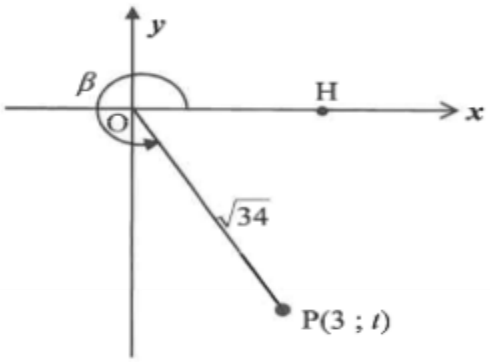
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QUESTION 5

5.1 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, P(3 ; t) is a point in the Cartesian plane. OP $=\sqrt{34}$ and $\hat{HOP}=\beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan\beta$ (1)

5.2.3 $\cos 2\beta$ (4)

5.3 Prove:

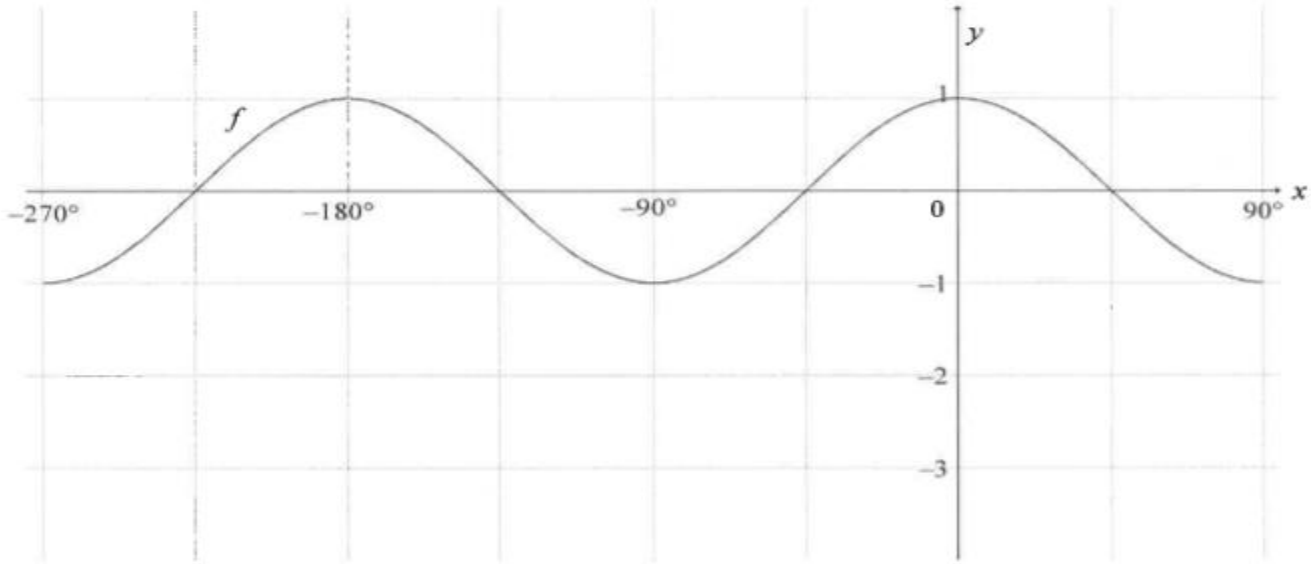
5.3.1 $\sin(A+B)-\sin(A-B)=2\cos A.\sin B$ (2)

5.3.2 **Without using a calculator,** that $\sin 77° - \sin 43° = \sin 17°$ (4)
[19]

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QUESTION 6

In the diagram, the graph of $f(x)=\cos 2x$ is drawn for the interval $x\in[-270°;90°]$.



6.1 Draw the graph of $g(x)=2\sin x-1$ for the interval $x\in[-270°;90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

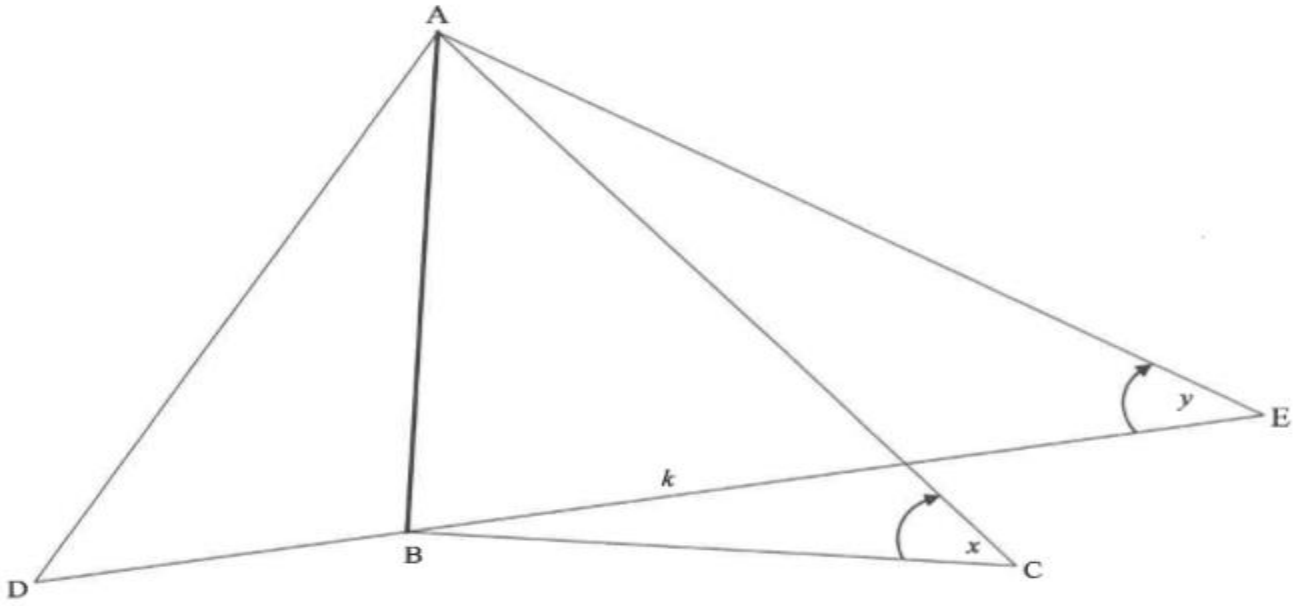
6.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x=\frac{-1+\sqrt{5}}{2}$. (4)

6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x\in[-270°;90°]$. (4)
[12]

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QUESTION 7

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



7.1 Write down the size of $A\hat{B}C$. (1)

7.2 Show that $AC = \dfrac{k.\tan y}{\sin x}$ (4)

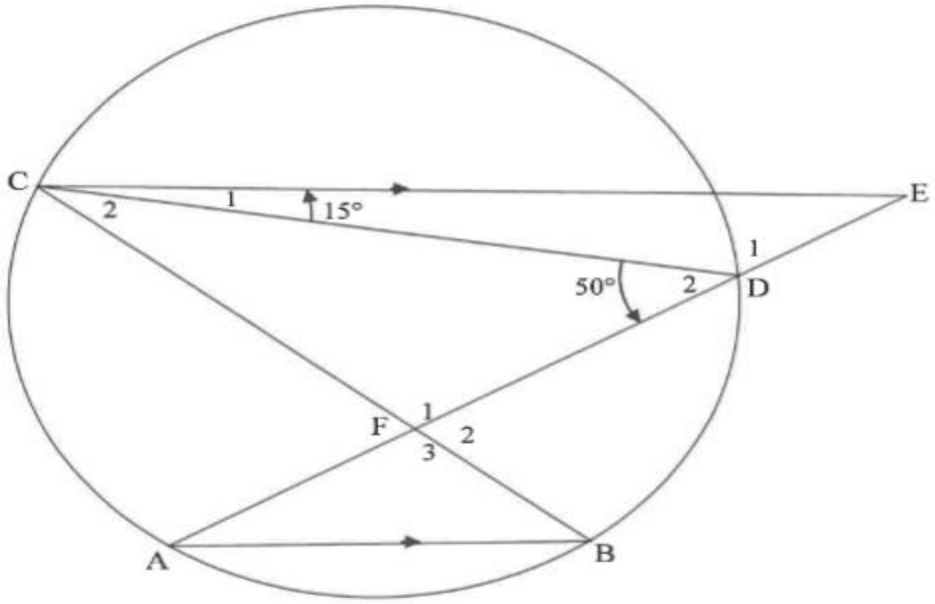
7.3 If it is further given that $D\hat{A}C = 2x$ and AD = AC, show that the distance DC between the players at D and C is $2k\tan y$. (5)
[10]

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Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



8.1 Calculate, with reasons, the size of:

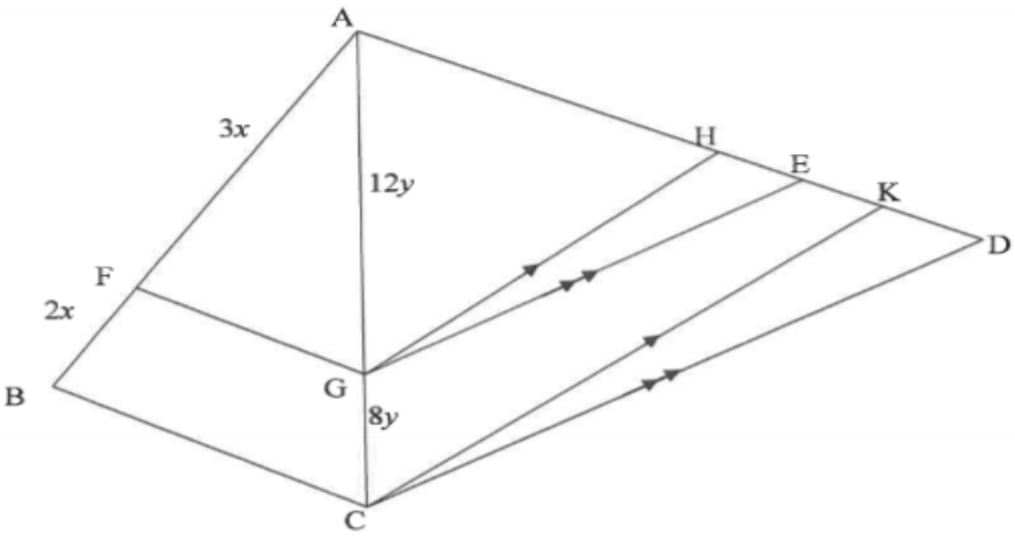
8.1.1 $\hat{A}$ (3)

8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)
[7]

QUESTION 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



9.1 Prove that:

9.1.1 FG || BC (2)

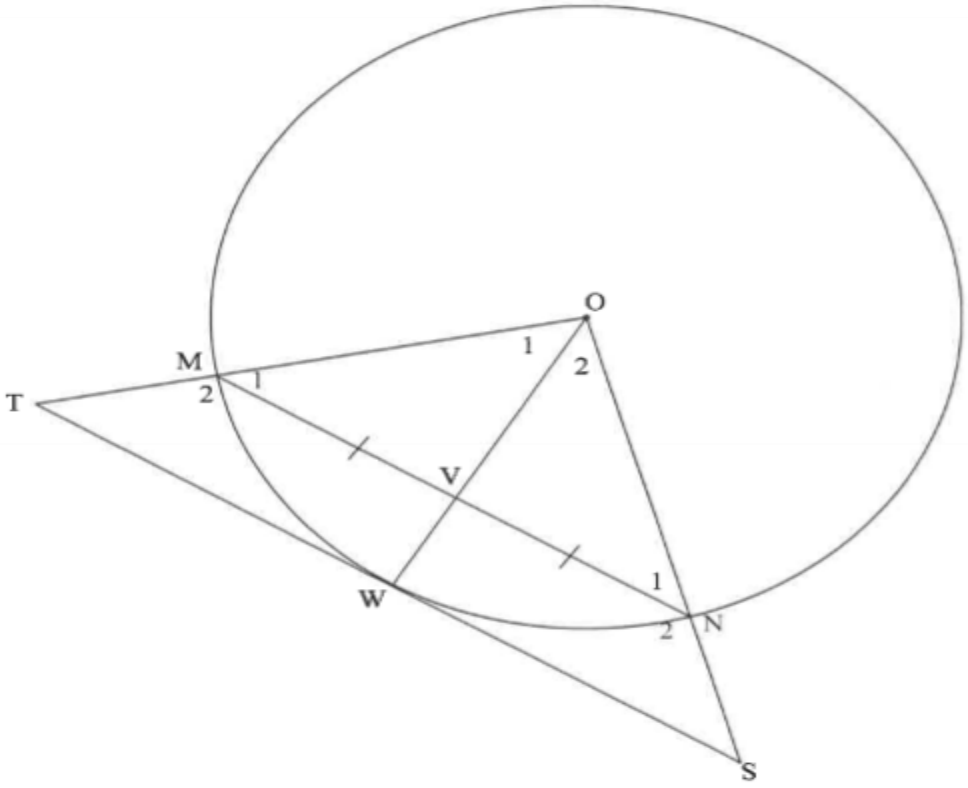
9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)
[10]

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QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why OV ⊥ MN. (1)

10.2 Prove that:

10.2.1 MN ∥ TS (2)

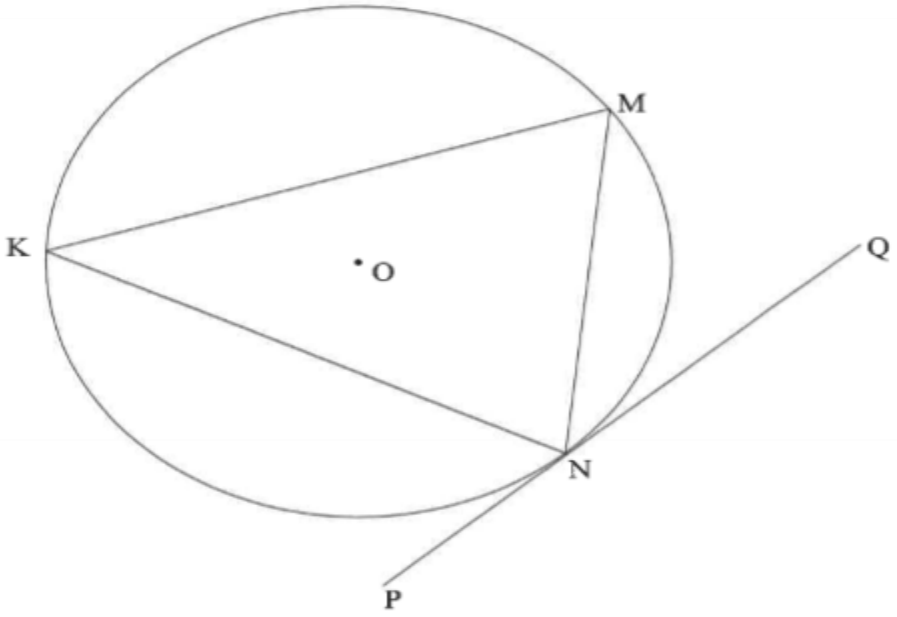
10.2.2 TMNS is a cyclic quadrilateral (4)

10.2.3 OS . MN = 2ON . WS (5)
[12]

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QUESTION 11

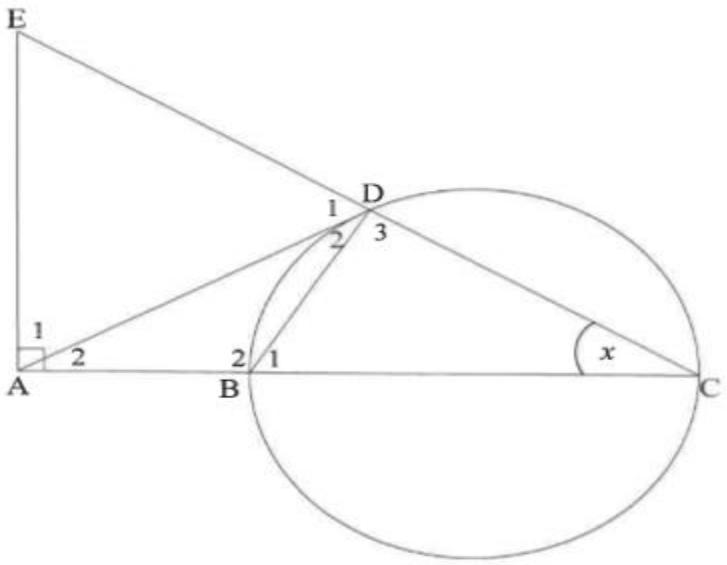
11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



Prove the theorem which states that $M\hat{N}Q = \hat{K}$. (5)

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11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA ⊥ AC. BD is drawn.
Let $\hat{C} = x$.



11.2.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

11.2.2 Prove that:

(a) AD = AE (3)

(b) ΔADB ||| ΔACD (3)

11.2.3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$ (2)

(b) Hence, prove that ΔADE is equilateral. (4)
[20]

TOTAL: 150

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GRADE 12/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE:* 150

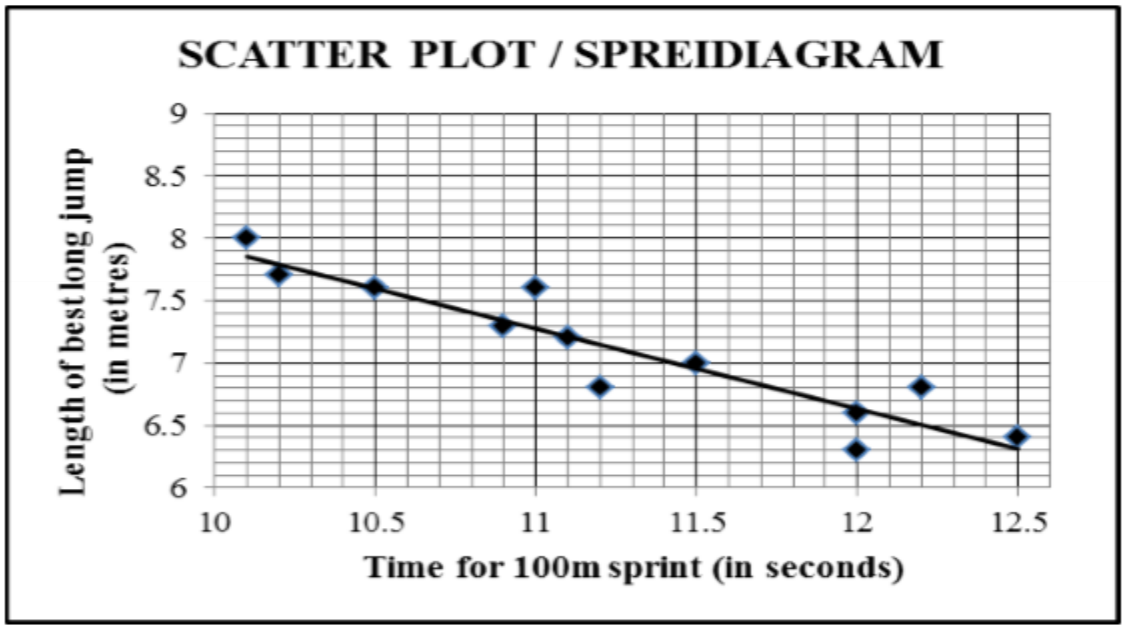
These marking guidelines consist of 29 pages.
Hierdie nasienriglyne bestaan uit 28 bladsye.

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Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 1

Time for 100 m sprint (in seconds) ***Tyd vir 100 m-naelloop (in sekondes)***	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) ***Afstand van beste sprong in verspring (in meter)***	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4



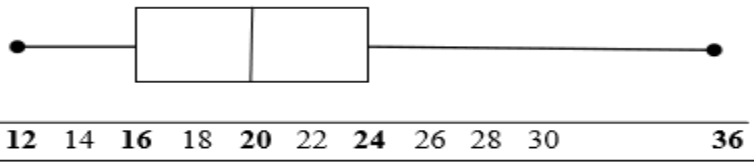
1.1	$a = 14{,}343\ldots = 14{,}34$ $b = -0{,}642\ldots = -0{,}64$	✓✓ value of a ✓ value of b (3)
1.2	$y = 14{,}34 - 0{,}64(11{,}7)$ $= 6{,}85$ **OR/OF** $y = 6{,}83$ (calculator / *sakrekenaar*)	✓ substitution correctly ✓ answer (2) ✓✓answer (2)
1.3	The gradient increases / *Die gradient neem toe* The point (12,3 ; 7,6) lies some distance above the current data. */Die punt (12,3 ; 7,6) lê bokant die huidige data.*	✓ increases/*neem toe* ✓ reasoning in words/ *redenasie in woorde* (2) **[7]**

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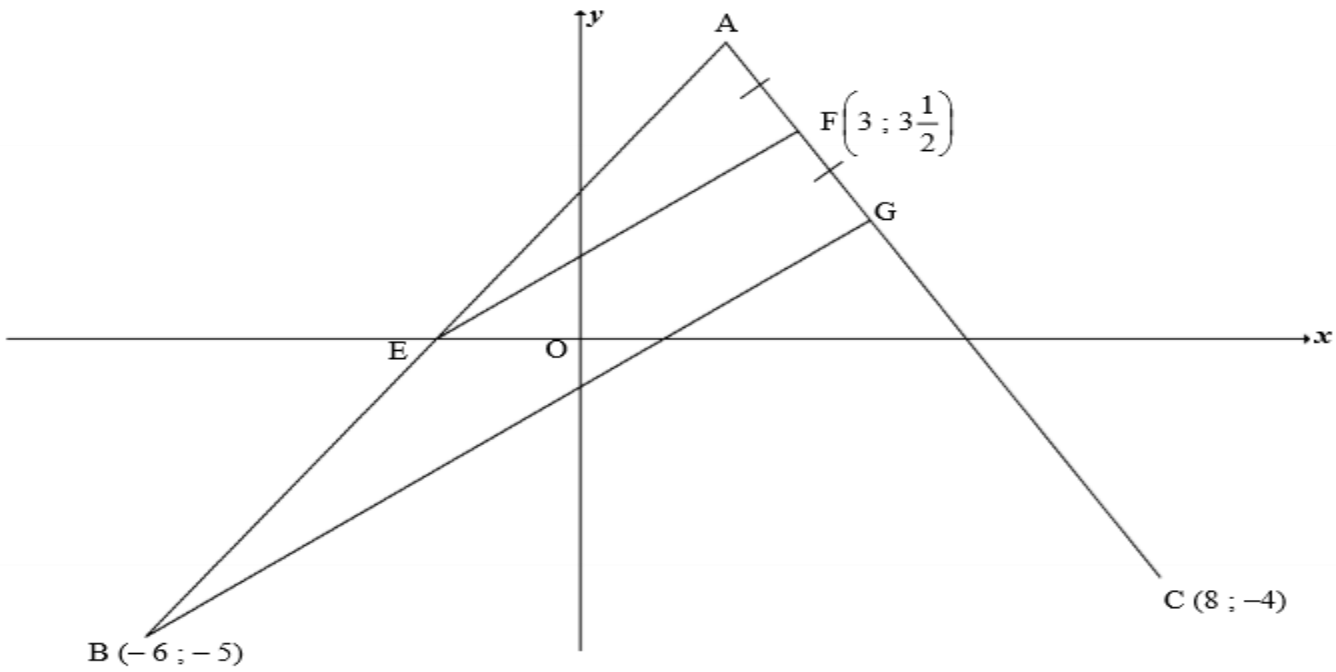
KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

QUESTION/*VRAAG* 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20{,}52$ seconds / *sekonde*	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ IQR/*IKO* $= Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20{,}52 + 5{,}94 = 26{,}46$ $\therefore > 26{,}46$ $\therefore$ 4 girls/*dogters*	✓ 26,46 ✓ answer (2)
2.3	 **12** 14 **16** 18 **20** 22 **24** 26 28 30 **36**	✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / *Meisies*	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) **None** of the boys / ***Nie een*** *van die seuns* ***nie*** 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. *5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.*	✓ answer ✓ reason/*rede* (2) **[13]**

QUESTION/*VRAAG* 3



3.1.1	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3\frac{1}{2} - (-4)}{3 - 8}$ $= -\frac{3}{2}$ $y = mx + c$ $\qquad$ $y - y_1 = m(x - x_1)$ $y = -\frac{3}{2}x + c$ $-4 = -\frac{3}{2}(8) + c$ **OR/*OF*** $(y - (-4)) = -\frac{3}{2}(x - 8)$ $c = 8$ $\qquad$ $y + 4 = -\frac{3}{2}x + 12$ $y = -\frac{3}{2}x + 8$ $\qquad$ $y = -\frac{3}{2}x + 8$ **OR/*OF***	✓ substitution of (8 ; –4) & $\left(3\ ;\ 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and (8 ; –4) ✓ equation of AC (4)

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	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - \left(3\frac{1}{2}\right)}{8 - 3}$ $= -\frac{3}{2}$ $y = mx + c$ $3\frac{1}{2} = -\frac{3}{2}(3) + c$ $c = 8$ $y = -\frac{3}{2}x + 8$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}(x - 3)$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}x + \frac{9}{2}$ $y = -\frac{3}{2}x + 8$	✓ substitution of (8 ; –4) & $\left(3 ; 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and $\left(3 ; 3\frac{1}{2}\right)$ ✓ equation of AC (4)
3.1.2	AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$ $15x + 10y = 80$ $\underline{7x - 10y = 8}$ $22x = 88$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2) **OR/*OF*** BG: $7x - 10y = 8 \quad \therefore y = \frac{7}{10}x - \frac{8}{10}$ $\therefore \frac{7}{10}x - \frac{8}{10} = -\frac{3}{2}x + 8$ [CA from 3.1.1] $\frac{11}{5}x = \frac{44}{5}$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2)	✓ method /*metode*: solving simultaneously / *los gelyktydig op* ✓ x coordinate ($x > 0$) ✓ y coordinate (3) ✓ method: equating *metode: stel vgls gelyk* ✓ x coordinate ($x > 0$) ✓ y coordinate (3)
3.2	$\frac{x_A + 4}{2} = 3$ and $\frac{y_A + 2}{2} = 3\frac{1}{2}$ $\therefore$ A(2 ; 5) **OR/*OF*** by translation/*deur translasie*: $x_A = 3 - (4 - 3) = 2$ $y_A = 3\frac{1}{2} + (3\frac{1}{2} - 2) = 5$ $\therefore$ A(2 ; 5)	✓ equation ito x ✓ equation ito y (2) ✓ equation ito x ✓ equation ito y (2)

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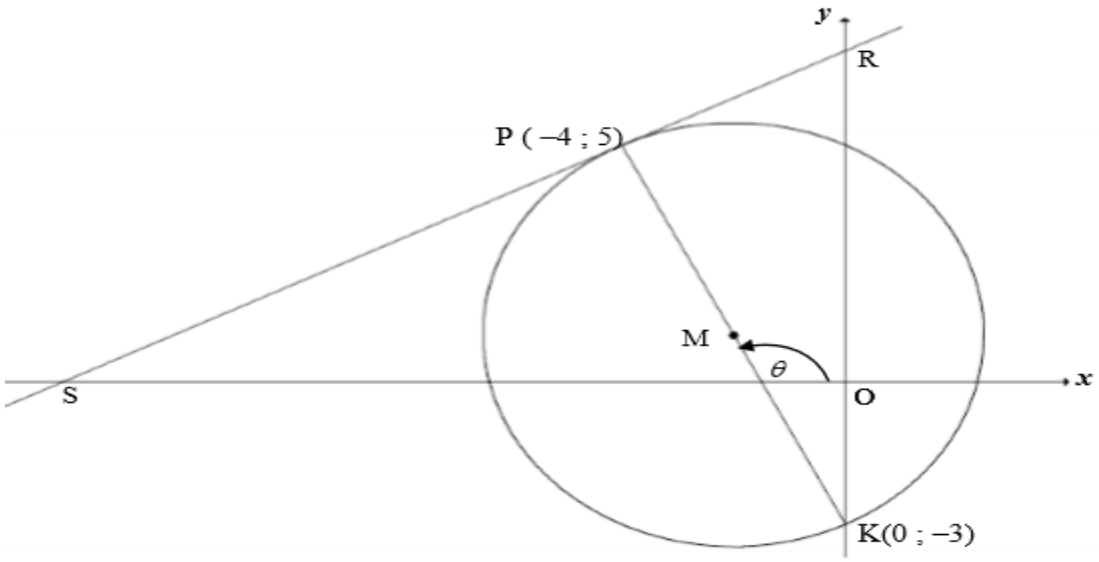
KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

3.3	The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ **But the *y*-coordinate of E is 0** **∴ E(–2 ; 0) is the midpoint of AB** ∴ EF ‖ BG [midpoint theorem/*middelpuntst* ***OR/OF*** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*]	✓ subst A & B into midpt formula ✓ *y* coordinate = 0 ✓ E = midpt ✓ Reason (4)
	OR/*OF* The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ $AE=\sqrt{(-2-2)^2+(0-5)^2}=\sqrt{41}$ $EB=\sqrt{(-2-(-6))^2+(0-(-5))^2}=\sqrt{41}$ ∴ In ΔABG: **AE = EB** and AF = FG ∴ EF ‖ BG [midpoint theorem/*middelpuntst*]	✓ subst A & B into midpt formula ✓ lengths of AE & EB ✓ AE = EB or E = midpt ✓ Reason (4)
	OR/*OF* Equation of AB: $y-(-5)=\left(\frac{5-(-5)}{2-(-6)}\right)(x-(-6))$ $y+5=\frac{10}{8}x+\frac{15}{2}\qquad\therefore y=\frac{5}{4}x+\frac{5}{2}$ x-intercept of AB: $0=\frac{5}{4}x+\frac{5}{2}\quad\therefore x=-2$ ∴ E(–2 ; 0) $m_{EF}=\frac{3\frac{1}{2}-0}{3-(-2)}=\frac{7}{10}$ $m_{EF}=m_{BG}=\frac{7}{10}$ ∴ EF ‖ BG BG: $7x-10y=8$ $\therefore\ y=\frac{7}{10}x-\frac{8}{10}$ $\therefore\ m_{BG}=\frac{7}{10}$	✓ equation of AB ✓ coordinates of E ✓ gradient of EF ✓ gradient EF = gradient BG (4)

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3.4	Midpoint of AC $= \left(5 ; \frac{1}{2}\right)$ $\frac{x_D + (-6)}{2} = 5$ and $\frac{y_D + (-5)}{2} = \frac{1}{2}$ $\therefore$ D(16 ; 6)	✓✓ $\left(5 ; \frac{1}{2}\right)$ ✓ x value ✓ y value (4)
	OR/*OF* by translation/*dmv translasie*: D(16 ; 6)	✓ method finding x ✓ method finding y ✓ x value ✓ y value (4)
	OR/*OF* $m_{BC} = \frac{-5-(-4)}{-6-8} = \frac{1}{14}$ and $m_{AB} = \frac{5-(-5)}{2-(-6)} = \frac{5}{4}$ AD: $y - 5 = \frac{1}{14}(x-2) \Rightarrow y = \frac{1}{14}x + \frac{34}{7}$ CD: $y + 4 = \frac{5}{4}(x-8) \Rightarrow y = \frac{5}{4}x - 14$ $\frac{5}{4}x - 14 = \frac{1}{14}x + \frac{34}{7}$ $\therefore \quad x = 16$ $y = 6$	✓✓ equating ✓ x value ✓ y value (4) **[17]**

QUESTION/*VRAAG* 4

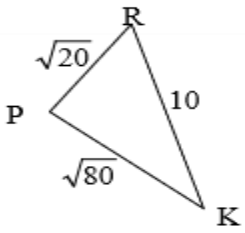


4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK ⊥ SR [radius ⊥ tangent/*raaklyn*] $\therefore m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK ⊥ SR **OR** r ⊥ tangent ✓ answer (4)
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of m and P ✓ equation (2)

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4.1.3	$M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore M(-2;1)$ $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right) \therefore M(-2;1)$ $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right) \therefore M(-2;1)$ $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2=20$ ✓ equation (4) ✓✓ M $(-2;1)$ $r^2=20$ ✓ equation (4) ✓✓ M $(-2;1)$ $r^2=20$ ✓ equation (4)

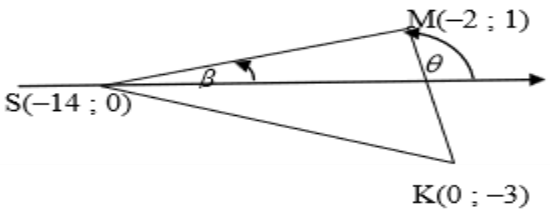
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4.1.4	$\tan\theta = m_{PK} = -2$ $\therefore\ \theta = 180° - 63{,}43°$ $= 116{,}57°$ $P\hat{K}R = 116{,}57° - 90°$ [ext ∠of ΔMOK] $= 26{,}57°$	✓ $\tan\theta = -2$ ✓ size of θ ✓ answer (3)
	OR/*OF*	
		
	In ΔRPK: $PK = \sqrt{(0-(-4))^2 + (-3-5)^2} = \sqrt{80}$ $PR = \sqrt{(-4-0)^2 + (5-7)^2} = \sqrt{20}$ $RK = 10$ $\cos P\hat{K}R = \frac{PK^2 + KR^2 - PR^2}{2.PK.KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $P\hat{K}R = 26{,}57°$	✓ lengths of PK, PR & RK ✓ correct values into cos rule ✓ answer (3)
	OR/*OF*	
	$\sin P\hat{K}R = \frac{\sqrt{20}}{10}$ **OR/*OF*** $\cos P\hat{K}R = \frac{\sqrt{80}}{10}$ $P\hat{K}R = 26{,}57°$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)
	OR/*OF*	
	$\tan P\hat{K}R = \frac{\sqrt{20}}{\sqrt{80}}$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)

4.1.5	RS ∥ tangent at K(0 ; −3) $\therefore m_{PS} = m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ **OR/*OF*** $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{tang} = -1$ [radius ⊥ tangent/*raaklyn*] $\therefore m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
4.2	$t \in (-3 ; 7)$ **OR/*OF*** $-3 < t < 7$	✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
4.3	RS: $y = \frac{1}{2}x + 7$ ∴ S(−14 ; 0) $SP = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ Area ΔSMK $= \frac{1}{2}$. MK . SP $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

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OR/OF



Let β = inclination of SM/ *inklinasie van SM*

RS: $y = \frac{1}{2}x + 7$ $\therefore$ S(–14 ; 0)

✓ coordinates of S

$SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$

✓ length of SM

$\tan\beta = \frac{1-0}{-2-(-14)} = \frac{1}{12}$ $\therefore$ $\beta = 4{,}76°$

$\therefore S\hat{M}K = 116{,}57° - 4{,}76°$ [ext $\angle$ of Δ]
$= 111{,}81°$

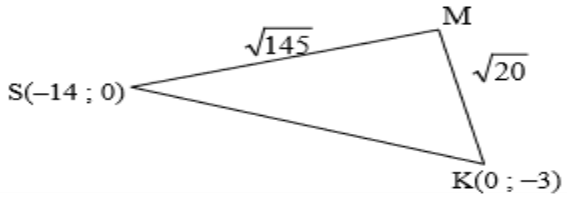
✓ size of/*grootte v* $S\hat{M}K$

$\text{Area } \Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$
$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$
$= 24{,}9985 = 25$ square units

✓ correct substitution into area rule
✓ answer

(5)

OR/*OF*



RS: $y = \frac{1}{2}x + 7$ $\therefore$ S(–14 ; 0)

✓ coordinates of S

$SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$

✓ length of SK

$\cos S\hat{M}K = \frac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\frac{2\sqrt{29}}{29}$

$S\hat{M}K = 111{,}80°$

✓ size of /*grootte v* $S\hat{M}K$

$\text{Area } \Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$
$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$
$= 24{,}9985 = 25$ square units

✓ correct substitution into area rule
✓ answer

(5)

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	OR/*OF*	
	Produce KS to T	
	RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(−14 ; 0)	✓ coordinates of S
	$SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$	✓ length of SK & SM
	$SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$	
	$m_{SK} = -\frac{3}{14} \Rightarrow T\hat{S}O = 167{,}91°$	
	$m_{SM} = \frac{1}{12} \Rightarrow M\hat{S}O = 4{,}76°$	
	$M\hat{S}K = 180° - 167{,}91° + 4{,}76° = 16{,}85°$	✓ size of /*grootte v* $M\hat{S}K$
	$\text{Area } \Delta SMK = \frac{1}{2}(SM)(SK).\sin M\hat{S}K$	
	$= \frac{1}{2}(\sqrt{145})(\sqrt{205}).\sin 16{,}85°$	✓ correct substitution into area rule
	$= 24{,}9985 = 25$ square units	✓ answer
		(5)

QUESTION/*VRAAG* 5

5.1	$\frac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$ $=\frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $=\frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$ $=\cos A$	✓ sin A ✓ –sin A ✓ sin A ✓ –tan A ✓ $\tan A=\frac{\sin A}{\cos A}$ ✓ answer (6)
5.2.1	$t^2=(\sqrt{34})^2-(3)^2$ $\therefore t=-5$	✓ substitution ✓ answer (2)
5.2.2	$\tan\beta=\frac{-5}{3}$	✓ correct ratio (1)
5.2.3	$\cos 2\beta = 2\cos^2\beta-1$ $=2\left(\frac{3}{\sqrt{34}}\right)^2-1$ $=2\left(\frac{9}{34}\right)-1$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta=1-2\sin^2\beta$ $=1-2\left(-\frac{5}{\sqrt{34}}\right)^2$ $=1-2\left(\frac{25}{34}\right)$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta=\cos^2\beta-\sin^2\beta$ $=\left(\frac{3}{\sqrt{34}}\right)^2-\left(-\frac{5}{\sqrt{34}}\right)^2$ $=\frac{9}{34}-\frac{25}{34}$ $=-\frac{16}{34}$ OR $-\frac{8}{17}$	✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4)

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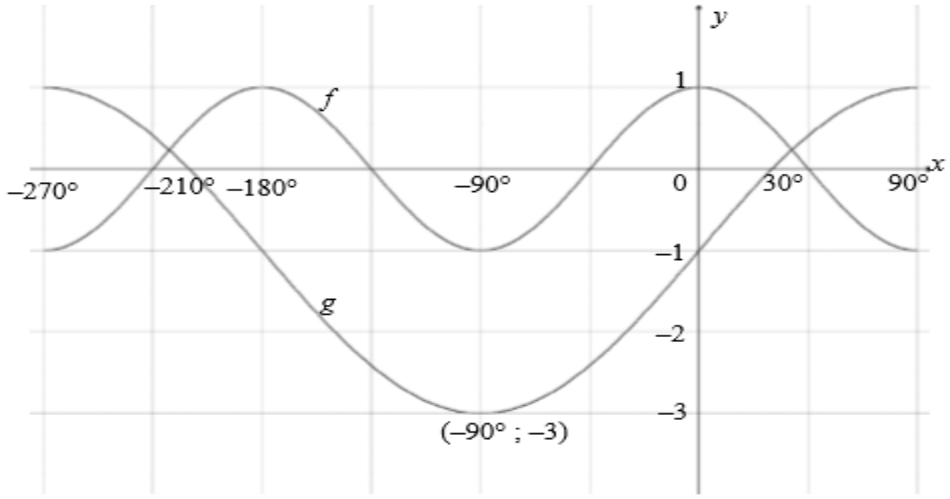
5.3.1	LHS = sin(A + B) – sin(A – B) = sin A.cos B + cos A.sin B – (sin A. cos B – cos A. sin B) = sin A.cos B + cos A.sin B – sin A. cos B + cos A. sin B = 2cos A.sin B = RHS	✓ compound formula ✓ compound formula (2)
5.3.2	$\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= 2\cos 60°.\sin 17°$ $= 2 \times \frac{1}{2} \times \sin 17°$ $= \sin 17°$ **OR/*OF*** $\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= (\sin 60° \cos 17° + \cos 60° \sin 17°) -$ $(\sin 60° \cos 17° - \cos 60° \sin 17°)$ $= \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17° - \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17°$ $= \sin 17°$	✓ 60° + 17° ✓ 60° – 17° ✓ simplify ✓ $\frac{1}{2}$ (4) ✓ 60° + 17° ✓ 60° – 17° ✓ expansion ✓ $\frac{1}{2}$ (4) **[19]**

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QUESTION/*VRAAG* 6

6.1



✓ (–90° ; –3)
✓ (0 ; –1)
✓ x – intercepts: –210° & 30°
✓ shape

(4)

6.2

$\cos 2x = 2\sin x - 1$
$1 - 2\sin^2 x = 2\sin x - 1$
$2\sin^2 x + 2\sin x - 2 = 0$
$\sin^2 x + \sin x - 1 = 0$
$\sin x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$= \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$
$\sin x = \dfrac{-1+\sqrt{5}}{2}$, since $\sin x = \dfrac{-1-\sqrt{5}}{2} < -1$ has no solution

✓ $\cos 2x = 1 - 2\sin^2 x$
✓ standard form
✓ using quadratic formula
✓ substitution into quadratic formula

(4)

6.3

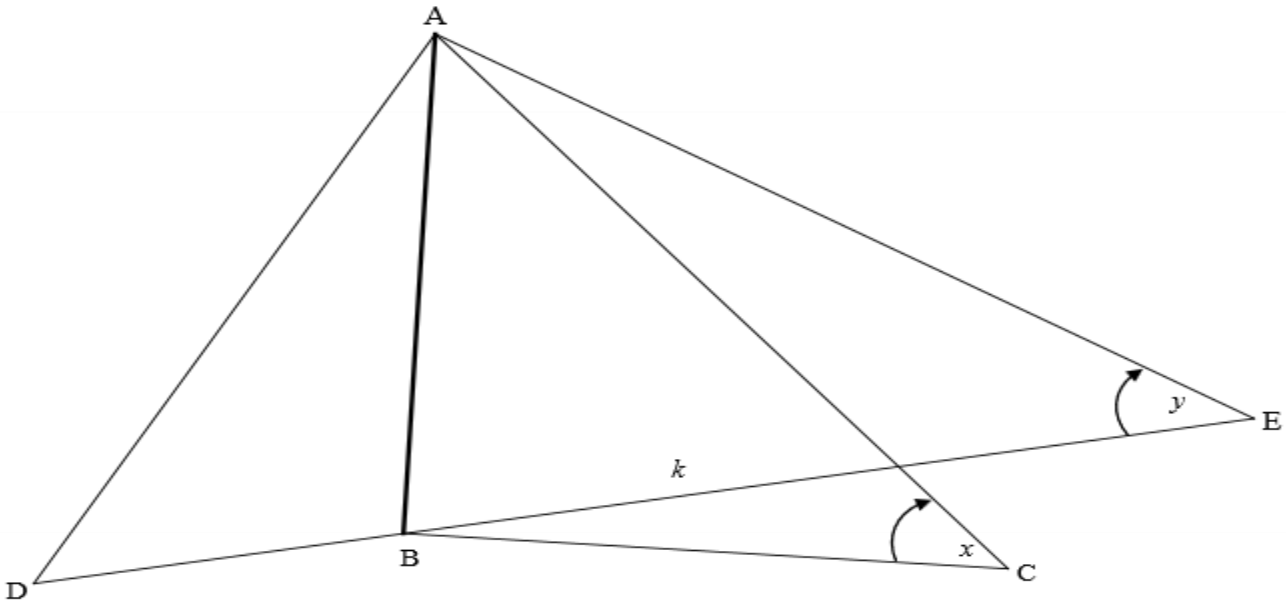
$\sin x = \dfrac{-1+\sqrt{5}}{2} = 0{,}618\ldots$
Reference $\angle = 38{,}17°$
$\therefore x = 38{,}17° + k.360°$ or $x = 141{,}83° + k.360°$; $k \in Z$
$\therefore x = 38{,}17°$ or $-218{,}17°$
$y = 0{,}24$
∴ Points of intersection/*snypunte*:
$(38{,}17° ; 0{,}24)$ and $(-218{,}17° ; 0{,}24)$

✓ 38,17°
✓ 141,83°
✓ –218,17°
✓ 0,24

(4)
[12]

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QUESTION/*VRAAG* 7



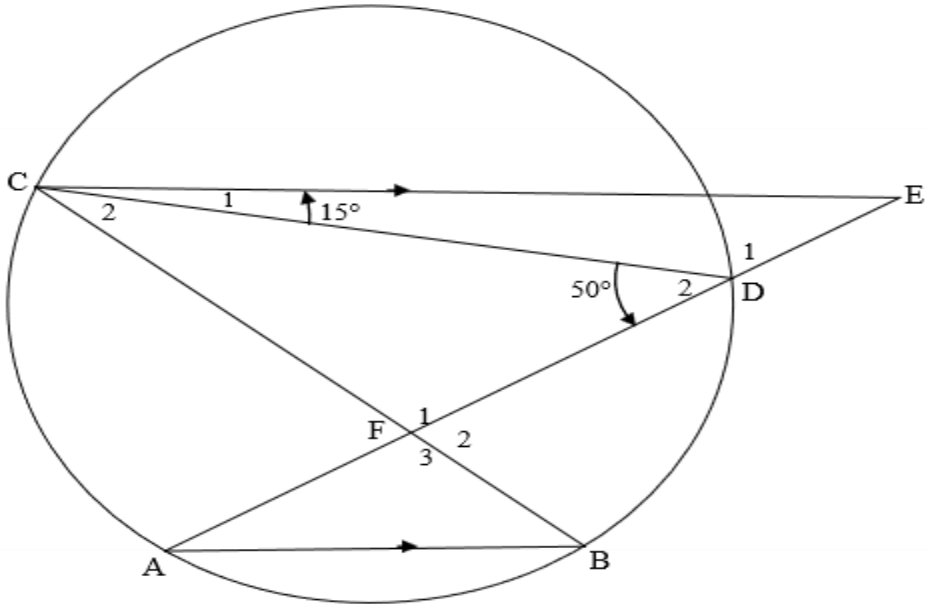
7.1	$A\hat{B}C = 90°$	✓ answer (1)
7.2	In Δ ABE: $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ In Δ ABC: $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	✓ correct ratio ✓ value AB ✓ correct ratio ✓ AC as subject and substitution (4)

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7.3	$\hat{ADC} = \hat{ACD} = \frac{180° - 2x}{2} = 90° - x$	✓ $90° - x$
	$\frac{DC}{\sin 2x} = \frac{AC}{\sin(90° - x)}$	✓subst into sine rule
	$\frac{DC}{2\sin x \cos x} = \frac{AC}{\cos x}$	✓ $2\sin x \cos x$
	$DC = \frac{AC(2\sin x \cos x)}{\cos x}$	✓ $\cos x$
	$= \frac{k\tan y}{\sin x} \cdot \frac{2\sin x \cos x}{\cos x}$	✓ substitution
	$= 2k\tan y$	(5)
	OR/*OF*	
	$DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$	
	$= AC^2 + AC^2 - 2AC^2\cos 2x$	✓substitution into cos rule
	$= 2AC^2(1 - \cos 2x)$	✓ factorisation
	$= 2AC^2(1 - 1 + \sin^2 x)$	✓ $1 - 2\sin^2 x$
	$= 4AC^2\sin^2 x$	✓ DC ito AC and $\sin x$
	$DC = 2AC.\sin x$	
	$= 2\left(\frac{k.\tan y}{\sin x}\right).\sin x$	✓ substitution
	$= 2k.\tan y$	(5)
	OR/*OF*	
	$DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$	✓correct cos rule
	$= 2\left(\frac{k\tan y}{\sin x}\right)^2 - 2\left(\frac{k\tan y}{\sin x}\right)^2 \cos 2x$	✓ substitution
	$= \frac{2k^2\tan^2 y}{\sin^2 x} - \frac{2k^2\tan^2 y}{\sin^2 x}(1 - 2\sin^2 x)$	✓ $1 - 2\sin^2 x$
	$= \frac{2k^2\tan^2 y}{\sin^2 x} - \frac{2k^2\tan^2 y}{\sin^2 x} + 4k^2\tan^2 y$	✓ squaring and multiplication
	$DC = \sqrt{4k^2\tan^2 y}$	✓ $\sqrt{4k^2\tan^2 y}$
	$= 2k\tan y$	(5) **[10]**

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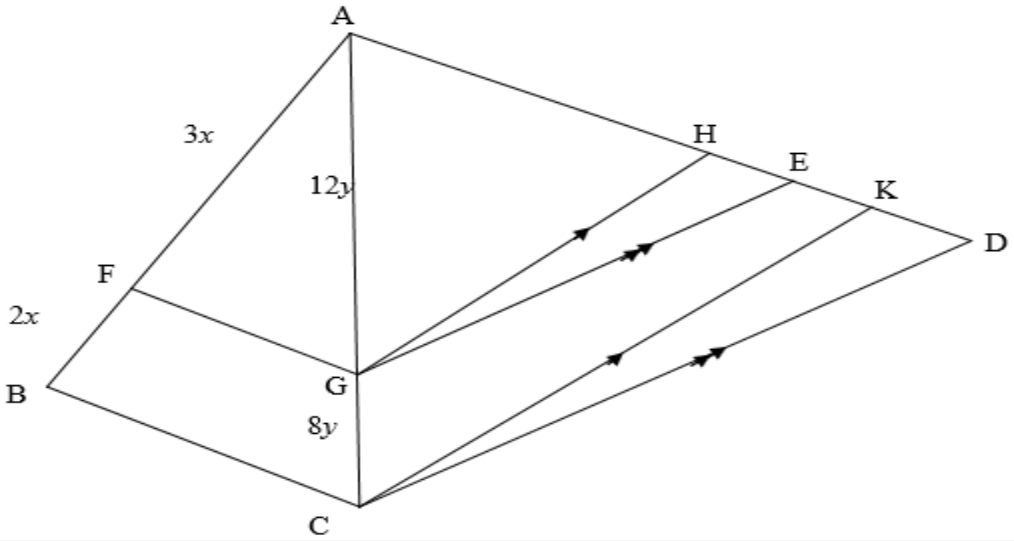
QUESTION/*VRAAG* 8



8.1.1	$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ/*buite ∠ van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] **OR/*OF*** $\hat{E} = 180° - (130° + 15°) = 35°$ [str line; ∠s of Δ/*rt lyn; ∠e van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] **OR/*OF*** $\hat{B} = 50°$ [∠s in same segment/*∠e in dieselfde segment*] $\hat{C}_2 + 15° = 50°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] $\therefore \hat{C}_2 = 35°$ $\hat{A} = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3)
8.1.2	$\hat{C}_2 = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓ R (2)
8.2	$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] ∴ CF is a tangent to the circle [converse tan chord theorem] ∴ *CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]*	✓ S ✓ R (2) **[7]**

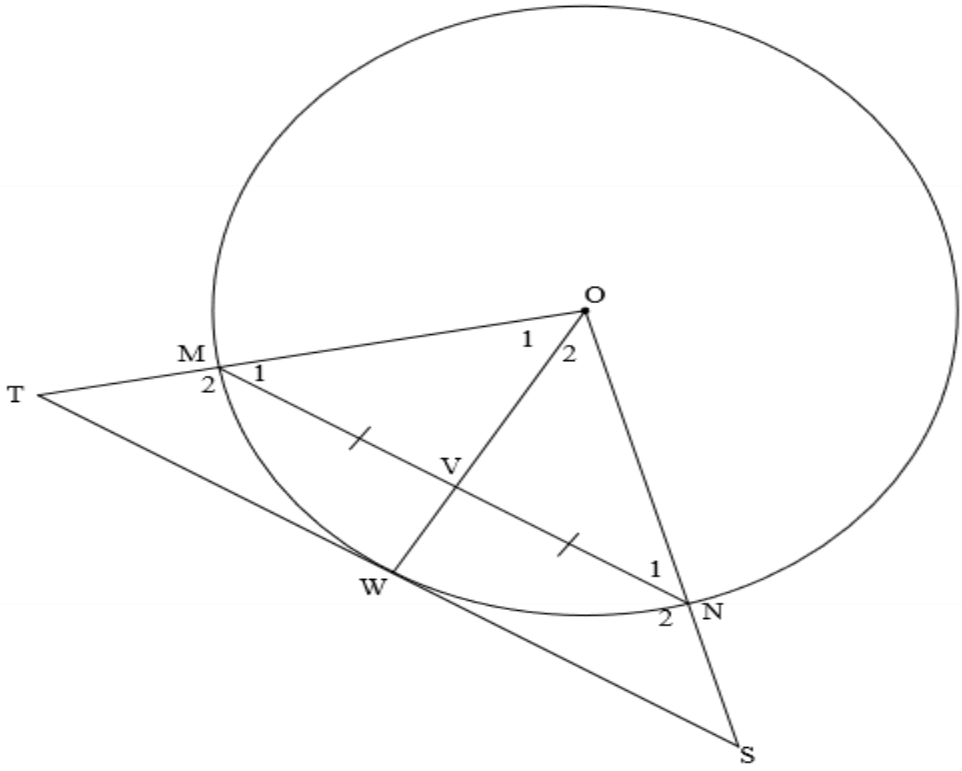
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QUESTION/*VRAAG* 9



9.1.1	$\frac{AF}{BF}=\frac{3x}{2x}=\frac{3}{2}$ & $\frac{AG}{CG}=\frac{12y}{8y}=\frac{3}{2}$ $\therefore \frac{AF}{BF}=\frac{AG}{CG}$ $\therefore$ FG ‖ BC [conv prop th/*omg ewveredigh st.* **OR** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye v Δ in dies verh*]	✓ $\frac{AF}{BF}=\frac{AG}{CG}$ ✓R (2)
9.1.2	$\frac{AG}{GC}=\frac{AH}{HK}$ [prop theorem/*eweredigh st*; GH ‖ CK **OR** line ‖ to 1 side of Δ/*lyn ‖ 1 sy van Δ*] $\frac{AG}{GC}=\frac{AE}{ED}$ [prop theorem/*eweredigh st*; GE ‖ CD] $\therefore \frac{AH}{HK}=\frac{AE}{ED}$	✓S ✓ R ✓ S (3)
9.2	$\frac{AE}{ED}=\frac{3}{2}$ and $\frac{AH}{HK}=\frac{3}{2}$ $\frac{AE}{12}=\frac{3}{2}$ and $\frac{15}{HK}=\frac{3}{2}$ $\therefore$ AE = 18 and HK = 10 $\therefore$ HE = AE – AH = 18 – 15 = 3 $\therefore$ EK = HK – HE = 10 – 3 = 7 **OR/*OF*** AD = 30 KD = AD – AH – HK = 30 – 15 – 10 = 5 EK = ED – KD = 12 – 5 = 7	✓ use of ratios ✓ AE = 18 ✓ HK = 10 ✓ HE = 3 **or** KD = 5 ✓ EK = 7 (5) **[10]**

QUESTION/*VRAAG* 10



10.1	Line from centre to midpoint of chord/ *lyn vanaf midpt na midpt van koord*	✓ R (1)
10.2.1	$O\hat{W}T = O\hat{W}S = 90°$ [radius ⊥ tangent/*raaklyn*] ∴ MN ∥ TS [corresp ∠s =/*ooreenkomstige ∠e* = **OR** co-int ∠s 180°/*ko-binne∠e 180°* **OR** alternate ∠s/*verwiss ∠e*]	✓ R ✓ R (2)
10.2.2	$\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/ *∠e teenoor = sye*] $\hat{M}_1 = \hat{T}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] ∴ $\hat{N}_1 = \hat{T}$ ∴ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*] **OR/*OF*** $\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/ *∠e teenoor = sye*] $\hat{N}_1 = \hat{S}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] ∴ $\hat{S} = \hat{M}_1$ ∴ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*]	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)

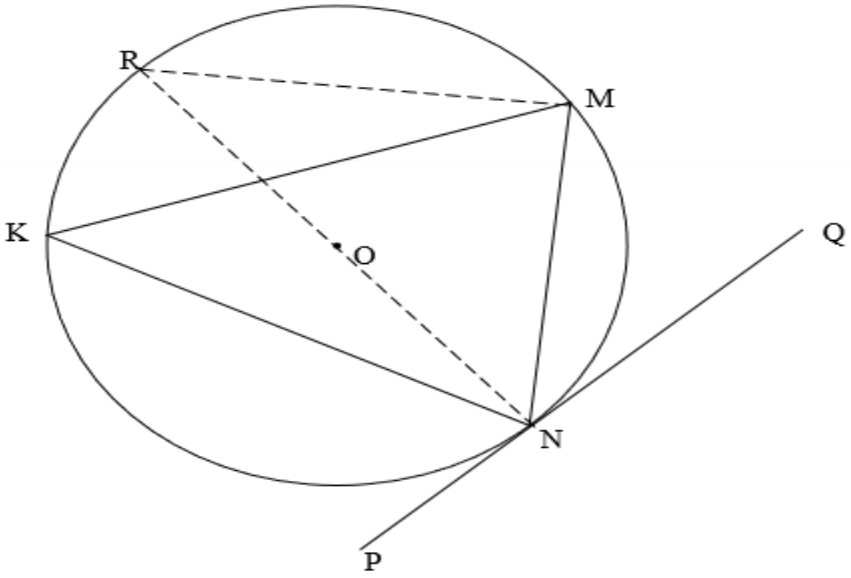
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10.2.3	In ΔOVN and ΔOWS $\hat{O}_2 = \hat{O}_2$ [common/*gemeenskaplik*] $O\hat{V}N = O\hat{W}S = 90°$ [from 10.1] $O\hat{N}V = O\hat{S}W$ [sum ∠s Δ/*som ∠e Δ*] $\therefore \Delta OVN \mid\mid\mid \Delta OWS$ [∠, ∠, ∠] $\therefore \frac{VN}{WS} = \frac{ON}{OS}$ But $VN = \frac{1}{2}MN$ [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ $\therefore OS.MN = 2ON.\ WS$ **OR/*OF*** In ΔOVM and ΔOWS $O\hat{V}M = O\hat{W}S = 90°$ [from 10.1] $O\hat{M}V = O\hat{S}W$ [sum ∠s Δ/*som ∠e Δ*] $\therefore \Delta OVM \mid\mid\mid \Delta OWS$ [∠, ∠, ∠] $\therefore \frac{OM}{OS} = \frac{VM}{WS}$ But $VN = \frac{1}{2}MN$ [given] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{OM}{OS}$ $\therefore OS.MN = 2ON.\ WS$ [VM = VN] **OR/*OF*** If any other 2 Δs are used, first need to prove that TW = WS by proving ΔOWT ≡ ΔOWS In ΔOVM and ΔOWT $\hat{O}_1 = \hat{O}_1$ [common/*gemeenskaplik*] $O\hat{V}M = O\hat{W}T = 90°$ [from 10.1] $O\hat{M}V = O\hat{T}W$ [sum ∠s Δ/*som ∠e Δ*] $\therefore \Delta OVM \mid\mid\mid \Delta OWT$ [∠, ∠, ∠] $\therefore \frac{VM}{WT} = \frac{OM}{OT}$ But $VN = VM = \frac{1}{2}MN$ [given] and WT = WS and OT = OS [ΔOWT ≡ ΔOWS] $\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$ $\therefore OS.MN = 2ON.\ WS$	✓ S; S; S **OR** S; S; R ✓ ΔOVN ||| ΔOWS ✓ $\frac{VN}{WS} = \frac{ON}{OS}$ ✓ $VN = \frac{1}{2}MN$ ✓ substitution (5) ✓ S; S; S **OR** S; S; R ✓ ΔOVM ||| ΔOWS ✓ $\frac{OM}{OS} = \frac{VM}{WS}$ ✓ $VN = \frac{1}{2}MN$ ✓ substitution (5) ✓✓ similarity ✓✓ congruency ✓ $VN = VM = \frac{1}{2}MN$ (5) **[12]**

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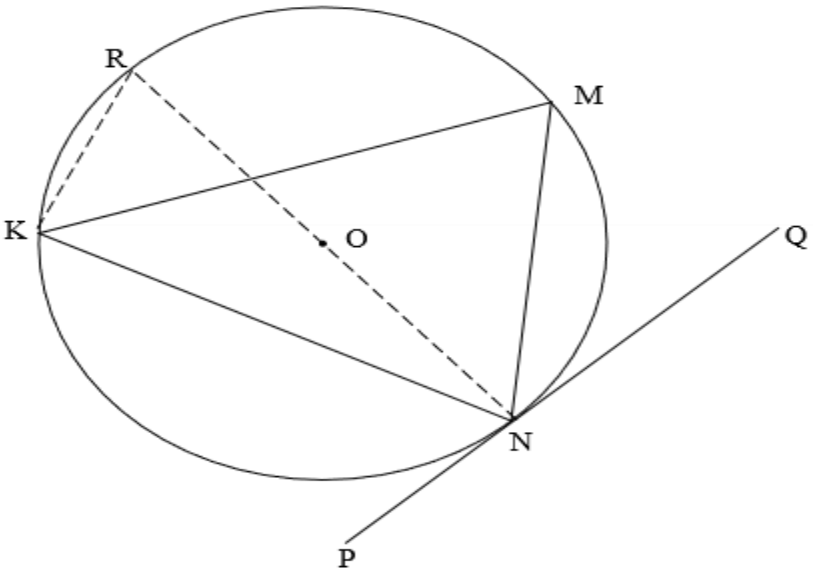
KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

QUESTION/*VRAAG* 11

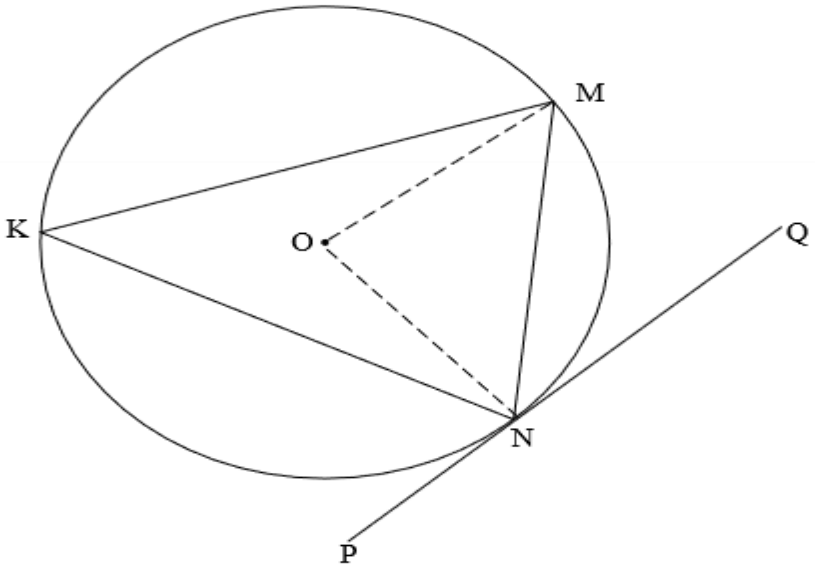


11.1	Construction: Draw diameter NR and draw RM *Konstruksie: Trek middellyn NR en verbind RM* $O\hat{N}M + M\hat{N}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{M}R = 90°$ [∠ in semi circle/*semi-sirkel*] $\therefore M\hat{R}N = 180° - (90° + 90° - M\hat{N}Q)$ [sum ∠s Δ] $= M\hat{N}Q$ but $M\hat{R}N = M\hat{K}N$ [∠s same segment/*∠e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$ **OR/*OF***	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)

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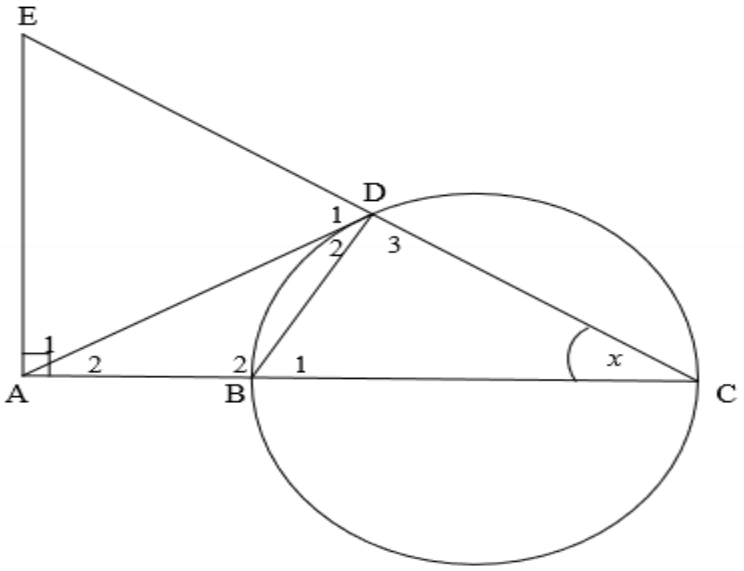
11.1	Construction: Draw diameter NR and draw RK *Konstruksie: Trek middellyn NR en verbind RK* $M\hat{N}Q + R\hat{N}M = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{K}R = 90°$ [∠ in semicircle/*semi-sirkel*] $\therefore M\hat{K}N = 90° - R\hat{K}M$ $= 90° - R\hat{N}M$ [∠s same segment/*∠e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)



11.1	Construction: Draw radii ON and OM *Konstruksie: Trek radiusse ON en OM* $M\hat{O}N = 2\hat{K}$ [∠ at centre = 2∠ at circumf/*midpts∠ = 2 omtreks∠*] $O\hat{N}M + O\hat{M}N = 180° - 2\hat{K}$ [∠s of Δ/*∠e van Δ*] $O\hat{N}M = O\hat{M}N = \frac{180° - 2\hat{K}}{2} = 90° - \hat{K}$ [∠s opp = sides/*∠e teenoor = sye*] $O\hat{N}Q = 90°$ [radius⊥tangent/*radius⊥raaklyn*] $\therefore M\hat{N}Q = \hat{K}$	✓ construction ✓ S /R ✓ S ✓ S/ R ✓ S / R (5)

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11.2



11.2.1(a)	Angle in a semi circle/ *Hoek in halfsirkel*	✓ R (1)
11.2.1(b)	Exterior ∠of quad = opp interior ∠/ *Buite ∠van vierh = teenoorst binne ∠* **OR/*OF*** Opp ∠s of quad supplementary/ *Teenoorst ∠e van vierh is supplementêr*	✓ R (1)
11.2.1(c)	tangent chord theorem/ *raaklyn koord stelling*	✓ R (1)
11.2.2(a)	In ΔAEC $\hat{E} = 180° - (90° + x)$ [sum ∠s Δ] $= 90° - x$ $\hat{D}_1 = 180° - (90° + x)$ [∠s on a straight line] $= \hat{E} = 90° - x$ $\therefore$ AD = AE [sides opp = ∠s/ *sye teenoor = ∠e*]	✓ S ✓ S ✓ R (3)
11.2.2(b)	In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum $\angle^e$ Δ] $\therefore$ ΔADB ||| ΔACD **OR/*OF*** In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\therefore$ ΔADB ||| ΔACD [∠, ∠, ∠]	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ R (3)

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11.2.3(a)	$\frac{AD}{AC}=\frac{AB}{AD}$ [||| Δs] $AD^2 = AC \cdot AB$ $= 3r \times r$ $= 3r^2$	✓ ratio ✓ substitution (2)
11.2.3(b)	$AD = AE = \sqrt{3}r$ [from 11.2.2(a) &11.2.3(a)] $AB = r$ and $BC = 2r$ $\therefore AC = 3r$ In ΔACE: $\tan \hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ $\therefore \hat{E} = 60°$ $\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60°$ [$\angle$s of $\Delta = 180°$] $\therefore$ ΔADE is equilateral/*is gelyksydig* **OR/*OF*** $\frac{AD}{AC}=\frac{DB}{CD}$ [||| Δs] $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ $\tan x = \frac{1}{\sqrt{3}}$ $\therefore$ In ΔBDC: $x = 30°$ $\therefore \hat{E} = 60°$ $\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60°$ [$\angle$s of $\Delta = 180°$] $\therefore$ ΔADE is equilateral/*is gelyksydig* **OR/*OF*** $\frac{AD}{AC}=\frac{DB}{CD}$ [||| Δs] $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ $\therefore BD = \frac{CD}{\sqrt{3}}$ $DC^2 = BC^2 - DB^2$ $= 4r^2 - \frac{CD^2}{3}$ $3DC^2 = 12r^2 - CD^2$ $4CD^2 = 12r^2$ $DC = \sqrt{3}r$	✓ AC ito r ✓ trig ratio ✓ simplification ✓ all 3 $\angle$s = 60° (4) ✓ $\frac{\sqrt{3}r}{3r}=\frac{DB}{CD}$ ✓ $\frac{1}{\sqrt{3}} = \tan x$ ✓ $x = 30°$ ✓ all 3 $\angle$s = 60° (4) ✓ $BD = \frac{CD}{\sqrt{3}}$ ✓ $DC = \sqrt{3}r$

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	$EC^2 = EA^2 + AC^2$ $= 3r^2 + 9r^2$ $EC = 2\sqrt{3}r$ $\therefore ED = EC - DC$ $= \sqrt{3}r$ $\therefore ED = EA = AD$ $\therefore \Delta ADE$ is equilateral/*is gelyksydig*	✓ $EC = 2\sqrt{3}r$ ✓ $ED = EA = AD$ (4) **[20]**

TOTAL/*TOTAAL*: 150

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GRADE 12

MATHEMATICS P2

NOVEMBER 2018

MARKS: 150

TIME: 3 hours

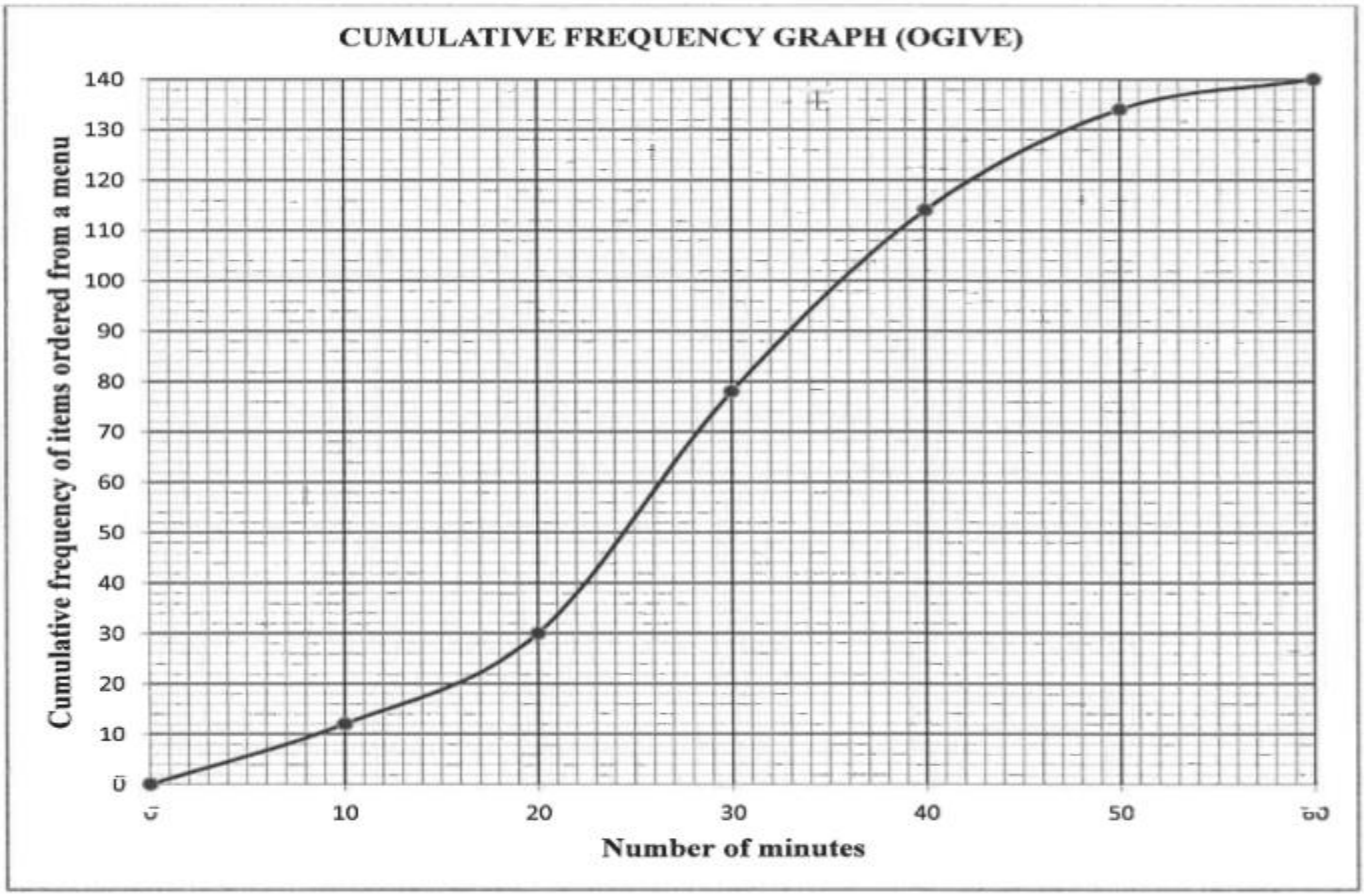
This question paper consists of 15 pages, 1 information sheet and an answer book of 31 pages.

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Please turn over

QUESTION 1

1.1 The cumulative frequency graph (ogive) drawn below shows the total number of food items ordered from a menu over a period of 1 hour.



1.1.1 Write down the total number of food items ordered from the menu during this hour. (1)

1.1.2 Write down the modal class of the data. (1)

1.1.3 How long did it take to order the first 30 food items? (1)

1.1.4 How many food items were ordered in the last 15 minutes? (2)

1.1.5 Determine the 75th percentile for the data. (2)

1.1.6 Calculate the interquartile range of the data. (2)

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1.2 Reggie works part-time as a waiter at a local restaurant. The amount of money (in rands) he made in tips over a 15-day period is given below.

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1 Calculate:

(a) The mean of the data (2)

(b) The standard deviation of the data (2)

1.2.2 Mary also works part-time as a waitress at the same restaurant. Over the same 15-day period Mary collected the same mean amount in tips as Reggie, but her standard deviation was R14.

Using the available information, comment on the:

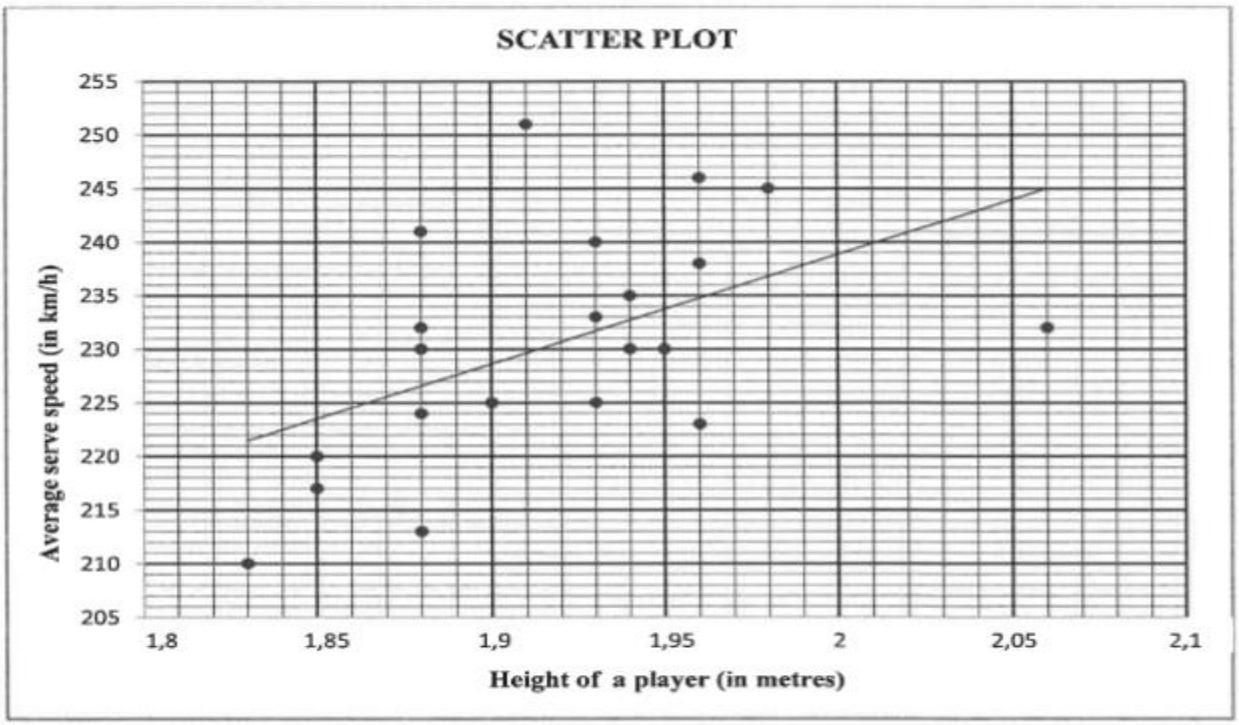
(a) Total amount in tips that they EACH collected over the 15-day period (1)

(b) Variation that EACH of them received in daily tips over this period (1)
[15]

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QUESTION 2

A familiar question among professional tennis players is whether the speed of a tennis serve (in km/h) depends on the height of a player (in metres). The heights of 21 tennis players and the average speed of their serves were recorded during a tournament. The data is represented in the scatter plot below. The least squares regression line is also drawn.



2.1 Write down the fastest average serve speed (in km/h) achieved in this tournament. (1)

2.2 Consider the following correlation coefficients:

A. $r = 0,93$ B. $r = -0,42$ C. $r = 0,52$

2.2.1 Which ONE of the given correlation coefficients best fits the plotted data? (1)

2.2.2 Use the scatter plot and least squares regression line to motivate your answer to QUESTION 2.2.1. (1)

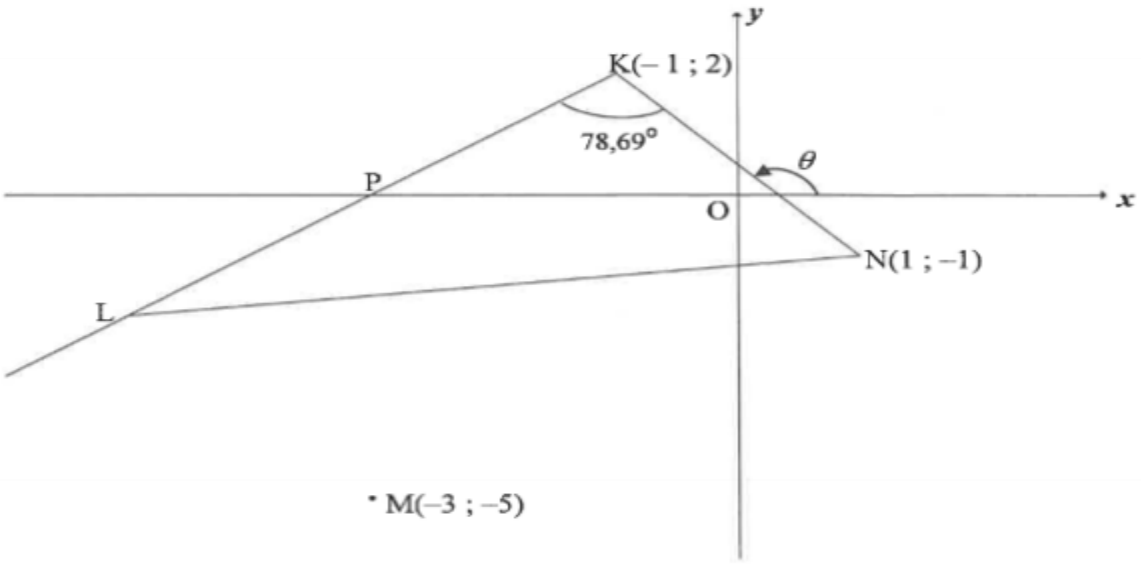
2.3 What does the data suggest about the speed of a tennis serve (in km/h) and the height of a player (in metres)? (1)

2.4 The equation of the regression line is given as $\hat{y} = 27,07 + bx$.
Explain why, in this context, the least squares regression line CANNOT intersect the y-axis at $(0 ; 27,07)$. (1)
[5]

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QUESTION 3

In the diagram, K(– 1 ; 2), L and N(1 ; – 1) are vertices of ΔKLN such that $L\hat{K}N = 78{,}69°$. KL intersects the x-axis at P. KL is produced. The inclination of KN is θ. The coordinates of M are (–3 ; –5).



3.1 Calculate:

3.1.1 The gradient of KN (2)

3.1.2 The size of θ, the inclination of KN (2)

3.2 Show that the gradient of KL is equal to 1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

3.4 Calculate the length of KN. (2)

3.5 It is further given that KN = LM.

3.5.1 Calculate the possible coordinates of L. (5)

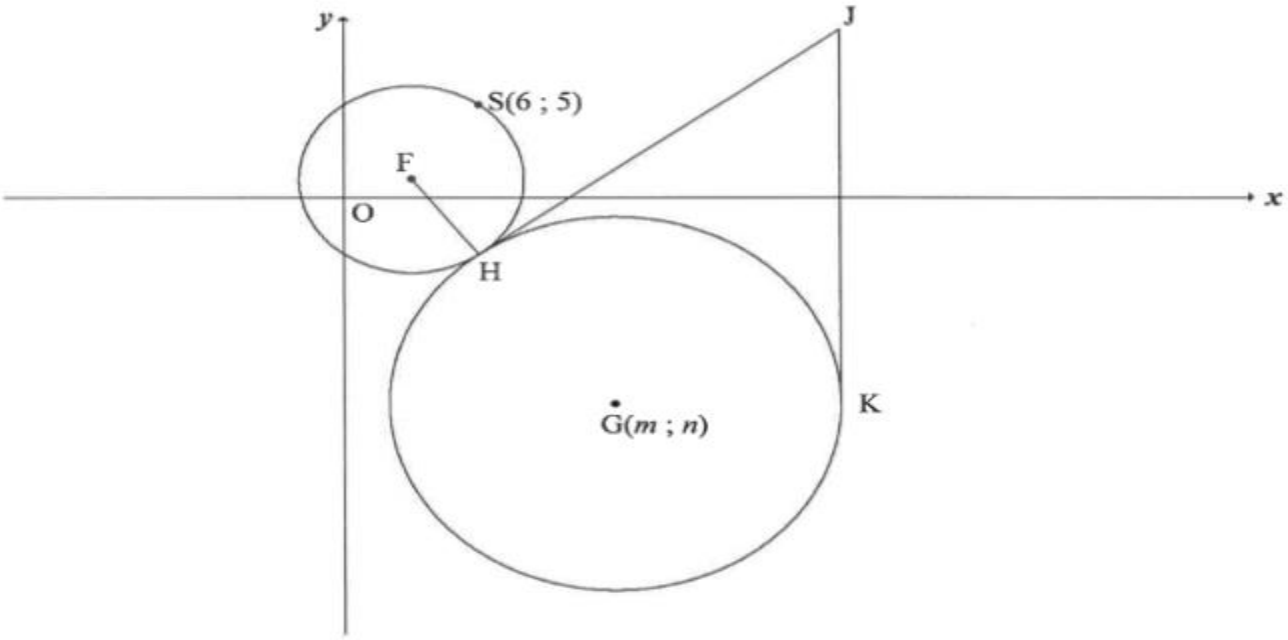
3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram. (3)

3.6 T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN. (4)
[22]

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QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G$(m\,;n)$ in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



4.1 Write down the coordinates of F. (2)

4.2 Calculate the length of FS. (2)

4.3 Write down the length of HG. (1)

4.4 Give a reason why JH = JK. (1)

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that JK = 20 (4)

4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

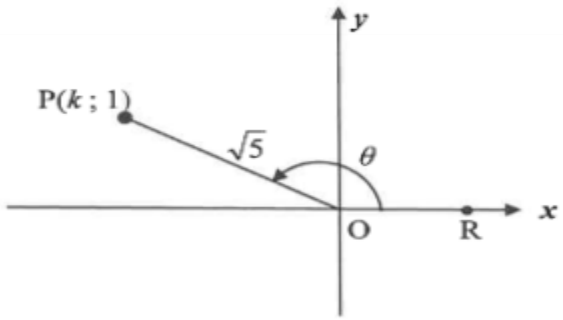
4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x=22$ (7)

[18]

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QUESTION 5

5.1 In the diagram, P(k ; 1) is a point in the 2nd quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x-axis and obtuse $\hat{ROP} = \theta$.



5.1.1 Calculate the value of k. (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $\tan\theta$ (1)

(b) $\cos(180° + \theta)$ (2)

(c) $\sin(\theta + 60°)$ in the form $\frac{a+b}{\sqrt{20}}$ (5)

5.1.3 **Use a calculator** to calculate the value of $\tan(2\theta - 40°)$ correct to ONE decimal place. (3)

5.2 Prove the following identity: $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$ (5)

5.3 Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$ (5)

[23]

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QUESTION 6

Consider: $f(x) = -2\tan\frac{3}{2}x$

6.1 Write down the period of f. (1)

6.2 The point A(t ; 2) lies on the graph. Determine the general solution of t. (3)

6.3 On the grid provided in the ANSWER BOOK, draw the graph of f for the interval $x \in [-120° ; 180°]$. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)

6.4 Use the graph to determine for which value(s) of x will $f(x) \geq 2$ for $x \in [-120° ; 180°]$. (3)

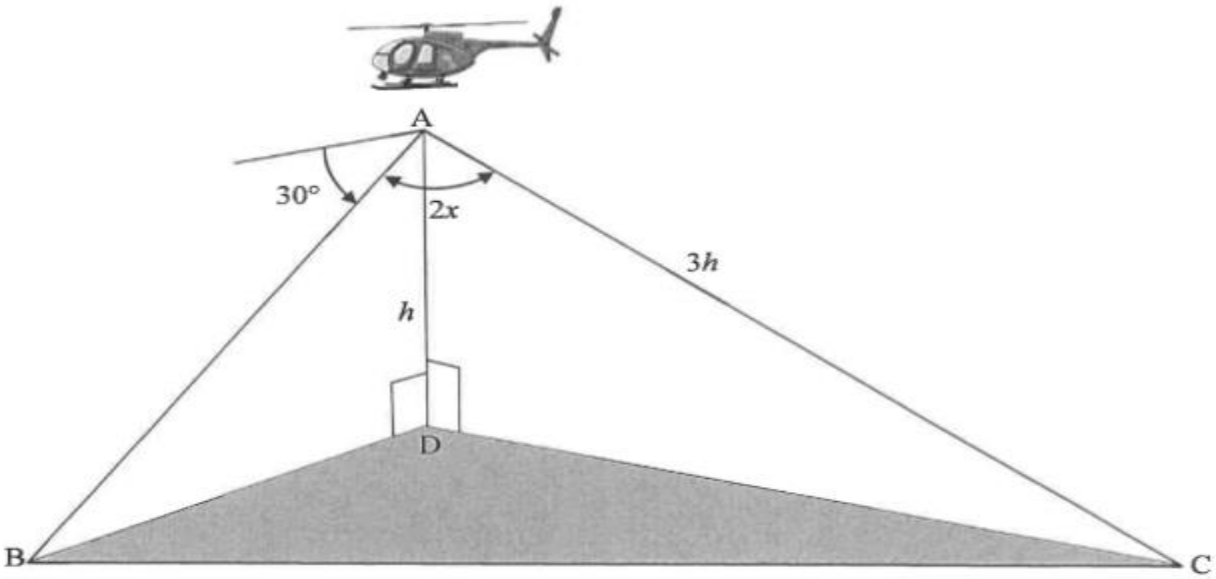
6.5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$. (2)

[13]

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QUESTION 7

A pilot is flying in a helicopter. At point A, which is h metres directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is 30°. He also determines that the control room at point C is $3h$ metres from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane. This scenario is shown in the diagram below.



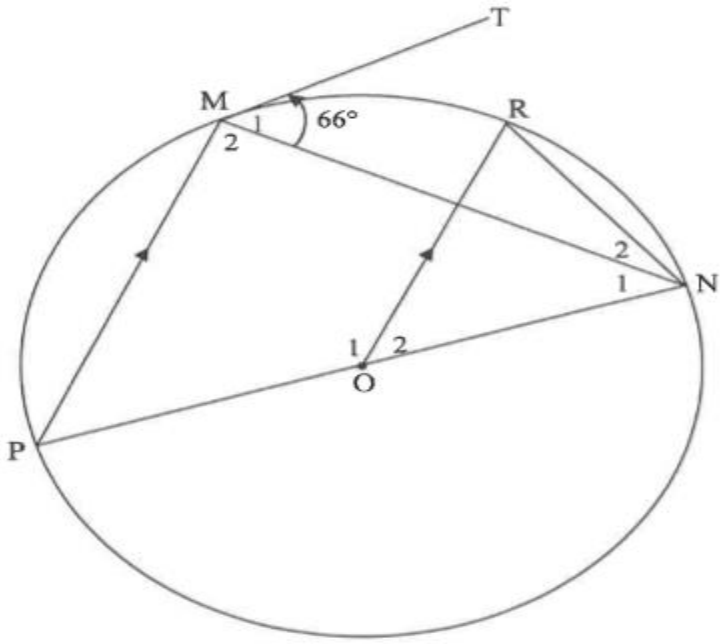
7.1 Determine the distance AB in terms of h. (2)

7.2 Show that the distance between the strange object at point B and the control room at point C is given by $BC = h\sqrt{25 - 24\cos^2 x}$. (4)
[6]

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QUESTION 8

8.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR || PM. NR and MN are drawn. Let $\hat{M}_1 = 66°$.



Calculate, with reasons, the size of EACH of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_2$ (2)

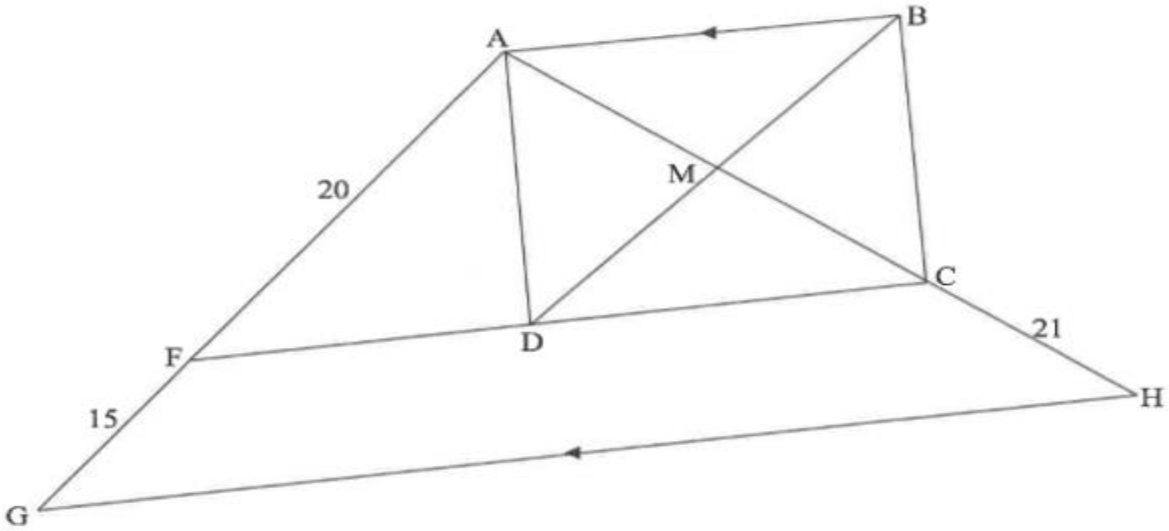
8.1.3 $\hat{N}_1$ (1)

8.1.4 $\hat{O}_2$ (2)

8.1.5 $\hat{N}_2$ (3)

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8.2 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that $AF = 20$ units, $FG = 15$ units and $CH = 21$ units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.



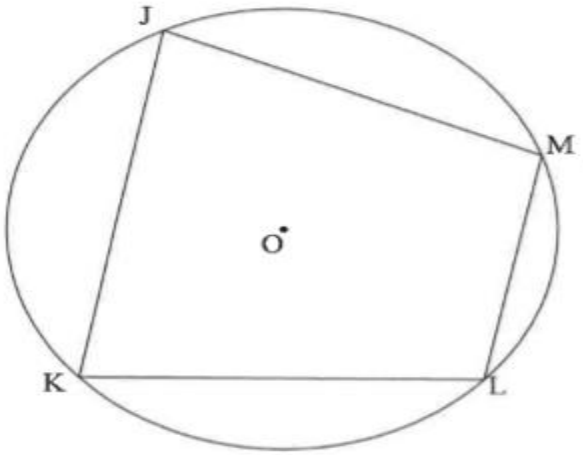
8.2.1 Explain why FC || GH. (1)

8.2.2 Calculate, with reasons, the length of DM. (5)
[16]

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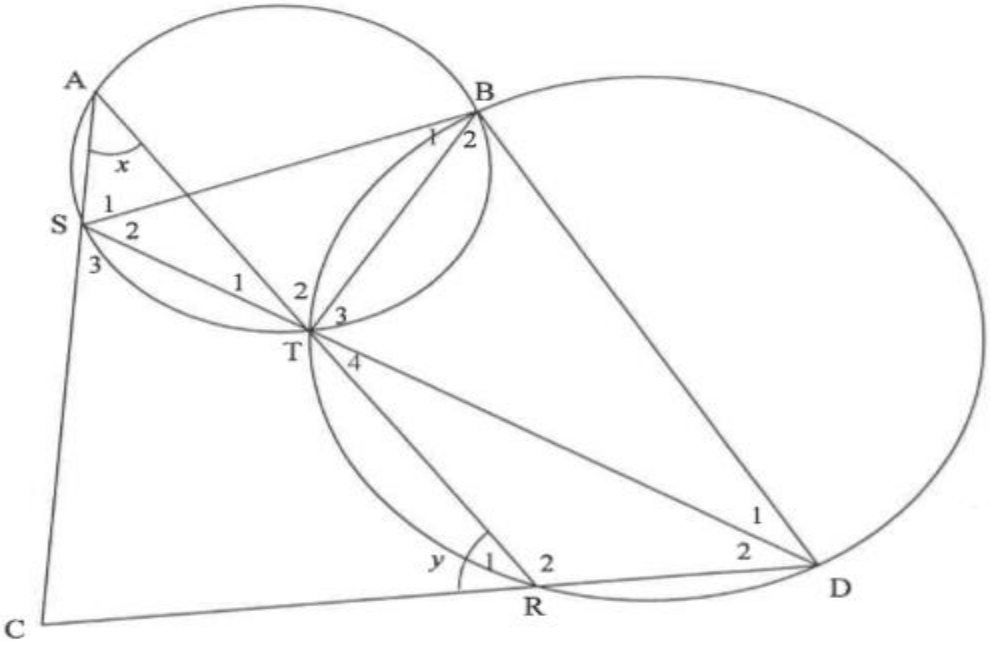
QUESTION 9

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.
Prove the theorem which states that $\hat{J} + \hat{L} = 180°$. (5)



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9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\hat{R}_1 = y$.



9.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

(b) y (2)

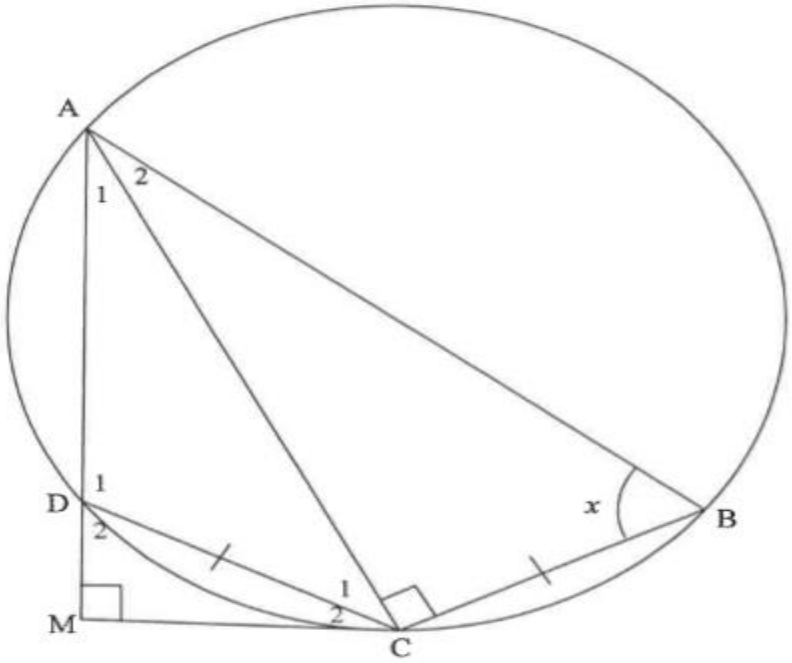
9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

9.2.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.
Prove that SD is not a diameter of circle BDS. (4)
[16]

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QUESTION 10

In the diagram, ABCD is a cyclic quadrilateral such that AC ⊥ CB and DC = CB. AD is produced to M such that AM ⊥ MC. Let $\hat{B} = x$.



10.1 Prove that:

10.1.1 MC is a tangent to the circle at C (5)

10.1.2 ΔACB ||| ΔCMD (3)

10.2 Hence, or otherwise, prove that:

10.2.1 $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ (6)

10.2.2 $\frac{AM}{AB} = \sin^2 x$ (2)

[16]

TOTAL: 150

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GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

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Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 1

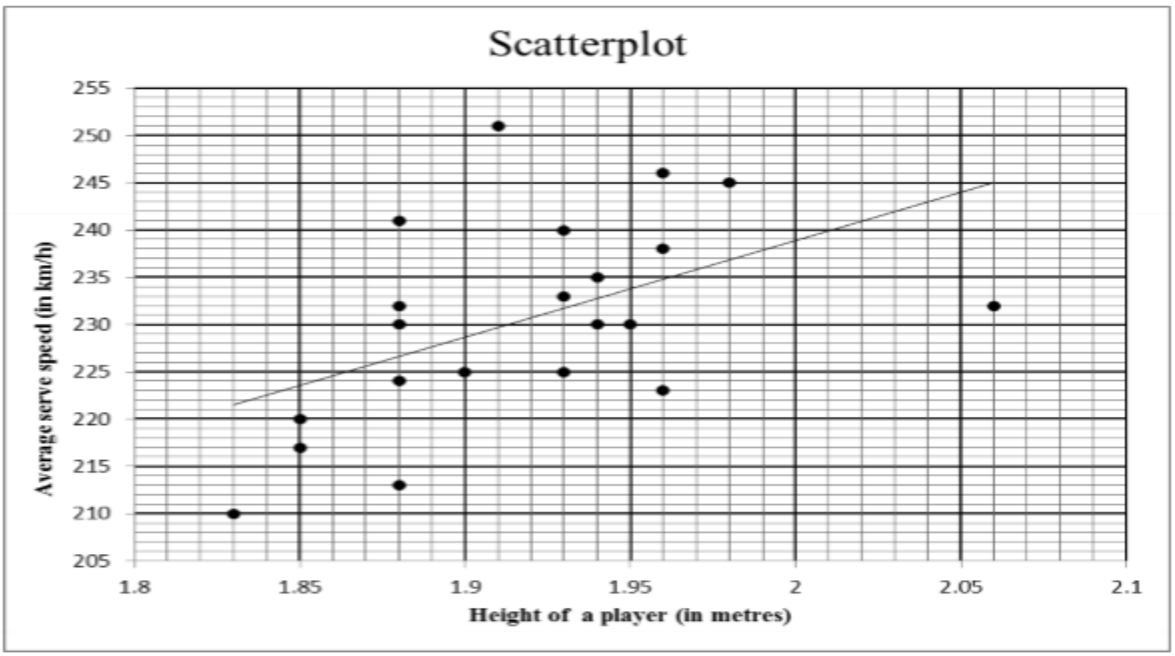
1.1.1	140 items	✓ answer (1)
1.1.2	Modal class/*modale klas:* $20 < x \leq 30$ minutes **OR/*OF*** $20 \leq x < 30$ minutes	✓ answer (1) ✓ answer (1)
1.1.3	Number of minutes taken = 20 minutes	✓ answer (1)
1.1.4	140 – 126 [Accept: 124 to 128] 14 orders (12 to 16) Answer only: Full marks	✓ 126 ✓ answer (2)
1.1.5	75th percentile is at 105 items =37 minutes [accept 36 – 38 minutes] Answer only: Full marks	✓ 105 ✓ answer (2)
1.1.6	Lower quartile is at 35 items =21,5 min [accept 21 – 23 min] IQR = 37 – 21,5 = 15,5 min [accept 13 – 17 min] Answer only: Full marks	✓ lower quartile (Q_1) ✓ answer (2)

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1(a)	$\bar{x} = \frac{1420}{15}$ Answer only: Full marks = R94,666.. = *R*94,67	✓ 1420 ✓ answer (2)
1.2.1(b)	σ = *R*22,691... = R22,69	✓✓ answer (2)
1.2.2(a)	They both collected the **same (equal) amount** in tips, i.e. R1 420 over the 15-day period. *Hulle albei het **dieselfde bedrag** met fooitjies ontvang, nl. R1 420 oor die 15 dae-tydperk*	✓ answer (1)
1.2.2(b)	Mary's standard deviation is smaller than Reggie's which suggests that there was **greater variation in the amount of tips that Reggie collected** each day compared to the number of tips that Mary collected each day. *Marie se standaardafwyking is kleiner as Reggie s'n wat beteken dat daar **groter variasie/verspreiding in die fooitjies was wat Reggie** elke dag **ontvang** het in vergelyking met die getal fooitjies wat Marie elke dag ontvang het.*	✓ explanation (1)
		[15]

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QUESTION/*VRAAG* 2



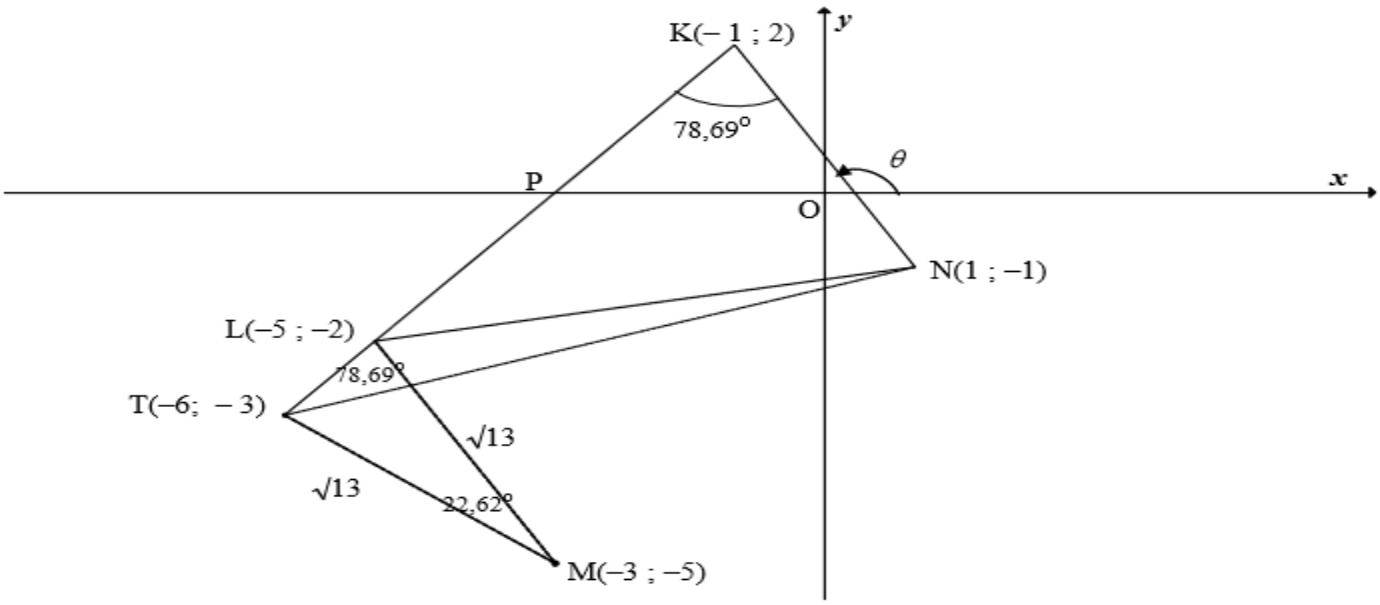
2.1	251 km/h	✓ answer (1)
2.2.1	$r = 0{,}52$ **OR** C	✓ answer (1)
2.2.2	The points are **fairly scattered** and the least squares regression line is increasing. *Die punte is **redelik verspreid** en die kleinstekwadrate-regressielyn neem toe.*	✓ reason (1)
2.3	There is a weak positive relation hence the height could have an influence *Daar is 'n swak positiewe verband, tog kan die lengte 'n invloed hê.* **OR/*OF*** There is no conclusive evidence that the height of a player will influence his/her tennis serve speed. *Daar is geen duidelike bewys dat die lengte van die speler sy/haar afslaanspoed kan beïnvloed nie.* **OR/*OF*** There is no conclusive evidence that a taller person will serve faster than a shorter person. *Daar is geen duidelike bewys dat 'n langer speler vinniger sal afslaan as 'n korter een nie.*	✓ answer (1) ✓ answer (1) ✓ answer (1)

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2.4	For (0 ; 27,07), it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h. **It is impossible for a person to have a height of 0 m.** *(0 ; 27,07) beteken dat 'n speler 'n lengte van 0* m *kan hê en teen 'n spoed van 27,07* km/h *kan afslaan.* ***Dit is onmoontlik om 'n lengte van 0 m te hê.*** **OR/OF** This means that the **player does not exist and therefore cannot serve and have a serve speed.** *Dit beteken dat die* ***speler nie bestaan nie en daarom nie kan afslaan en 'n afslaanspoed hê nie.***	✓ explanation (1) ✓ explanation (1)
		[5]

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QUESTION/*VRAAG* 3



3.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2-(-1)}{-1-1}$ $= -\frac{3}{2}$ Answer only: Full marks	✓ correct substitution ✓ answer (2)
3.1.2	$\tan\theta = m_{KN} = -\frac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$ Answer only: Full marks	✓ $\tan\theta = m_{KN} = -\frac{3}{2}$ ✓ answer (2)
3.2	Inclination KL = $123{,}69° - 78{,}69° = 45°$ [ext ∠Δ] $\tan 45° = m_{KL} = 1$	✓ S ✓ $\tan 45° = m_{KL} = 1$ (2)
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$	✓ substitute (– 1 ; 2) and m ✓ equation (2) ✓ substitute (– 1 ; 2) and m ✓ equation (2)

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3.4	$KN = \sqrt{(1+1)^2 + (-1-2)^2}$ $KN = \sqrt{13}$ or 3,61 Answer only: Full marks	✓ substitute K and N into distance formula ✓ answer (2)
3.5.1	$(x+3)^2 + (y+5)^2 = 13$ ...(1) L is a point on KL $y = x + 3$ ...(2) (2) in (1): $(x+3)^2 + (x+3+5)^2 = 13$ $x^2 + 6x + 9 + x^2 + 16x + 64 = 13$ $2x^2 + 22x + 60 = 0$ $x^2 + 11x + 30 = 0$ $(x+5)(x+6) = 0$ $x = -5$ or $x = -6$ $y = -2$ or $y = -3$ L(–5 ; –2) or (–6 ; –3) **OR/OF** $(x+3)^2 + (y+5)^2 = 13$ ...(1) L is a point on KL $y = x + 3 \quad \therefore x = y - 3$ ...(2) (2) in (1): $(y-3+3)^2 + (y+5)^2 = 13$ $y^2 + y^2 + 10y + 25 = 13$ $2y^2 + 10y + 12 = 0$ $y^2 + 5y + 6 = 0$ $(y+2)(y+3) = 0$ $y = -2$ or $y = -3$ $x = -5$ or $x = -6$ L(–5 ; –2) or (–6 ; –3)	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ x-values ✓ y-values (5) ✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ y-values (both) ✓ x-values (both) (5)
3.5.2	Midpoint of KM: (– 2 ; – 1,5) $\therefore \frac{x_L + 1}{2} = -2$ and $\frac{y_L - 1}{2} = -\frac{3}{2}$ $\therefore$ L(– 5 ; – 2) **OR/*OF*** $m_{KN} = m_{LM}$ $\frac{y-(-5)}{x-(-3)} = -\frac{3}{2}$ $2(x+3+5) = -3(x+3)$ $2x + 16 = -3x - 9$ $5x = -25$ $x = -5$ $\therefore$ L(– 5 ; – 2) Answer only: Full marks	✓ midpoint of KM ✓x value ✓ y value (3) ✓ $m_{LM} = m_{KN}$ ✓x value ✓ y value (3)

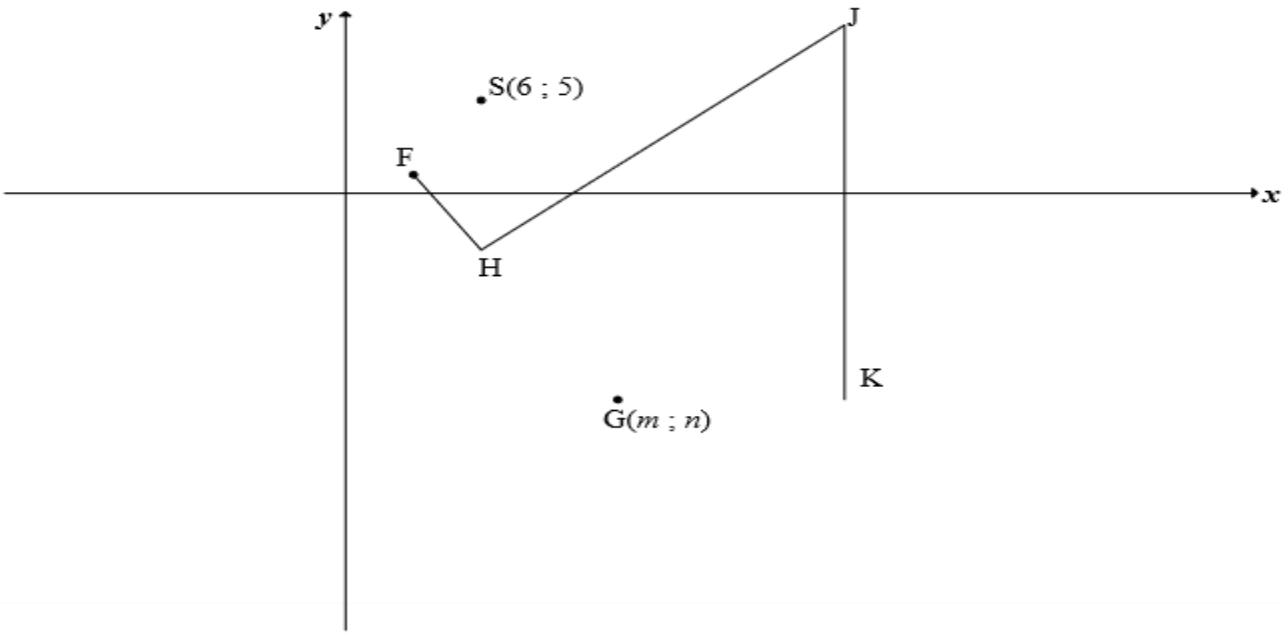
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	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	✓ transformation ✓x value ✓ y value (3)
3.6	T$(-6\,;-3)$ (from Question 3.5.1) $KT = \sqrt{(-1-(-6))^2 + (2-(-3))^2}$ $= \sqrt{50}$ $KN = \sqrt{13}$ (CA from 3.4) Area of $\Delta KTN = \frac{1}{2}KT.KN \sin L\hat{K}N$ $= \frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$ $= 12{,}50$ square units	✓ coordinates of T ✓ length of KT ✓ substitution into area rule ✓ answer (4)

	OR/*OF* In ΔKLM: $\frac{TL}{\sin 22{,}62°} = \frac{\sqrt{13}}{\sin 78{,}69°}$ $TL = 1{,}414..$ $KL = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$ $= \sqrt{32}$ $\therefore KT = 7{,}0708...$ Area of $\Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$ $= \frac{1}{2}(7{,}0708).\sqrt{13}\sin 78{,}69°$ $= 12{,}50$ square units	✓ length of TL ✓ length of KT ✓ substitution into area rule ✓ answer (4)
		[22]

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QUESTION/*VRAAG* 4



4.1	F(3 ; 1)	✓*x* value ✓ *y* value (2)
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ $FS = 5$	✓ substitution of F & S ✓ answer (2)
4.3	FH(FS) : HG = 1 : 2 $\therefore$ HG = 2 FH = 10	✓ HG = 10 (1)
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*	✓ answer (1)
4.5.1	$F\hat{H}J = 90°$ [tan ⊥ radius / *rkl ⊥ radius*] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*] $FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62	✓ S ✓ R ✓ S ✓ answer (4)
4.5.2	$(x-m)^2 + (y-n)^2 = 100$	✓ answer (1)

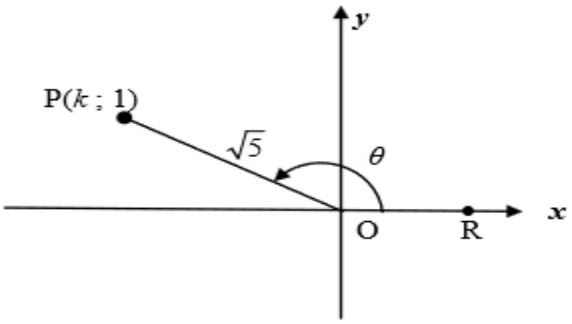
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4.5.3	K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ F, H and G are collinear [HJ is a common tangent] *F, H en G is saamlynig* *[HJ is 'n gemeemskaplike raaklyn]* $FG^2 = (12-3)^2 + (n-1)^2$ $15^2 = 81 + (n-1)^2$ $(n-1)^2 = 144$ $n - 1 = \pm 12$ **OR/*OF*** $n \neq 13$ or $n = -11$ $\therefore$ G(12; −11) $n^2 - 2n - 143 = 0$ $(n+11)(n-13) = 0$ $n = -11$ or $n \neq 13$ **OR/*OF*** K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ Let J(22 ; y): $FJ^2 = (22-3)^2 + (y-1)^2$ $425 = 361 + y^2 - 2y + 1$ $0 = y^2 - 2y - 63$ $0 = (y-9)(y+7)$ $\therefore y = 9$ or/of $y \neq -7$ $\therefore n = 9 - 20 = -11$ $\therefore$ G(12; −11)	✓ K(22; n) ✓ value of m ✓ subst. of F and G in distance formula ✓ FG = 15 ✓ simplification/ standard form ✓ value of n ✓ coordinates of G (7) ✓ K(22; n) ✓ value of m ✓ subst. of F and J in distance formula ✓ FJ = $\sqrt{425}$ ✓ standard form ✓ value of n ✓ coordinates of G (7)
		[18]

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QUESTION/*VRAAG* 5



5.1.1	$k^2 = (\sqrt{5})^2 - 1^2$ $= 4$ $k = -2$ Answer only: full marks	✓ substitution into theorem of Pythagoras ✓ answer (2)
5.1.2(a)	$\tan\theta = -\frac{1}{2}$	✓answer (1)
5.1.2(b)	$\cos(180° + \theta) = -\cos\theta$ $= \frac{2}{\sqrt{5}}$ Answer only: full marks	✓ reduction ✓ answer (2)
5.1.2(c)	$\sin(\theta + 60°) = \frac{a+b}{\sqrt{20}}$ LHS = $\sin\theta\cos 60° + \cos\theta\sin 60°$ $= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1-2\sqrt{3}}{2\sqrt{5}}$ $= \frac{1-2\sqrt{3}}{\sqrt{20}}$	✓ expansion ✓ subst of sin θ ✓ subst of cos θ ✓ both special ∠s ✓ $\frac{1-2\sqrt{3}}{2\sqrt{5}}$ (5)
5.1.3	$\tan\theta = -\frac{1}{2}$ $\therefore \theta = 180° - 26{,}57°$ $\therefore \theta = 153{,}43°$ $\tan(2\theta - 40°) = \tan[(2\times 153{,}43°) - 40°]$ $= \tan 266{,}87°$ $= 18{,}3$	✓ θ ✓ substitution ✓ answer (3)

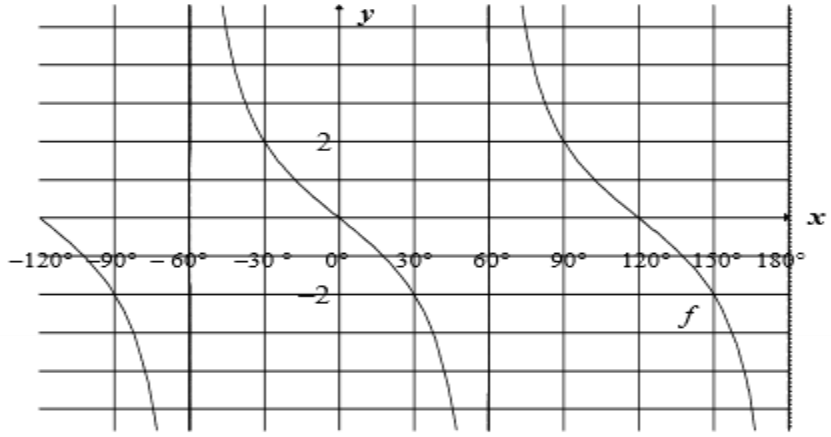
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5.2	LHS $=\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\cos x-\sin x}{\cos x+\sin x}$ RHS $=2\tan 2x$	
	$=\frac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$	✓ single fraction
	$=\frac{\cos^2 x+2\sin x\cos x+\sin^2 x-\cos^2 x+2\sin x\cos x-\sin^2 x}{\cos^2 x-\sin^2 x}$	✓ expansion
	$=\frac{2(2\sin x\cos x)}{\cos^2 x-\sin^2 x}$	✓ simplification (both)
	$=\frac{2\sin 2x}{\cos 2x}$	✓ double ∠ identity
	$=2\tan 2x$	✓ double ∠ identity
	= RHS	(5)
	OR/*OF*	
	LHS $=\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\cos x-\sin x}{\cos x+\sin x}$ RHS $=2\tan 2x$	
	$=\frac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$	✓ single fraction
	$=\frac{(\cos x+\sin x+\cos x-\sin x)(\cos x+\sin x-\cos x+\sin x)}{\cos^2 x-\sin^2 x}$	✓ difference of two squares
	$=\frac{(2\cos x)(2\sin x)}{\cos^2 x-\sin^2 x}$	✓ simplification (both)
	$=\frac{2(2\sin x\cos x)}{\cos^2 x-\sin^2 x}$	✓ double ∠ identity
	$=\frac{2\sin 2x}{\cos 2x}$	✓ double ∠ identity
	$=2\tan 2x$	(5)
	= RHS	
	OR/*OF*	
	RHS $=2\tan 2x$	
	$=\frac{2\sin 2x}{\cos 2x}$	
	$=\frac{2(2\sin x.\cos x)}{\cos^2 x-\sin^2 x}$	✓ double ∠ identity
	$=\frac{4\sin x.\cos x}{\cos^2 x-\sin^2 x}$	✓ double ∠ identity
	$=\frac{1+2\sin x.\cos x-(1-2\sin x.\cos x)}{\cos^2 x-\sin^2 x}$	✓ identity & method
	$=\frac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$	✓ factorising numerator and denominator
	$=\frac{(\cos x+\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}-\frac{(\cos x-\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$	✓ writing as 2 terms
	$=\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\cos x-\sin x}{\cos x+\sin x}=$ LHS	(5)

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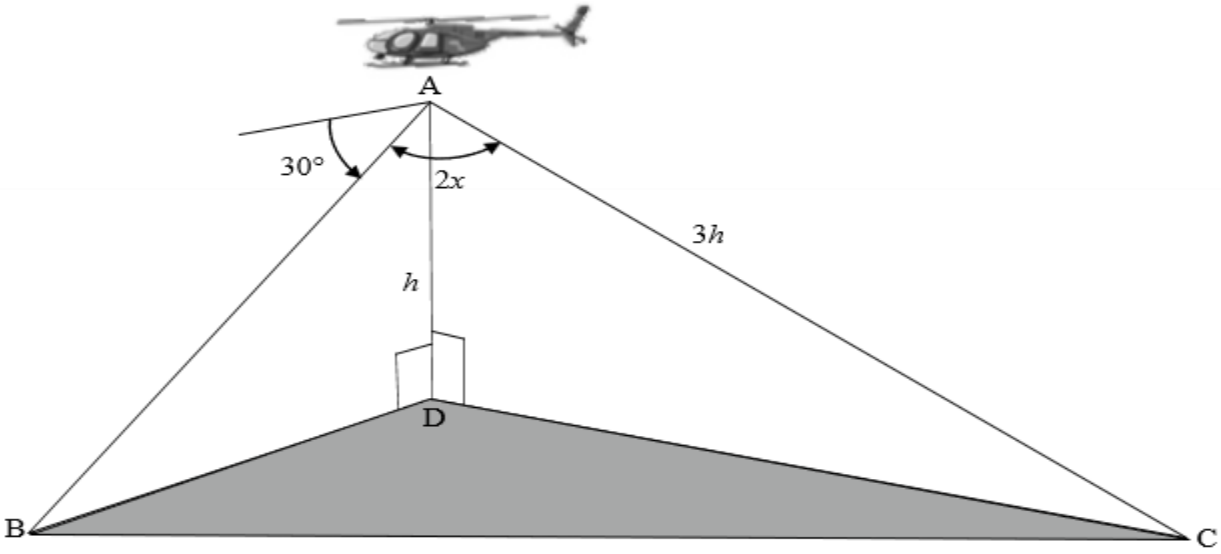
5.3	$\sum_{A=38°}^{52°} \cos^2 A$ $= \cos^2 38° + \cos^2 39° + \cos^2 40° + ... + \cos^2 51° + \cos^2 52°$ $= \sin^2 52° + \sin^2 51° + \sin^2 50° + ... + \cos^2 51° + \cos^2 52°$ $= 7(1) + \cos^2 45°$ $= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$ $= 7\frac{1}{2}$ **OR/*OF*** $\sum_{A=38°}^{52°} \cos^2 A$ $= \cos^2 38° + \cos^2 39° + \cos^2 40° + ... + \cos^2 51° + \cos^2 52°$ $= (\cos^2 38° + \sin^2 52°) + (\cos^2 39° + \sin^2 51°) ... + \cos^2 45°$ $= 7(1) + \cos^2 45°$ $= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$ $= 7\frac{1}{2}$	✓ expansion ✓ co ratio ✓ $\cos^2 45°$ ✓ $7 \times$ identity ✓ answer (5) ✓ expansion ✓ pairing ✓ $\cos^2 45°$ ✓ $7 \times$ identity ✓ answer (5)
		[23]

QUESTION/*VRAAG* 6

6.1	Period = 120°	✓ answer (1)
6.2	$2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.180°$ **OR/*OF*** $\frac{3}{2}t = -45° + k.180°$ $t = 90° + k.120°$; $k \in Z$ $t = -30° + k.120°$; $k \in Z$ **OR/*OF*** $2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.360°$ **or**/of $\frac{3}{2}t = 315° + k.360°$ $t = 90° + k.240°$ **or**.of $t = 210° + k.240°$; $k \in Z$	✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3) ✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3)
6.3		✓asymptotes: $x = \pm 60°$; $x = 180°$ ✓x-intercepts 0°; ±120° ✓ negative shape ✓ (90° ; 2) or (− 30° ; 2) or (30° ; − 2) or (− 90° ; − 2) (4)
6.4	$x \in (-60° ; -30°]$ or $(60° ; 90°]$ **OR/*OF*** $-60° < x \leq -30°$ or $60° < x \leq 90°$	✓ interval ✓ interval ✓ notation (3) ✓ interval ✓ interval ✓ notation (3)
6.5	$g(x) = -2\tan\left[\frac{3}{2}(x + 40°)\right] = f(x + 40°)$ Translation of 40° to the left / skuif met 40° links	✓ Translation of 40° ✓ to the left (2)
		[13]

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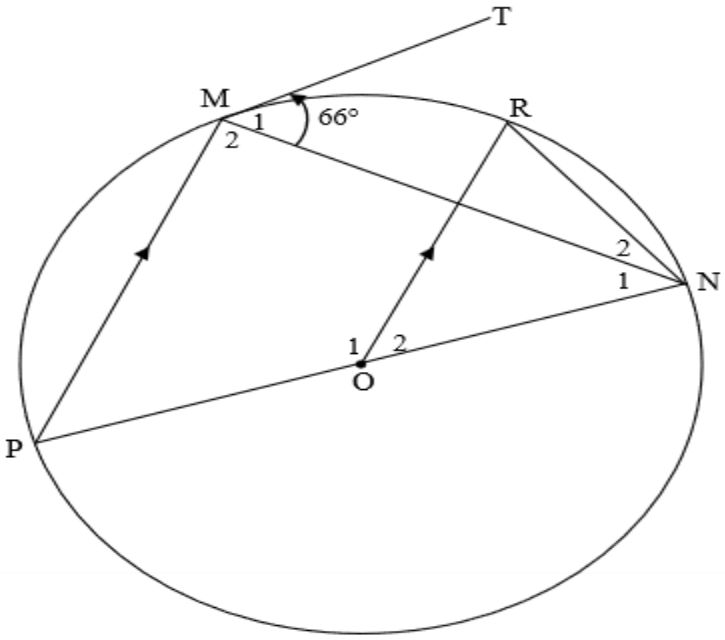
QUESTION/*VRAAG* 7



7.1	$A\hat{B}D = 30°$ $\sin 30° = \frac{h}{AB}$ $AB = \frac{h}{\sin 30°}$ **OR** $AB = \frac{h}{\frac{1}{2}}$ **OR** $AB = 2h$ **OR/*OF*** $B\hat{A}D = 60°$ $cos60° = \frac{h}{AB}$ $AB = \frac{h}{\cos 60°}$ **OR** $AB = \frac{h}{\frac{1}{2}}$ **OR** $AB = 2h$	✓ $A\hat{B}D = 30°$ ✓ answer (2) ✓ $B\hat{A}D = 60°$ ✓ answer (2)
7.2	$BC^2 = AB^2 + AC^2 - 2AB.AC\cos B\hat{A}C$ $= (2h)^2 + (3h)^2 - 2(2h)(3h)\cos 2x$ $= 13h^2 - 12h^2(2\cos^2 x - 1)$ $= 13h^2 - 24h^2\cos^2 x + 12h^2$ $= 25h^2 - 24h^2\cos^2 x$ $BC = h\sqrt{25 - 24\cos^2 x}$	✓ use of cosine rule in ΔABC ✓ substitution ✓ double angle identity ✓ $25h^2 - 24h^2\cos^2 x$ (4)
		[6]

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QUESTION/*VRAAG* 8

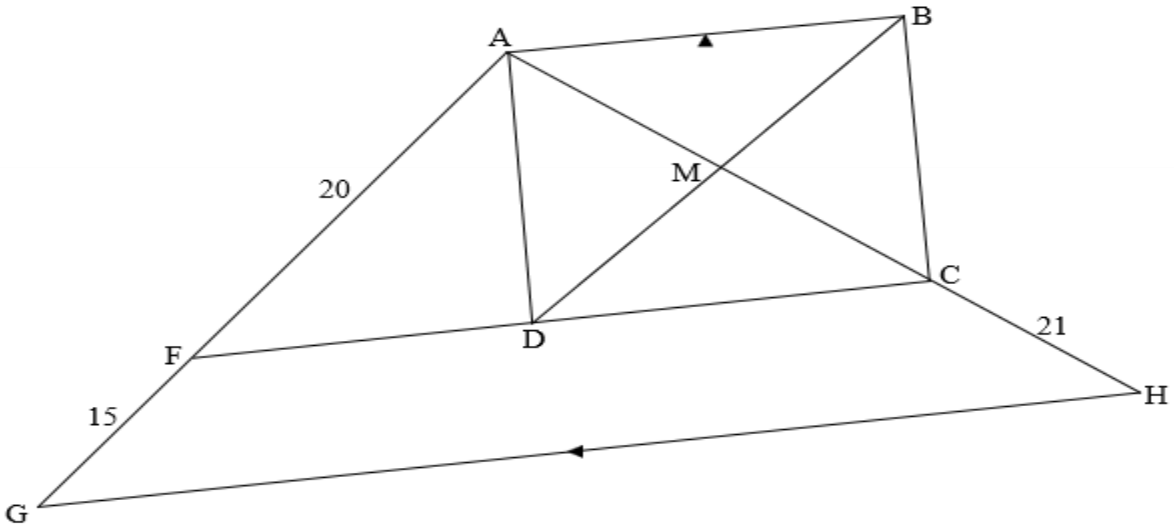


8.1.1	$\hat{P} = \hat{M}_1 = 66°$	[tan chord theorem/*raaklyn koordst*]	✓S ✓R (2)
8.1.2	$\hat{M}_2 = 90°$	[∠ in semi circle/*∠ in halfsirkel*]	✓S ✓R (2)
8.1.3	$\hat{N}_1 = 180° - (90° + 66°)$ $= 24°$	[sum of ∠s of /*som van ∠e* ΔMNP]	✓S (1)
8.1.4	$\hat{O}_2 = \hat{P} = 66°$	[corres. ∠s;/*ooreenk ∠e*, PM ‖ OR]	✓S ✓R (2)
8.1.5	$\hat{R} + \hat{N}_1 + \hat{N}_2 = 180° - 66°$ $= 114°$ $\hat{R} = \hat{N}_1 + \hat{N}_2 = 57°$ $\therefore\ \hat{N}_2 = 33°$	[sum of ∠s of/*som van ∠e* ΔRNO] [∠s opposite = radii/*∠e teenoor = radii*]	✓S ✓S/R ✓S (3)
	OR/*OF* $P\hat{O}R = 114°$ $P\hat{N}R = 57°$ $\therefore\ \hat{N}_2 = 33°$	[∠s on straight line/*∠e op reguitlyn*] [∠ at centre = twice ∠ at circumference/ *midpts∠ = 2 × omtreks∠*]	✓S ✓S/R ✓S (3)

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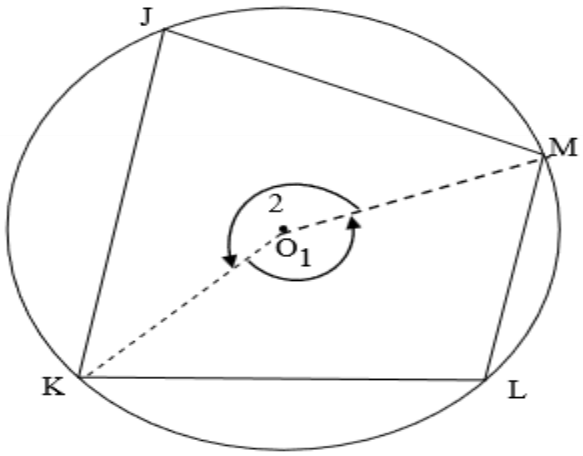
8.2



8.2.1	FC ∥ AB ∥ GH [opp sides of **rectangle**/*teenoorst sye v reghoek*]	✓ R (1)
8.2.2	$\frac{AC}{CH} = \frac{AF}{FG}$ [line ∥ one side of Δ] **OR** [prop theorem; FC ∥ GH] *[lyn ∥ een sy van Δ]* ***OF*** *[eweredighst; FC ∥ GH]* $\frac{AC}{21} = \frac{20}{15}$ $AC = \frac{20 \times 21}{15}$ $= 28$ DB = AC = 28 [diags of rectangle =/*hoeklyne v reghoek* =] $DM = \frac{1}{2}DB = 14$ [diags of rectangle bisect/*hoekl v reghoek halveer*]	✓S ✓R ✓ AC ✓ S ✓ S (5)
		[16]

QUESTION/*VRAAG* 9

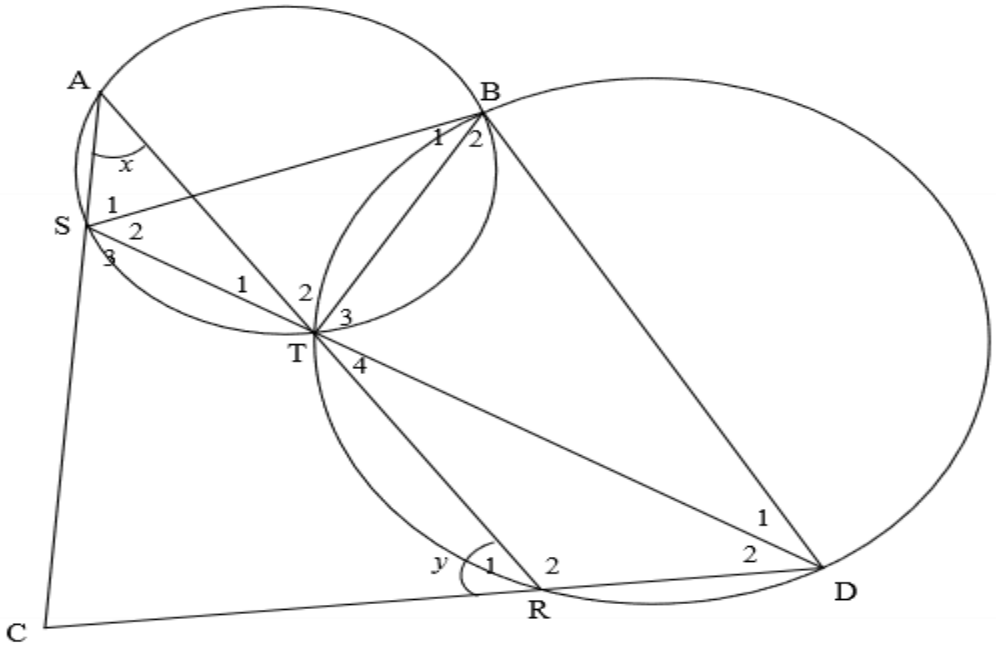
9.1



9.1	Constr/*Konstr.*: Draw KO and MO/*Trek KO en MO* Proof: $\hat{O}_1 = 2\hat{J}$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]* $\hat{O}_2 = 2\hat{L}$ [∠ at centre = twice ∠ at circumference] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s around a point /*∠e om 'n punt*] $\therefore 2\hat{J} + 2\hat{L} = 360°$ $\therefore 2(\hat{J} + \hat{L}) = 360°$ $\therefore \hat{J} + \hat{L} = 180°$	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S (5)
	OR/*OF*	
	Constr/*Konstr.*: Draw KO and MO/*Trek KO en MO* Proof: Let $\hat{J} = x$ $\hat{O}_1 = 2x$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]* $\hat{O}_2 = 360° - 2x$ [∠s around a point /*∠e om 'n punt*] $\therefore \hat{L} = 180° - x$ [∠ at centre = twice ∠ at circumference] $\therefore \hat{J} + \hat{L} = 180°$	✓ construction ✓ S ✓ R ✓ S/R ✓ S (5)

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

9.2

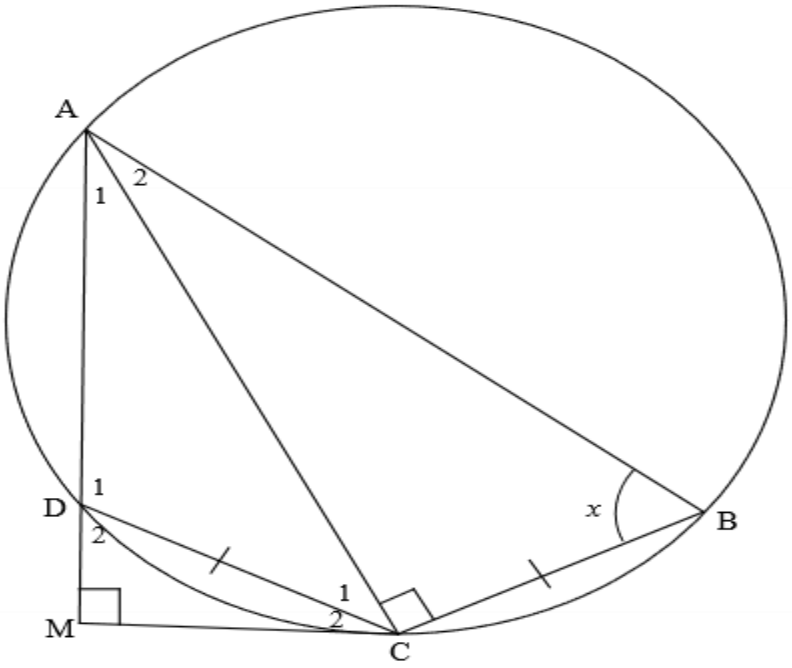


9.2.1(a)	$\hat{B}_1 = x$ [∠s in same seg/*∠e in dieselfde segm*]	✓ S ✓ R (2)
9.2.1(b)	$\hat{B}_2 = y$ [ext ∠ of cyclic quad/*buite∠ koordevh*]	✓ S ✓ R (2)
9.2.2	$\hat{C} = 180° - (x + y)$ [sum of ∠s of/*som v ∠e*, ΔACR] $S\hat{B}D + \hat{C} = x + y + 180° - (x + y)$ $S\hat{B}D + \hat{C} = 180°$ SCDB is a cyclic quad [converse opp angles of cyclic quad] [*omgekeerde teenoorst ∠e koordevh*] **OR/*OF*** $\hat{S}_1 = \hat{T}_2$ [∠s in same segment/*∠e in dies. segment*] $\hat{T}_2 = \hat{D}_1 + \hat{D}_2 = B\hat{D}R$ [ext ∠ of cyc quad/*buite∠ koordevh*] $\therefore \hat{S}_1 = B\hat{D}R$ $\therefore$ SCDB is cyc quad [ext ∠ of quad = opp ∠/*buite∠ = tos ∠*]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3)

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9.2.3	$\hat{T}_4 = y - 30°$ [ext ∠ of/*buite* ∠ Δ TDR] $\hat{T}_1 = y - 30°$ [vert opp ∠s =/*regoorst* ∠*e* =] $y - 30° + x + 100° = 180°$ [sum of ∠s of/*som v* ∠*e*, Δ AST] $\therefore x + y = 110°$ $S\hat{B}D = 110°$ ∴ SD not diameter [line does not subtend 90° ∠] *SD nie 'n middellyn [lyn onderspan nie 90°∠]* **OR/*OF*** $A\hat{S}T = \hat{C} + \hat{D}_2$ [ext ∠ of/*buite* ∠ Δ SCD] $\hat{C} = 100° - 30° = 70°$ $S\hat{B}D = 180° - 70°$ [opp ∠s cyclic quad/ teenoorst ∠e kdvh] $= 110°$ ∴ SD not diameter [line does not subtend 90° ∠] *SD nie 'n middellyn [lyn onderspan nie 90°∠]*	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)
		[16]

QUESTION/*VRAAG* 10



10.1.1	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s = *kde onderspan =∠e*]	✓ S ✓R
	$\hat{D}_2 = x$ [exterior angle of cyclic quad/*buite∠ koordevh.*]	✓ S/R
	$\therefore \hat{C}_2 = 90° - x$ [sum of ∠s of/*som v ∠e*, ΔDCM]	✓ $\hat{C}_2 = 90° - x$
	$\therefore \hat{C}_2 = \hat{A}_1 = 90° - x$	
	∴ MC is a tangent to the circle at C [converse: tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	OR/*OF*	
	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan =∠e*]	✓ S ✓ R
	$\hat{C}_1 + \hat{C}_2 = x$ [sum of ∠s of/*som v ∠e*, ΔACM]	✓✓ $\hat{C}_1 + \hat{C}_2 = x$
	$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$	
	∴ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	OR/*OF*	
	In ΔAMC and ΔACB:	
	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan =∠e*]	✓ S ✓ R
	$A\hat{M}C = A\hat{C}B = 90°$ [given]	
	$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$	✓✓ $\hat{C}_1 + \hat{C}_2 = x$

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	∴ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior ∠ of cyclic quad.] *[bewys **OF** buite ∠ v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of ∠s in Δ] *[Bewys **OF** som v ∠e in Δ]* ΔACB ||| ΔCMD [∠, ∠, ∠] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior ∠ of cyclic quad.] *[bewys **OF** buite ∠ v koordevh]* $A\hat{C}B = A\hat{M}C = 90°$ [given/*gegee*] ΔACB ||| ΔCMD [∠, ∠, ∠] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior ∠ of cyclic quad] *[bewys **OF** buite ∠ v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of ∠s in Δ] *[Bewys **OF** som v ∠e in Δ]* $A\hat{C}B = A\hat{M}C = 90°$ [given **OR** sum of ∠s in Δ] *[gegee **OF** som v ∠e in Δ]* ΔACB ||| ΔCMD	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.1	$\frac{BC}{MD} = \frac{AB}{DC}$ [ΔACB ||| ΔCMD] $\frac{DC}{MD} = \frac{AB}{DC}$ [BC = DC] $\therefore DC^2 = AB \times MD$ In ΔAMC and/*en* ΔCMD $\hat{M}$ is common/*gemeen* $\hat{A}_1 = \hat{C}_2$ [tan chord th /*raaklyn koordst*] **OR/*OF*** $\hat{C}_1 + \hat{C}_2 = \hat{B} = \hat{D} = x$ [tan chord th /*raaklyn koordst* **OR/*OF*** exterior ∠ of cyclic quad/ *buite ∠ v kdvh*] ΔAMC ||| ΔCMD [∠, ∠, ∠] $\frac{AM}{CM} = \frac{CM}{MD}$ $\therefore CM^2 = AM \times MD$ $\therefore \frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$ $= \frac{AM}{AB}$	✓ $\frac{BC}{MD} = \frac{AB}{DC}$ ✓ $DC^2 = AB \times MD$ ✓ S ✓ S ✓ $CM^2 = AM \times MD$ ✓ $\frac{AM \times MD}{AB \times MD}$ (6)

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	OR/*OF*	
	$\frac{AC}{MC}=\frac{AB}{DC}$ [ΔACB ||| ΔCMD] $\therefore CM\times AB = AC\times DC$	✓ $\frac{AC}{MC}=\frac{AB}{DC}$
	In ΔAMC and/*en* ΔACB $\hat{C}=\hat{M}=90°$ [given] $\hat{A}_1=\hat{A}_2$ [proven]	✓ S ✓ S
	OR/*OF*	
	$A\hat{C}M=\hat{B}=x$ [proven] ΔAMC ||| ΔACB [∠, ∠, ∠] $\frac{AC}{AM}=\frac{BC}{MC}$ $\therefore AC\times MC = AM\times BC$ $\therefore AC=\frac{BC.AM}{MC}$	✓ AC.MC = AM.BC
	$CM\times AB=\frac{BC.AM}{MC}\times DC$ $CM^2=\frac{DC.AM}{AB}\times DC$ [BC = DC] $\frac{CM^2}{DC^2}=\frac{AM}{AB}$	✓ equating ✓ S (6)
10.2.2	In ΔDMC: $\frac{CM}{DC}=\sin x$ $\frac{CM^2}{DC^2}=\sin^2 x$ $\frac{AC}{AB}=\frac{CM}{DC}$ $\therefore \frac{AM}{AB}=\sin^2 x$	✓ trig ratio ✓ square both sides (2)
	OR/*OF*	
	In ΔABC: $\sin x=\frac{AC}{AB}$ In ΔAMC: $\sin x=\frac{AM}{AC}$ $\sin x.\sin x=\frac{AC}{AB}\times\frac{AM}{AC}=\frac{AM}{AB}$	✓ 2 equations for $\sin x$ ✓ product (2)
		[16]

TOTAL/TOTAAL: 150

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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 24 pages.

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QUESTION 1

The table below shows the monthly income (in rands) of 6 different people and the amount (in rands) that each person spends on the monthly repayment of a motor vehicle.

MONTHLY INCOME (IN RANDS)	9 000	13 500	15 000	16 500	17 000	20 000
MONTHLY REPAYMENT (IN RANDS)	2 000	3 000	3 500	5 200	5 500	6 000

1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 If a person earns R14 000 per month, predict the monthly repayment that the person could make towards a motor vehicle. (2)

1.3 Determine the correlation coefficient between the monthly income and the monthly repayment of a motor vehicle. (1)

1.4 A person who earns R18 000 per month has to decide whether to spend R9 000 as a monthly repayment of a motor vehicle, or not. If the above information is a true representation of the population data, which of the following would the person most likely decide on:

A Spend R9 000 per month because there is a very strong positive correlation between the amount earned and the monthly repayment.

B NOT to spend R9 000 per month because there is a very weak positive correlation between the amount earned and the monthly repayment.

C Spend R9 000 per month because the point (18 000 ; 9 000) lies very near to the least squares regression line.

D NOT to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. (2)

[8]

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QUESTION 2

A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below:

AMOUNT PAID (IN RANDS)	FREQUENCY
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

2.1 How many people paid R200 or less on their monthly cellphone contracts? (1)

2.2 Use the information above to show that $a = 24$ and $b = 16$. (5)

2.3 Write down the modal class for the data. (1)

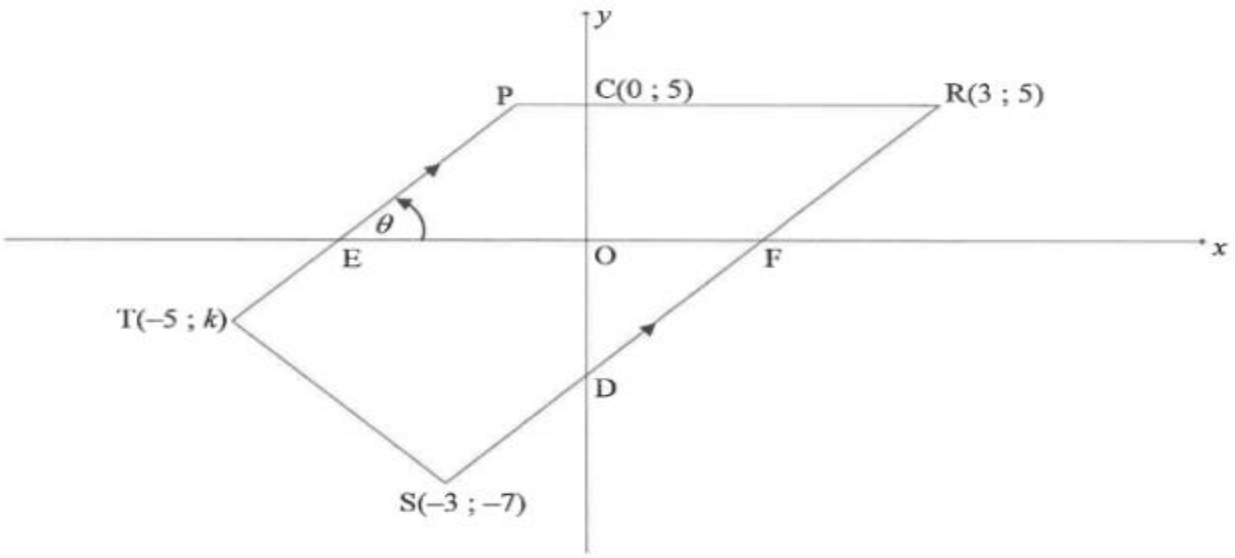
2.4 On the grid provided in the ANSWER BOOK, draw an ogive (cumulative frequency graph) to represent the data. (4)

2.5 Determine how many people paid more than R420 per month for their cellphone contracts. (2)
[13]

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QUESTION 3

In the diagram, P, R(3 ; 5), S(–3 ; –7) and T(–5 ; k) are vertices of trapezium PRST and PT ∥ RS. RS and PR cut the y-axis at D and C(0 ; 5) respectively. PT and RS cut the x-axis at E and F respectively. $P\hat{E}F = \theta$.



3.1 Write down the equation of PR. (1)

3.2 Calculate the:

3.2.1 Gradient of RS (2)

3.2.2 Size of θ (3)

3.2.3 Coordinates of D (3)

3.3 If it is given that $TS = 2\sqrt{5}$, calculate the value of k. (4)

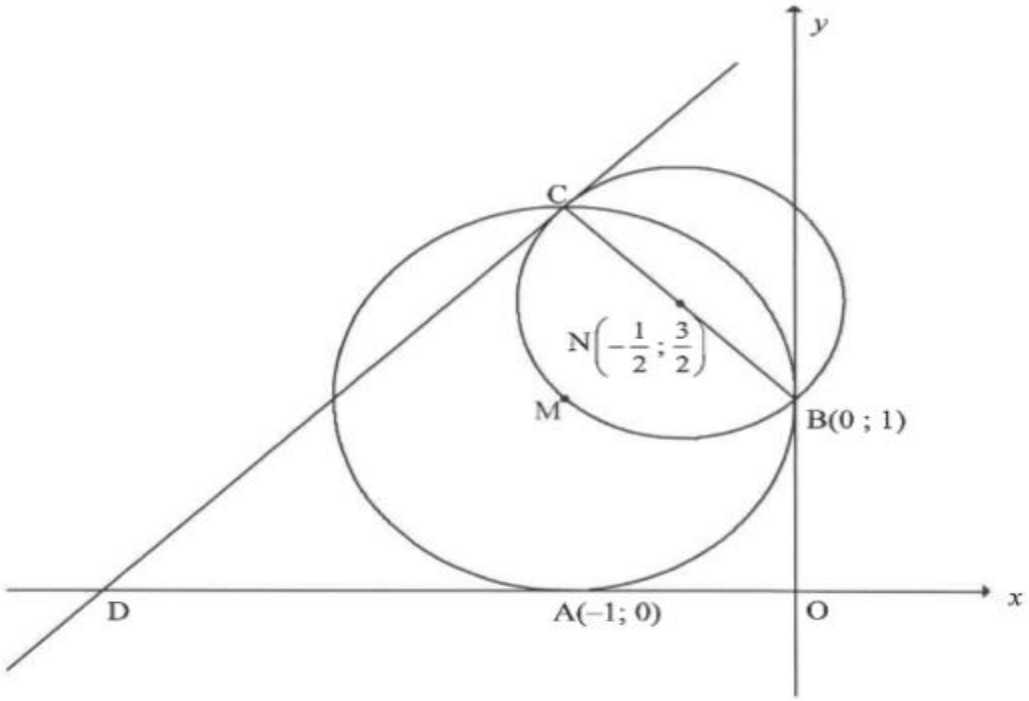
3.4 Parallelogram TDNS, with N in the 4th quadrant, is drawn. Calculate the coordinates of N. (3)

3.5 ΔPRD is reflected about the y-axis to form $\Delta P'R'D'$. Calculate the size of $R\hat{D}R'$. (3)
[19]

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QUESTION 4

In the diagram, a circle having centre M touches the x–axis at A(–1 ; 0) and the y-axis at B(0 ; 1). A smaller circle, centred at $N\left(-\frac{1}{2};\frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C. BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C, cuts the x-axis at D.



4.1 Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2 Calculate the coordinates of C. (2)

4.3 Show that the equation of the tangent CD is $y-x=3$. (4)

4.4 Determine the values of t for which the line $y=x+t$ will NOT touch or cut the smaller circle. (3)

4.5 The smaller circle centred at N is transformed such that point C is translated along the tangent to D. Calculate the coordinates of E, the new centre of the smaller circle. (3)

4.6 If it is given that the area of quadrilateral OBCD is $2a^2$ square units and $a>0$, show that $a=\frac{\sqrt{7}}{2}$ units. (5)

[20]

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QUESTION 5

5.1 Simplify the following expression to ONE trigonometric term:

$$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$$ (5)

5.2 **Without using a calculator**, determine the value of: $\frac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ (4)

5.3 Given: $\cos 26° = m$

Without using a calculator, determine $2\sin^2 77°$ in terms of m. (4)

5.4 Consider: $f(x)=\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°$

5.4.1 Determine the general solution of $f(x)=\tan 165°$ (6)

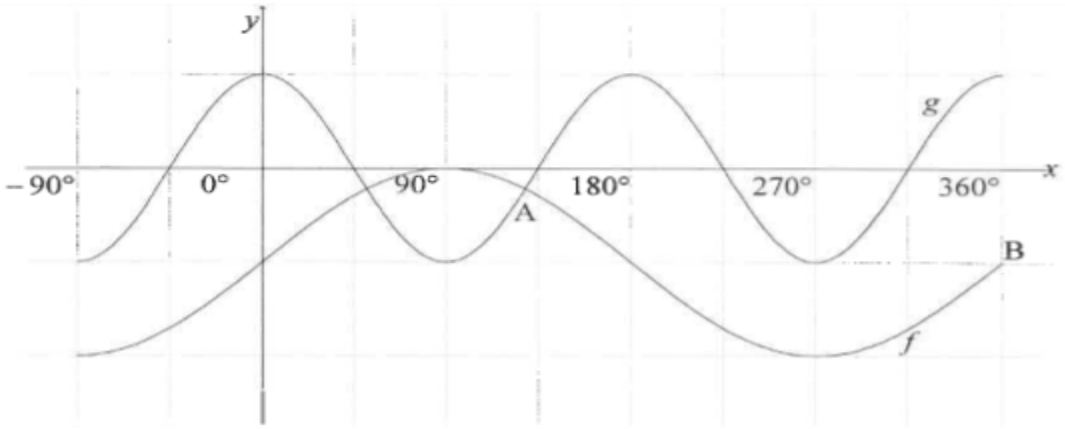
5.4.2 Determine the value(s) of x in the interval $x\in[0°;360°]$ for which $f(x)$ will have a minimum value. (3)

[22]

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QUESTION 6

In the diagram, the graphs of $f(x) = \sin x - 1$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-90°; 360°]$. Graphs f and g intersect at A. B(360° ; –1) is a point on f.



6.1 Write down the range of f. (2)

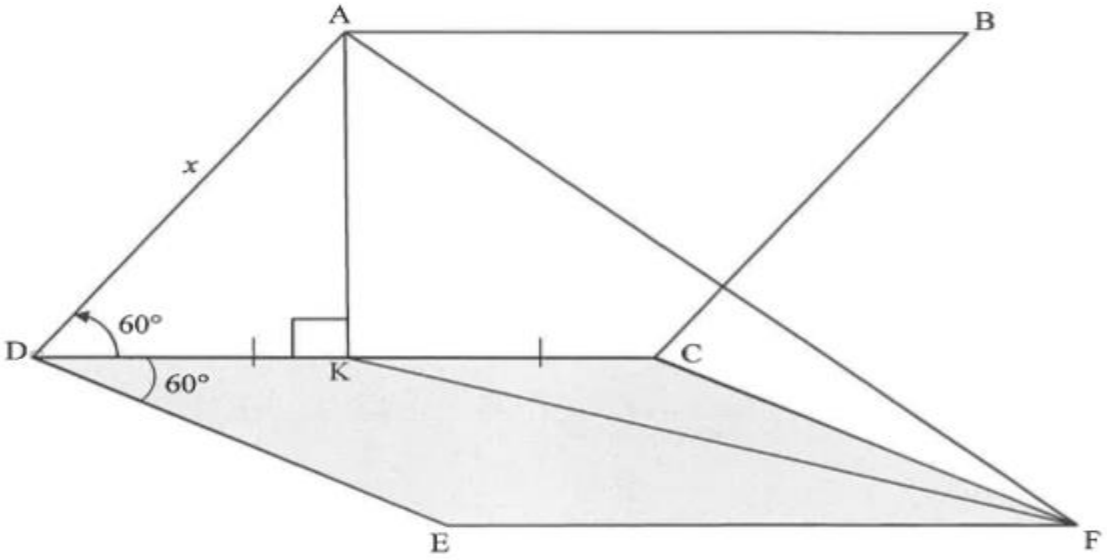
6.2 Write down the values of x in the interval $x \in [-90°; 360°]$ for which graph f is decreasing. (2)

6.3 P and Q are points on graphs g and f respectively such that PQ is parallel to the y-axis. If PQ lies between A and B, determine the value(s) of x for which PQ will be a maximum. (6)
[10]

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QUESTION 7

The diagram below shows a solar panel, ABCD, which is fixed to a flat piece of concrete slab EFCD. ABCD and EFCD are two identical rhombuses. K is a point on DC such that DK = KC and AK ⊥ DC. AF and KF are drawn. $A\hat{D}C = C\hat{D}E = 60°$ and AD = x units.



7.1 Determine AK in terms of x. (2)

7.2 Write down the size of $K\hat{C}F$. (1)

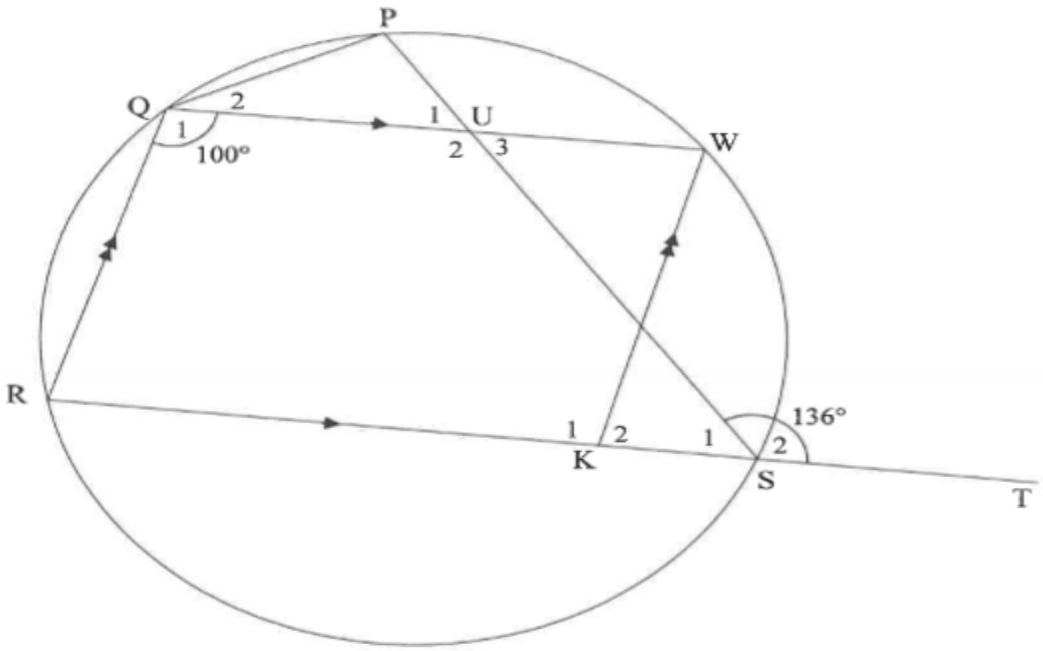
7.3 It is further given that $A\hat{K}F$, the angle between the solar panel and the concrete slab, is y. Determine the area of ΔAKF in terms of x and y. (7)

[10]

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QUESTION 8

8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.



Determine, with reasons, the size of:

8.1.1 $\hat{R}$ (2)

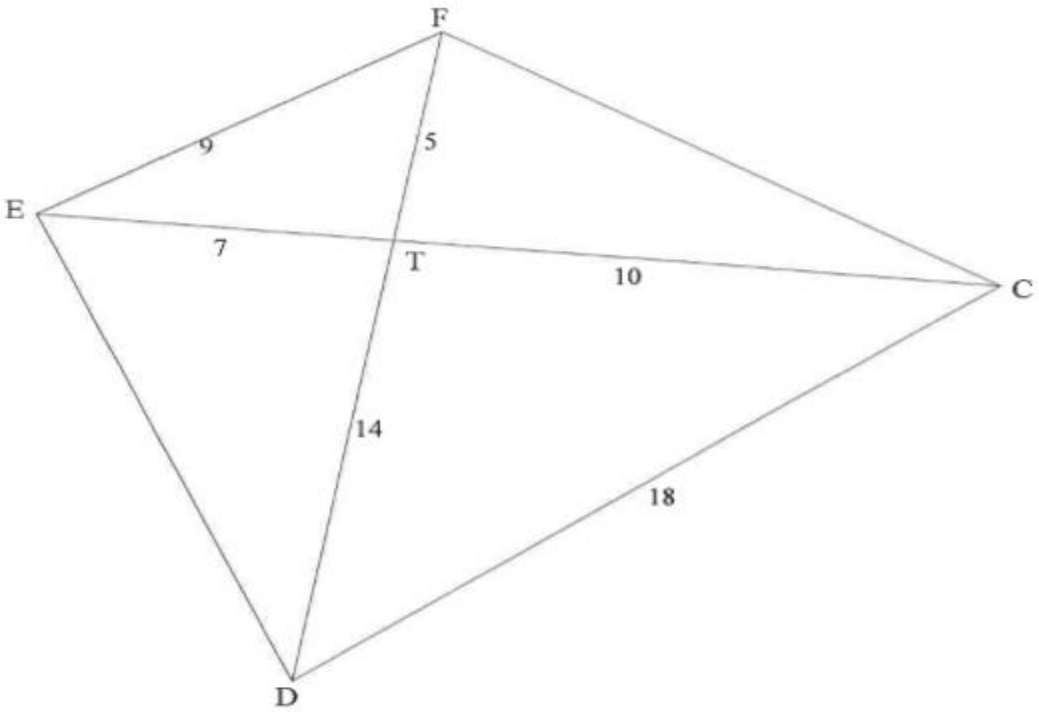
8.1.2 $\hat{P}$ (2)

8.1.3 $P\hat{Q}W$ (3)

8.1.4 $\hat{U}_2$ (2)

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8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T.
EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.



Prove, with reasons, that:

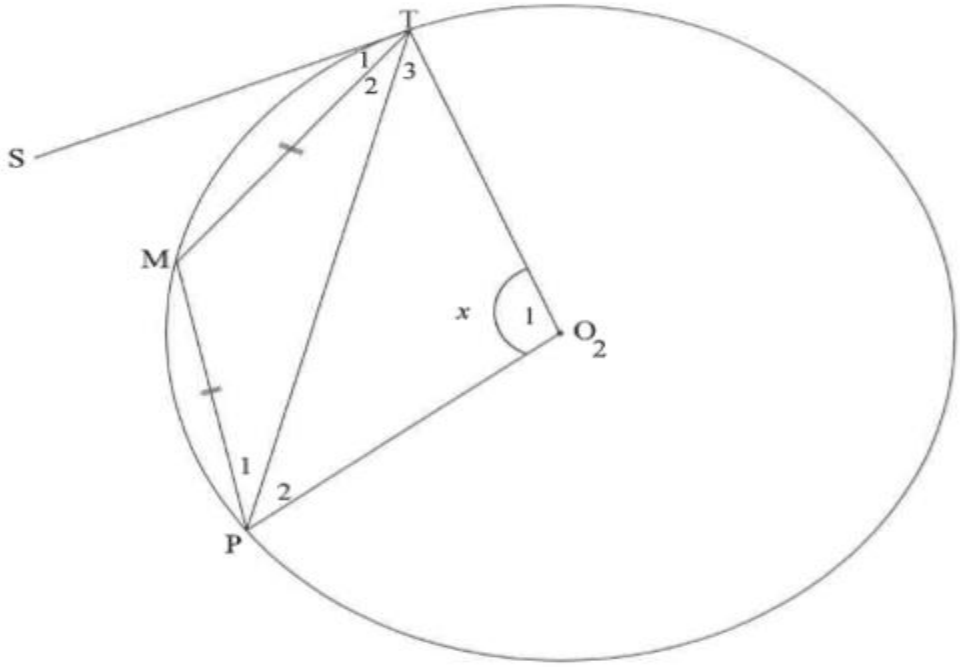
8.2.1 $E\hat{F}D = E\hat{C}D$ (4)

8.2.2 $D\hat{F}C = D\hat{E}C$ (3)
[16]

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QUESTION 9

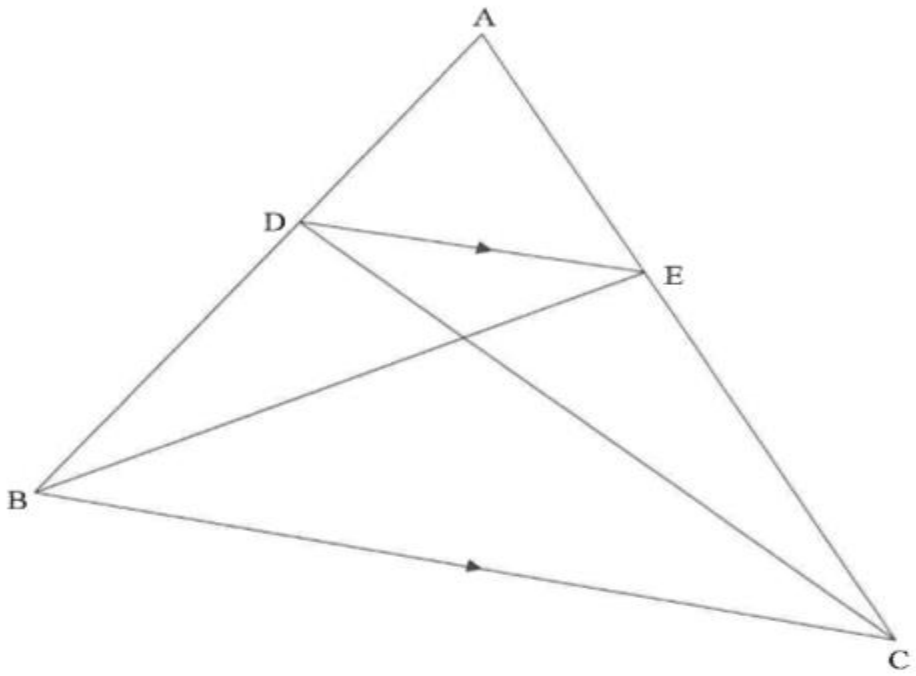
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.



Prove, with reasons, that $S\hat{T}M = \frac{1}{4}x$. [7]

QUESTION 10

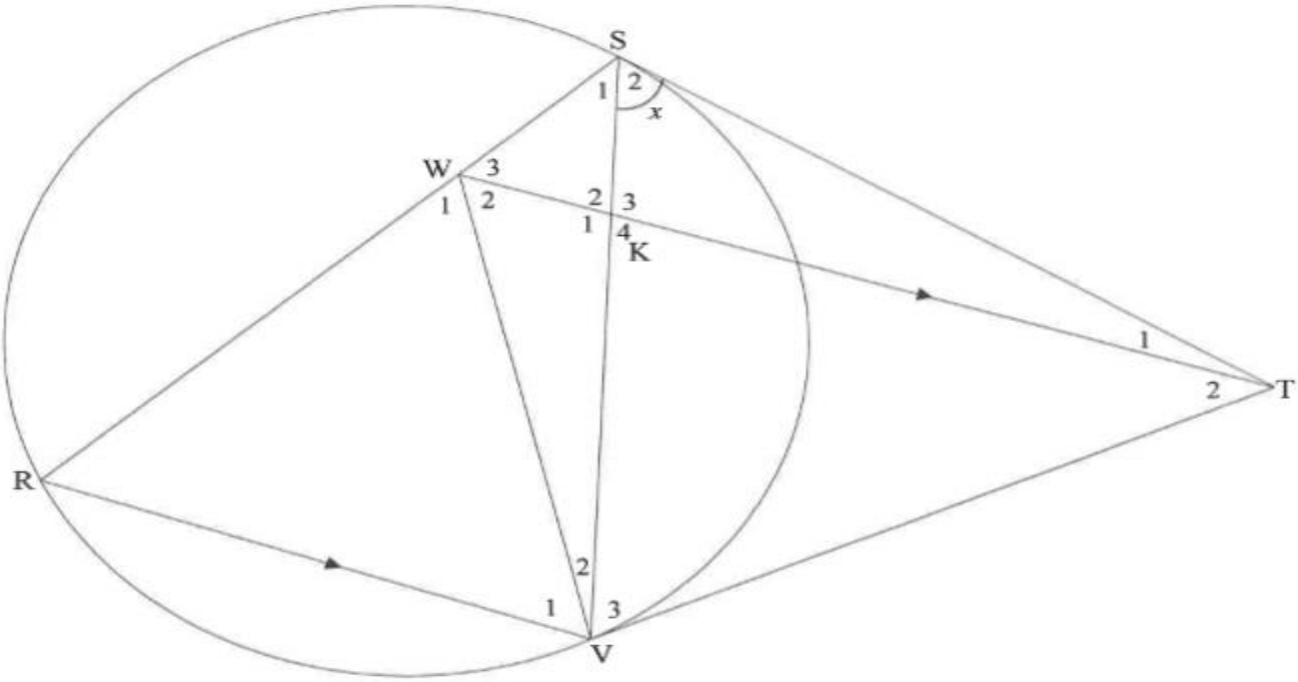
10.1 In the diagram, ΔABC is drawn. D is a point on AB and E is a point on AC such that DE || BC. BE and DC are drawn.



Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

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10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



10.2.1 Write down, with reasons, THREE other angles EACH equal to x. (6)

10.2.2 Prove, with reasons, that:

(a) WSTV is a cyclic quadrilateral (2)

(b) ΔWRV is isosceles (4)

(c) $\Delta WRV \,|||\, \Delta TSV$ (3)

(d) $\frac{RV}{SR} = \frac{KV}{TS}$ (4)

[25]

TOTAL: 150

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GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2019

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 26pages.
Hierdie nasienriglyne bestaan uit 26 bladsye.

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QUESTION/*VRAAG* 1

Monthly income (in rands) **Maandelikse inkomste (in rand)**	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) **Maandelikse paaiement (in rand)**	2 000	3 000	3 500	5 200	5 500	6 000

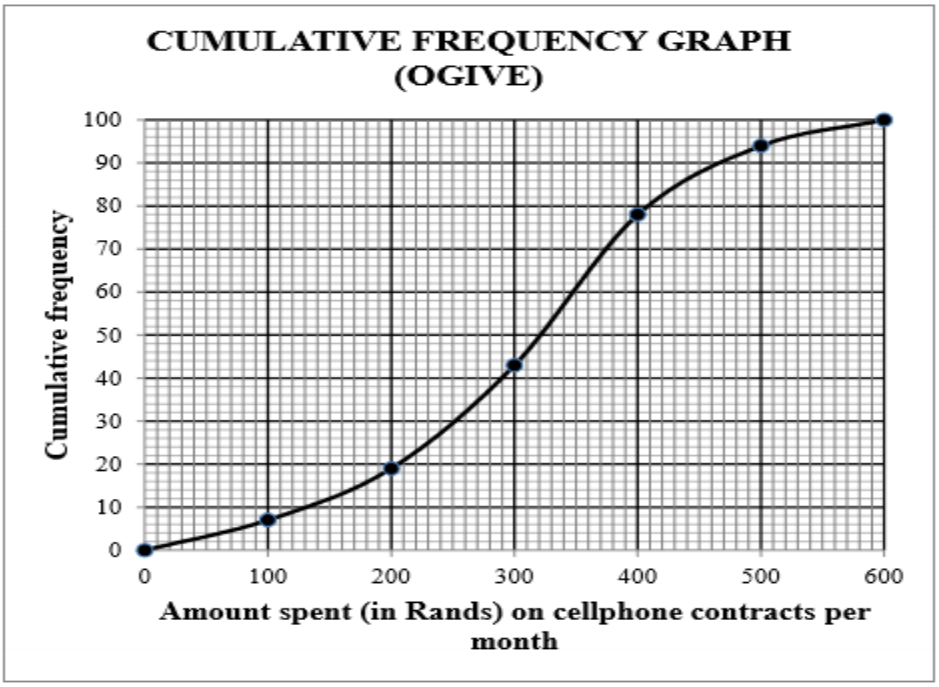
1.1	$a = -1946,875... = -1946,88$ $b = 0,41$ $\hat{y} = -1946,88 + 0,41x$ Answer only: Full marks	✓ $a = -1946,88$ ✓ $b = 0,41$ ✓ equation (3)
1.2	Monthly repayment ≈ R3 727,16 (calculator) *Maandelikse paaiement* ≈ R3 727,16 **OR** $\hat{y} = -1946,88 + 0,41(14000)$ ≈ R3 793,12	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.3	$r = 0,946$ .... $\approx 0,95$	✓ answer (1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR** D *Spandeer nie R9 000 per maand nie, want die punt (18 000 ;9 000) lê baie ver van die kleinste-kwadrate regressielyn.* ***OF*** *D*	✓✓ answer (2)
		[8]

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QUESTION/*VRAAG* 2

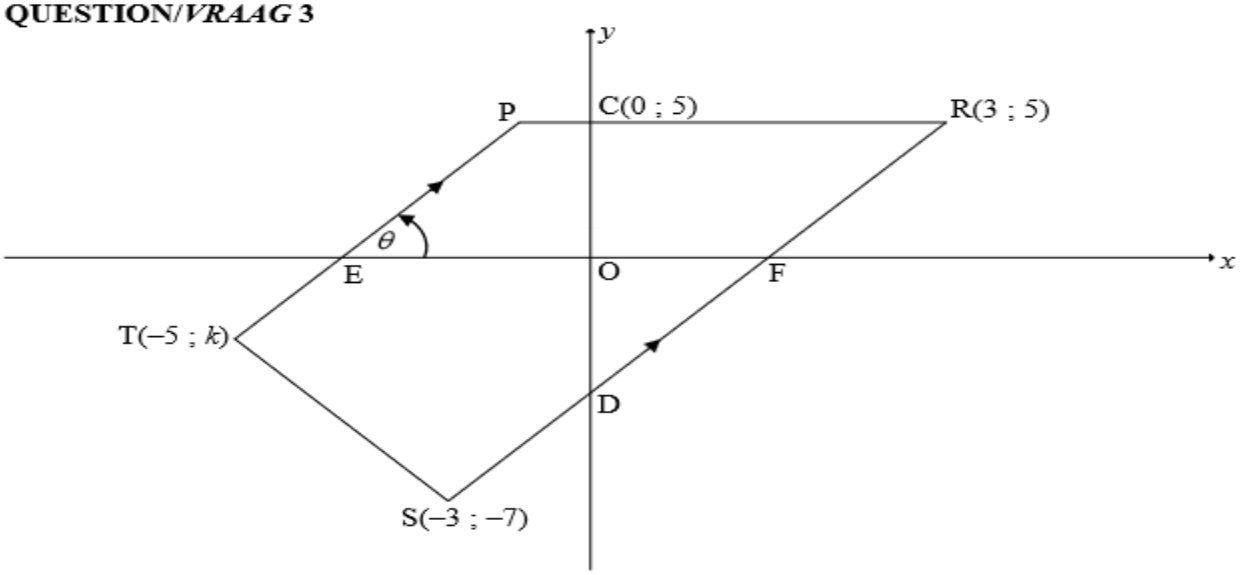
2.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*	✓ answer (1)
2.2	$7+12+a+35+b+6=100$ $a=40-b$ $309=\frac{(50\times 7)+(150\times 12)+(250\times a)+(350\times 35)+(450\times b)+(550\times 6)}{100}$ $309=\frac{(50\times 7)+(150\times 12)+(250\times (40-b))+(350\times 35)+(450\times b)+(550\times 6)}{100}$ $350+1800+10000-250b+12250+450b+3300=30900$ $200b=3200$ $b=16$ $a=24$ **OR/*OF*** $7+12+a+35+b+6=100$ $b=40-a$ $309=\frac{(50\times 7)+(150\times 12)+(250\times a)+(350\times 35)+(450\times b)+(550\times 6)}{100}$ $309=\frac{(50\times 7)+(150\times 12)+(250\times a)+(350\times 35)+(450\times (40-a))+(550\times 6)}{100}$ $350+1800+250a+12250+1800-450a=30900$ $200a=4\ 800$ $a=24$ $b=16$	✓ $\sum x=100$ ✓ $a=40-b$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n}=309$ ✓ $200b=3200$ (5) ✓ $\sum x=100$ ✓ $b=40-a$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n}=309$ ✓ $200a=4\ 800$ (5)
2.3	Modal class/*modale klas:* $300 < x \leq 400$	✓ answer (1)

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2.4		✓ grounded at (0 ; 0) ✓ (600 ; 100) ✓ cumulative frequencies for *y*-coordinates ✓ smooth shape (4)
2.5	Number of people/*Aantal mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people] *18 mense betaal meer as R420 per maand* Answer only: Full marks	✓ 82 ✓ answer (2)
		[13]

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QUESTION/*VRAAG* 3



3.1	Equation of PR: $y = 5$	✓ answer (1)
3.2.1	$m_{RS} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{RS} = \frac{5-(-7)}{3-(-3)} = \frac{12}{6}$ $= 2$ Answer only: Full marks	✓ substitution of R & S into gradient formula ✓ answer (2)
3.2.2	$m_{RS} = m_{PT}$ [PT ∥ RS] $\tan\theta = 2$ $\theta = 63{,}43°$	✓ $m_{RS} = m_{PT}$ ✓ $\tan\theta = 2$ ✓ $\theta = 63{,}43°$ (3)
3.2.3	Equation of RS: $y-5 = 2(x-3)$ or $y-(-7) = 2(x-(-3))$ or $5 = 2(3)+c$ $y-5 = 2x-6$ $y+7 = 2x+6$ $c = -1$ $y = 2x-1$ $y = 2x-1$ $y = 2x-1$ $\therefore$ D(0 ; –1) **OR/OF** $m_{RS} = m_{RD} = m_{DS}$ $2 = \frac{5-y}{3-0} = \frac{y+7}{0-(-3)}$ $\therefore y = -1$ $\therefore$ D(0 ; –1) Answer only: Full marks	✓ substitution ✓ equation of RS ✓ coordinates of D (3) ✓ equating gradients ✓ value of y ✓ coordinates of D (3)

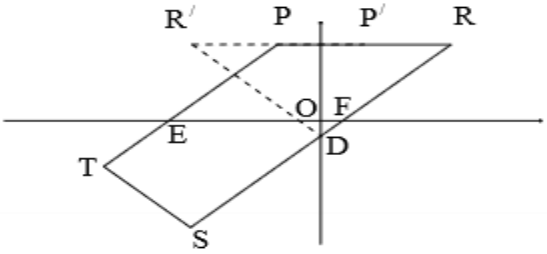
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3.3	$ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + (k+7)^2$ $(k+7)^2 = 16$ $k+7 = \pm 4$ $k = -11$ *or* $k = -3$ $\therefore k = -3$ **OR** $ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + k^2 + 14k + 49$ $k^2 + 14k + 33 = 0$ $(k+11)(k+3) = 0$ $k = -11$ *or* $k = -3$ $\therefore k = -3$	✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer (4) ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer (4)
3.4	Method: translation T→S: $(x\,;y) \rightarrow (x+2\,;y-4)$ $\therefore$ by symmetry: D→N: D(0 ; −1) → N(0 + 2 ; −1 − 4) $\therefore$ N(2 ; −5) Answer only: Full marks **OR** Midpoint of TN = Midpoint of SD $\frac{x+(-5)}{2} = \frac{-3+0}{2}$ and $\frac{y+(-3)}{2} = -\frac{-7+(-1)}{2}$ $x = 2$ and $y = -5$ $\therefore$ N(2 ; − 5) Answer only: Full marks	✓ method ✓ x-coordinate ✓ y-coordinate (3) ✓ method: midpoint of diagonals ✓ x-coordinate ✓ y-coordinate (3)

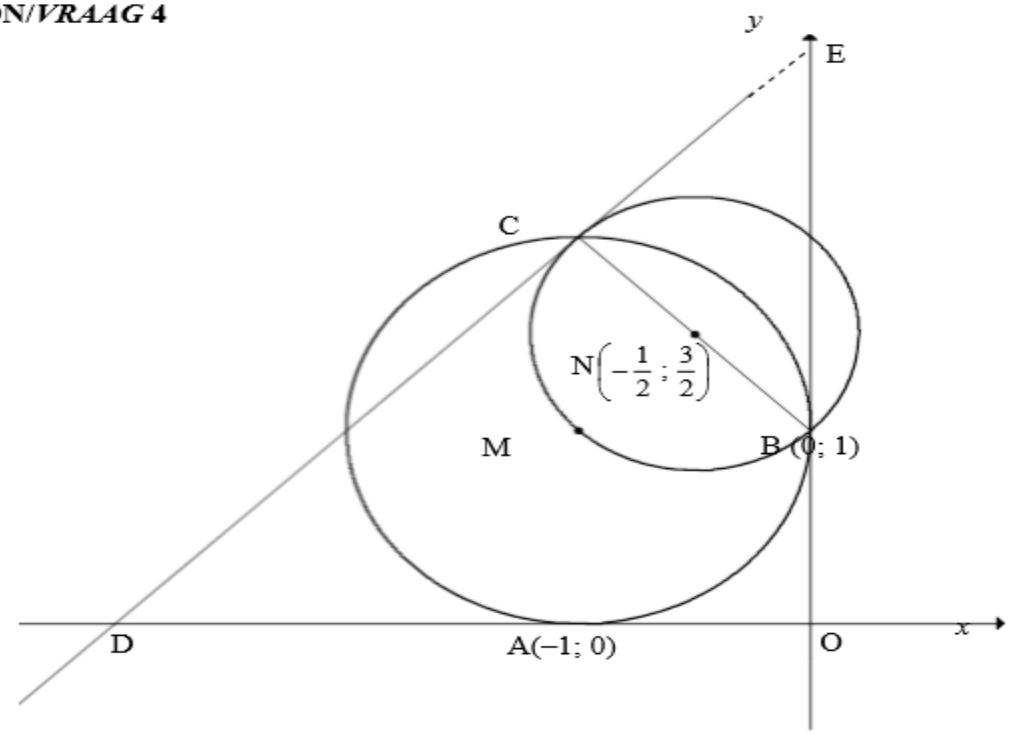
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3.5	 β is the inclination of RS $\therefore\ \beta = 63{,}434...°$ $O\hat{F}D = 63{,}434...°$ [vert opp $\angle$s] $O\hat{D}F = 90° - 63{,}434...° = 26{,}565...°$ $R\hat{D}R' = 2(26{,}565...°) = 53{,}13°$ **OR** PEFR is a $\|$m [both pairs of opp sides $\|$] $\therefore \hat{R} = \theta = 63{,}434...°$ [opp $\angle$s of $\|$m] $R\hat{R}'D = 63{,}434...°$ [$\angle$s opp = sides:RD = R/D] $R\hat{D}R' = 180° - (63{,}43° + 63{,}43°)$ [sum of $\angle$s in Δ] $R\hat{D}R' = 53{,}13°$ **OR** $\tan O\hat{D}F = \frac{3}{6}$ $O\hat{D}F = 26{,}565..°$ $R\hat{D}R' = 2(26{,}565...°) = 53{,}13°$ **OR** $R'(-3;5)$ [reflection of R(3 ; 5) about the y-axis] $RD = \sqrt{(3-0)^2 + (5-(-1)^2}$ $RD = \sqrt{45} = R'/D$ or $3\sqrt{5}$ or 6,71 $(RR')^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})(\cos R\hat{D}R')$ $6^2 = 45 + 45 - 2(45)(\cos R\hat{D}R')$ $\cos R\hat{D}R' = \frac{45+45-36}{2(45)}$ $\cos R\hat{D}R' = \frac{3}{5}$ $\therefore\ R\hat{D}R' = 53{,}13°$	✓ $\beta = 63{,}43°$ ✓ $O\hat{D}F = 26{,}57°$ ✓ answer (3) ✓ $\hat{R} = 63{,}43°$ ✓ $R\hat{R}'D = 63{,}43°$ ✓ answer (3) ✓ trig ratio ✓ $O\hat{D}F = 26{,}565..°$ ✓ answer (3) ✓ $R'(-3;5)$ **OR** $RD = \sqrt{45} = R'/D$ ✓ substitution into cosine rule ✓ answer (3)
		[19]

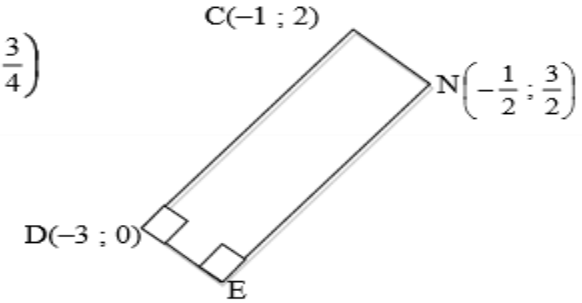
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QUESTION/*VRAAG* 4



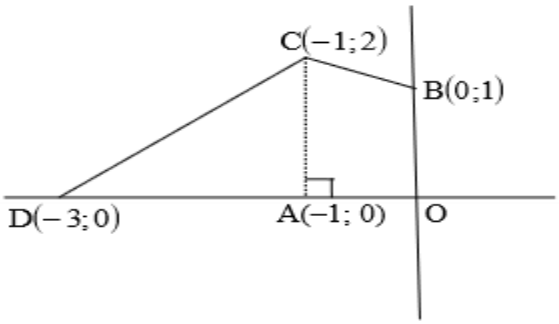
4.1	M(−1;1) $(x+1)^2+(y-1)^2=1$ Answer only: Full marks	✓M(−1;1) ✓LHS ✓ RHS (3)
4.2	Midpoint of CB, N: (− 0,5 ; 1,5) $\therefore \frac{x_C+0}{2}=-\frac{1}{2}$ and $\frac{y_C+1}{2}=\frac{3}{2}$ Answer only: Full marks ∴ C(− 1 ; 2) **OR** B→N: $(x\,;y)\rightarrow(x-0{,}5\,;y+0{,}5)$ N→C: $(x\,;y)\rightarrow(x-0{,}5\,;y+0{,}5)$ Answer only: Full marks ∴ C(− 0,5 − 0,5 ; 1,5 + 0,5) ∴ C(− 1 ; 2)	✓*x* value ✓ *y* value (2) ✓*x* value ✓ *y* value (2)

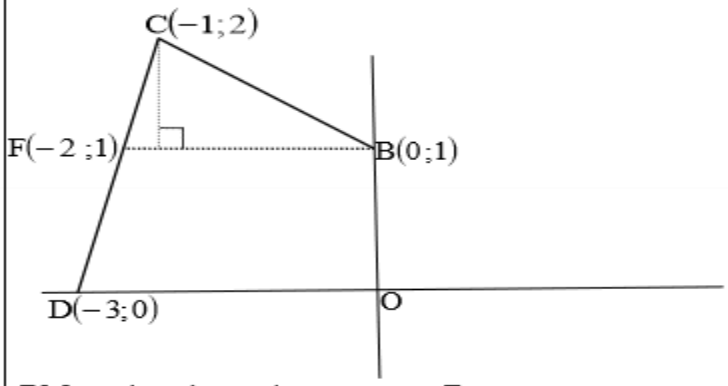
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4.3	$m_{\text{radius}} = \frac{2-1}{-1-0}$ OR $\frac{2-(-\frac{1}{2})}{-1-\frac{3}{2}}$ OR $\frac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$ **OR** $m_{\text{radius}} = \frac{2-1}{-1-0}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y - y_1 = m(x - x_1)$ $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y - 2 = x + 1$ $\therefore y = x + 3$ $y - x = 3$	✓ m_{radius} ✓ m_{tangent} ✓ substitute $(-1 ; 2)$ and m ✓ simplification (4) ✓ m_{radius} ✓ m_{tangent} ✓ substitute $(-1 ; 2)$ and m ✓ simplification (4)
4.4	Tangents to circle: $y = x + 3$ and $y = x + 1$ $\therefore t > 3$ or $t < 1$ Answers only: Full marks	✓ $y = x + 1$ ✓ $t > 3$ ✓ $t < 1$ (3)
4.5	Draw rectangle CNED: Midpt of DN $\left(-\frac{7}{4} ; \frac{3}{4}\right)$ $\therefore$ E$(-\frac{5}{2} ; -\frac{1}{2})$  **OR/*OF*** D(– 3 ; 0) C→N: $(x ; y) \rightarrow (x + 0{,}5 ; y - 0{,}5)$ D→E: D $(x ; y) \rightarrow$ E$(x + 0{,}5 ; y - 0{,}5)$ $\therefore$ E$(-3 + 0{,}5 ; 0 - 0{,}5)$ $\therefore$ E$(-2{,}5 ; -0{,}5)$ Answer only: Full marks	✓ midpt of DN ✓x value ✓ y value (3) ✓ coordinates of D ✓x value ✓ y value (3)

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4.6	 area of trapezium AOBC $= \frac{1}{2}(1+2)(1)$ $= 1\frac{1}{2}$ square units area of $\Delta ACD = \frac{1}{2}(2)(2)$ $= 2$ square units area of quadrilateral OBCD $= 3\frac{1}{2}$ square units $\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$ **OR**	✓ substitution into area of trapezium form ✓ area of trapezium ✓ area of triangle ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)



BM produced cuts the tangent at F.

area of $\Delta CFB = \frac{1}{2}(2)(1)$

$= 1$ square unit

area of trapezium BFDO $= \frac{1}{2}(2+3)(1)$

$= 2\frac{1}{2}$ square units

area of quadrilateral OBCD $= 3\frac{1}{2}$ square units

$\therefore 2a^2 = \frac{7}{2}$

$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$

✓ area of triangle

✓ substitution into area of trapezium

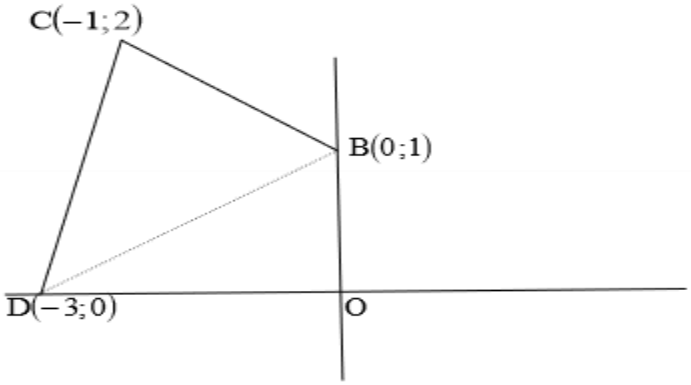
✓ area of trapezium

✓ area of OBCD

✓ equating area OBCD to $2a^2$

(5)

OR



Join DB

area of $\Delta ODB = \frac{1}{2}(3)(1)$

$= \frac{3}{2}$ square unit

area of $\Delta DCB = \frac{1}{2}(2\sqrt{2})(\sqrt{2})$

$= 2$ square unit

$\therefore$ area of OBCD $= \frac{3}{2} + 2 =$ square units

$2a^2 = \frac{7}{2}$

$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$

OR

✓ area of Δ

✓ subst into area of Δ

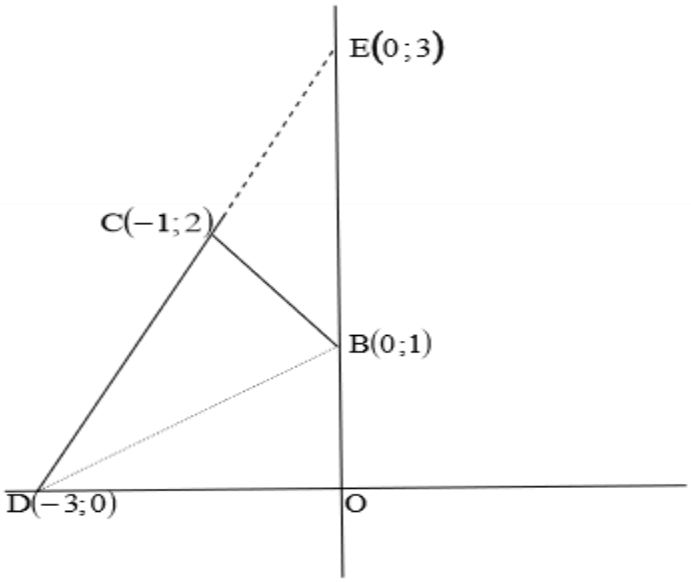
✓ area of Δ
✓ area of OBCD

✓ equating area OBCD to $2a^2$

(5)

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Let E be the point of intersection of DC with the positive y–axis.

area of $\Delta DEO = \frac{1}{2}(3)(3)$

$= \frac{9}{2}$ square unit

area of $\Delta ECB = \frac{1}{2}(2)(1)$ or $\frac{1}{2}(\sqrt{2})(\sqrt{2})$

$= 1$ square unit

area of quadrilateral OBCD $= \frac{9}{2} - 1 = 3\frac{1}{2}$ square units

$\therefore 2a^2 = \frac{7}{2}$

$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$

✓ area of Δ

✓ subst into area of Δ

✓ area of Δ

✓ area of OBCD

✓ equating area OBCD to $2a^2$

(5)

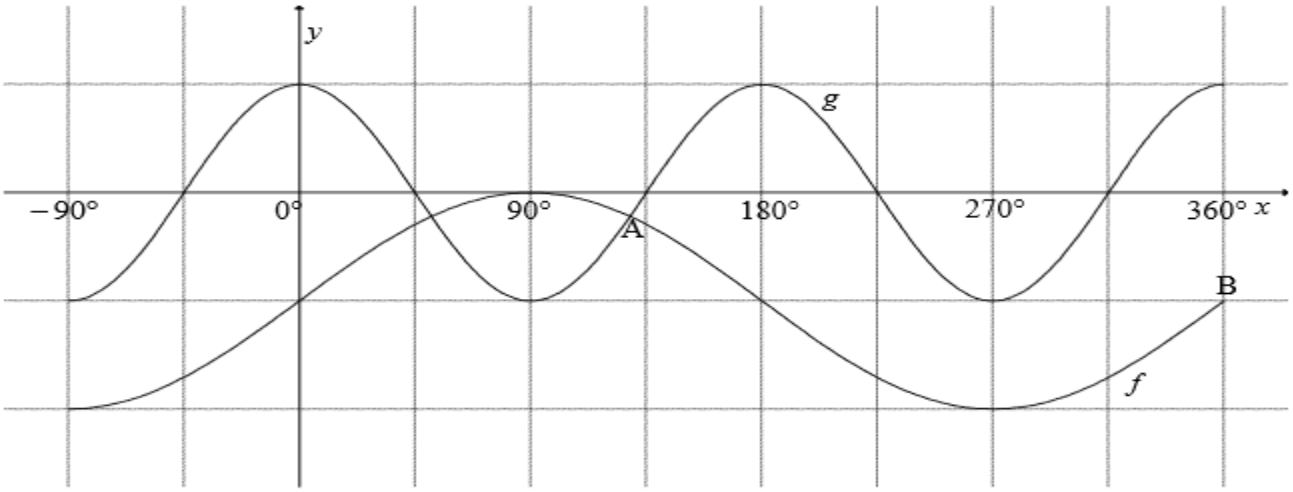
[20]

QUESTION/*VRAAG* 5

5.1	$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$ $=\frac{\sin x}{\cos x.\frac{\sin x}{\cos x}}+(-\sin x)\sin x$ $=1-\sin^2 x$ $=\cos^2 x$	✓ $-\sin x$ ✓ $\sin x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $1-\sin^2 x$ ✓ $\cos^2 x$ (5)
5.2	$\frac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ $=\frac{-\left(\cos^2 35°-\sin^2 35°\right)}{2(2\sin 10°\cos 10°)}$ $=\frac{-\cos 70°}{2\sin 20°}$ $=\frac{-\cos 70°}{2\cos 70°}$ **OR** $=\frac{-\sin 20°}{2\sin 20°}=-\frac{1}{2}$	✓ $-\left(\cos^2 35°-\sin^2 35°\right)$ ✓$-\cos 70°$ ✓ $2\sin 20°$ ✓ answer (4)
5.3	$2\sin^2 77°=2[\sin(90°-13°)]^2$ $=2\cos^2 13°$ $=2\cos^2 13°-1+1$ $=\cos 26°+1$ $=m+1$ **OR** $1-2\sin^2 77°=\cos 154°$ $2\sin^2 77°=1-\cos 154°$ $=1-(-\cos 26°)$ $=1+m$	✓ using co-ratio ✓ reduction ✓ $2\cos^2 13°-1=\cos 26°$ ✓ answer (4) ✓ $1-2\sin^2 77°=\cos 154°$ ✓ $2\sin^2 77°=1-\cos 154°$ ✓ reduction ✓ answer (4)
5.4.1	$\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°=\tan 165°$ $\sin(x+25°-15°)=-0,2679...\ OR\ -2+\sqrt{3}$ $\sin(x+10°)=-0,2679...OR\ -2+\sqrt{3}$ $x+10°=195,54°+k.360°$ or $x+10°=344,46°+k.360°$ $x=185,54°+k.360°;\ k\in Z$ or $x=334,46°+k.360°;k\in Z$ **OR/*OF***	✓✓ $\sin(x+10°)$ ✓ $-0,2679...$ ✓ $195,54°$ & $344,46°$ ✓ $185,54°$ & $334,46°$ ✓ $+k.360°;\ k\in Z$ (6)

	$\sin(x+25°)\sin 75° - \cos(x+25°)\cos 75° = \tan 165°$ $-(\cos(x+25°)\cos 75° - \sin(x+25°)\sin 75°) = -0,2679...$ $\cos(x+100°) = 0,2679...$ ref. $\angle = 74.4577...°$ $x+100° = 74,46° + k.360°$ *or* $x+100° = 285,54° + k.360°$ $x = -25,54° + k.360°;\ k \in Z$ or $x = 185,54° + k.360°;\ k \in Z$	✓✓ $\cos(x+100°)$ ✓ $-0,2679...$ ✓ $74,46°$ & $285,54°$ ✓ $-25,54°$ & $185,54°$ ✓ $+k.360°;\ k \in Z$ (6)
5.4.2	$f(x) = \sin(x+10°)$ **Answers only: Full marks** For minimum value of $\sin x$: $x = 270°$ For minimum value of $\sin(x+10°)$: $x = 260°$	✓ $f(x) = \sin(x+10°)$ ✓ $270°$ ✓ answer (3)
		[22]

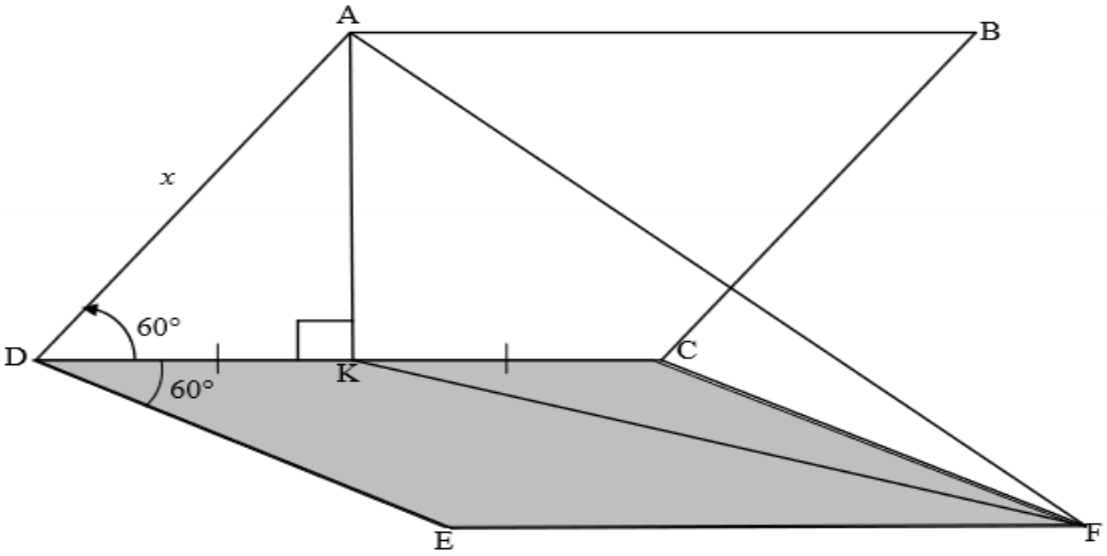
QUESTION/*VRAAG* 6



6.1	Range of f: $y \in [-2\,;0]$ **OR** $-2 \le y \le 0$	✓ critical values ✓ notation (2)
6.2	$x \in (90° \,;\, 270°)$ OR $x \in [90° \,;\, 270°]$	✓ critical values ✓ notation (2)
6.3	$PQ = \cos 2x - (\sin x - 1)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ $\sin x = -\frac{b}{2a}$ $= \frac{-(-1)}{2(-2)}$ $\sin x = -\frac{1}{4}$ $\therefore x = 194{,}48°$ or $x = 345{,}52°$	✓ $PQ = \cos 2x - (\sin x - 1)$ ✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ substitution into formula ✓ $\sin x = -\frac{1}{4}$ ✓ $194{,}48°$ ✓ $345{,}52°$ (6)
		[10]

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QUESTION/*VRAAG* 7

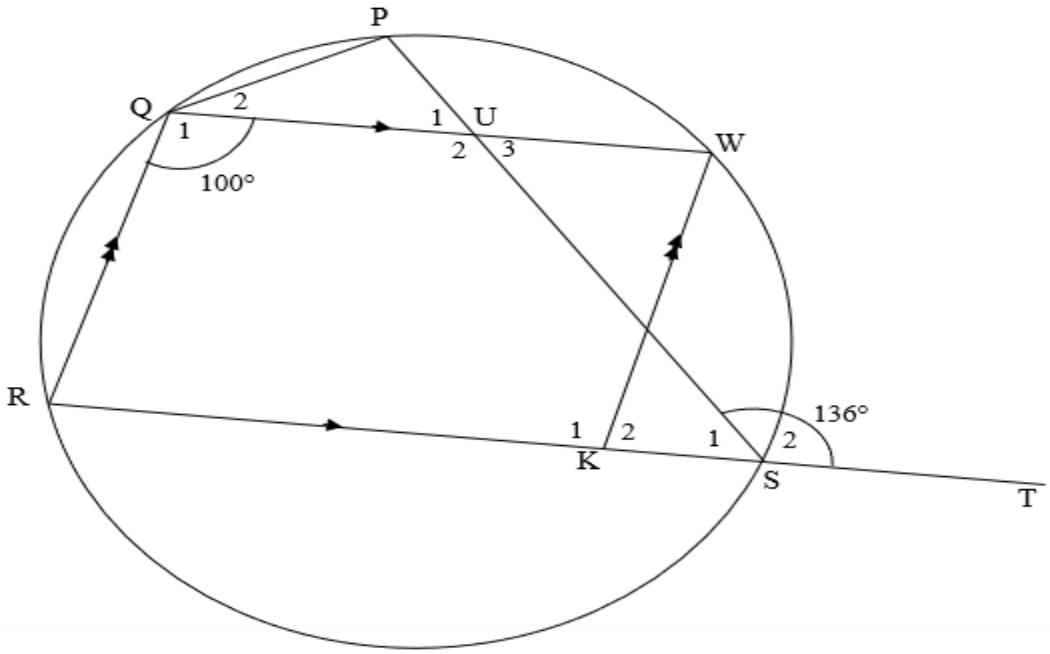


7.1	$\sin 60° = \frac{AK}{x}$ $AK = x\sin 60°$ or $\frac{\sqrt{3}}{2}x$ or $0{,}866x$	✓ trig ratio ✓ answer (2)
7.2	$K\hat{C}F = 120°$	✓ answer (1)
7.3	$KF^2 = CF^2 + CK^2 - 2CF.CK\cos K\hat{C}F$ $= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right)\cos 120°$ $= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$ $= \frac{7x^2}{4}$ $KF = \frac{\sqrt{7}x}{2}$ $A\hat{K}F = y$ Area $\Delta AKF = \frac{1}{2}.AK.KF\sin A\hat{K}F$ $= \frac{1}{2}.\frac{\sqrt{3}x}{2}.\frac{\sqrt{7}x}{2}\sin y$ $= \frac{x^2\sqrt{21}\sin y}{8}$	✓ correct use of cosine rule ✓ substitution ✓ $\cos 120° = -\frac{1}{2}$ ✓ $KF = \frac{\sqrt{7}x}{2}$ ✓ correct use of area rule ✓ substitution ✓ answer in terms of x and y (7)
		[10]

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QUESTION/*VRAAG* 8



8.1.1	$\hat{R} = 80°$ [co-int ∠s/*ko-binne ∠e*; QW ∥ RK]	✓S ✓R (2)
8.1.2	$\hat{P} = 100°$ [opp ∠s of cyclic quad/*teenoorst ∠e v koordevh*]	✓S ✓R (2)
8.1.3	$P\hat{Q}R = 136°$ [ext ∠ of cyclic quad/*buite ∠ v koordevh*] $\hat{Q}_2 = 36°$ **OR** $P\hat{U}W = \hat{S}_2 = 136°$ [corresp ∠s/*ooreenkomstige ∠e*; QW ∥ RK] $P\hat{Q}W + \hat{P} = P\hat{U}W$ [ext ∠s of/*buite ∠ van* Δ QPU] $P\hat{Q}W + 100° = 136°$ $P\hat{Q}W = 36°$ **OR** $\hat{U}_3 = 180° - 136° = 44°$ [co-int ∠s/*ko-binne ∠e*; QW ∥ RK] $\hat{U}_1 = \hat{U}_3 = 44°$ [vert opp ∠s/*regoorstaande ∠e*] $P\hat{Q}W = 180° - (100 + 44°)$ [sum of ∠s in Δ/*som ∠e van* Δ] $P\hat{Q}W = 36°$	✓S ✓R ✓S (3) ✓S ✓R ✓S (3) ✓S ✓R ✓S (3)

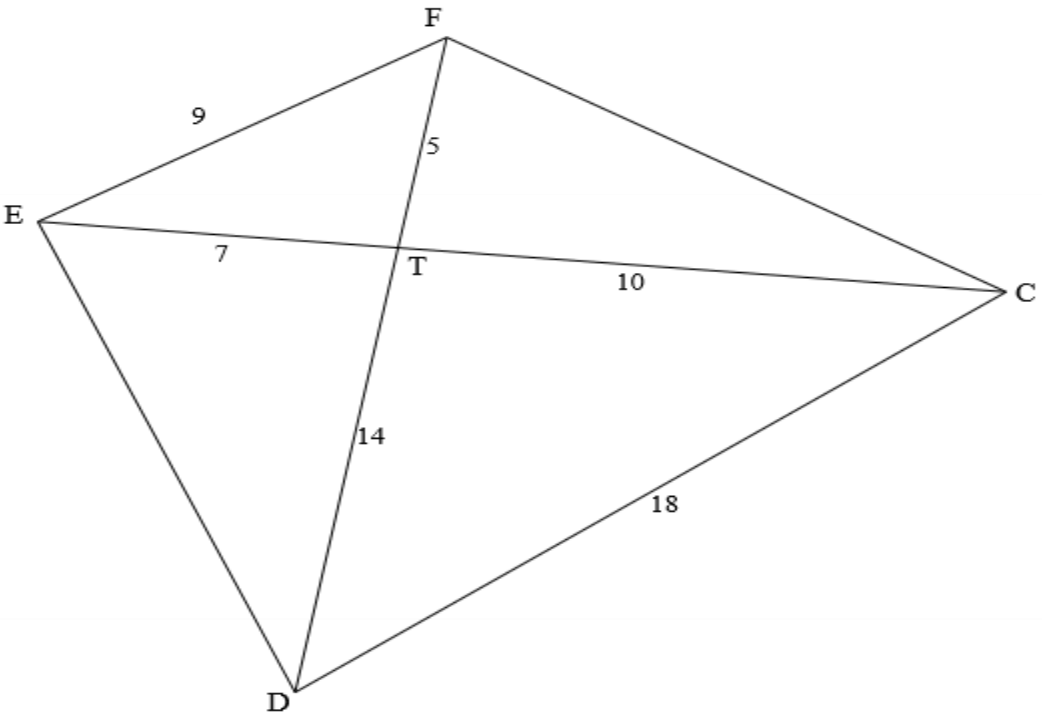
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8.1.4	$\hat{U}_2 = \hat{S}_2 = 136°$	[alt ∠s/*verwiss ∠e* ; QW ∥ RK]	✓S ✓R (2)
	OR		
	$\hat{U}_2 = 100° + 36°$ $= 136°$	[ext ∠s of/*buite ∠ van* ΔQPU]	✓S ✓R (2)
	OR		
	$\hat{U}_2 = P\hat{U}W = 136°$	[vert opp ∠s/*regoorstaande ∠e*]	✓S ✓R (2)
	OR		
	$\hat{U}_2 = 180° - \hat{U}_3$ $= 180° - 44°$ $= 136°$	[∠s on a str line/*∠e op reguitlyn*]	✓S ✓R (2)

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8.2



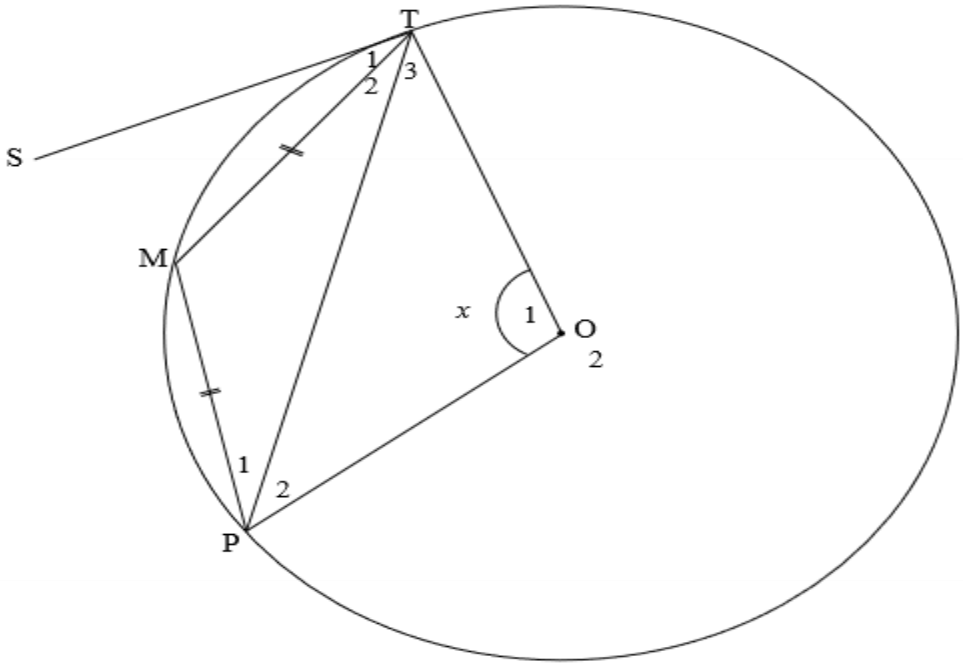
8.2.1	In ΔEFT and ΔDCT: $\frac{EF}{CD}=\frac{9}{18}=\frac{1}{2}$ $\frac{FT}{TC}=\frac{5}{10}=\frac{1}{2}$ $\frac{ET}{TD}=\frac{7}{14}=\frac{1}{2}$ $\therefore \Delta EFT \text{ ||| } \Delta DCT$ [Sides of Δ in prop/ *sye van Δ in dieselfde verh*] $\therefore E\hat{F}D = E\hat{C}D$ **OR** In ΔFET: $49 = 25 + 81 - 2(5)(9)\cos\hat{F}$ $\cos\hat{F} = \frac{19}{30}$ $\hat{F} = 50{,}7°$ In ΔTDC: $196 = 100 + 256 - 2(10)(18)\cos\hat{C}$ $\cos\hat{C} = \frac{19}{30}$ $\hat{C} = 50{,}7°$	✓✓ all 3 ratios $= \frac{1}{2}$ ✓ ΔEFT ||| ΔDCT ✓ R (4) ✓✓ $\hat{F} = 50{,}7°$ ✓✓ $\hat{C} = 50{,}7°$ (4)

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8.2.2	$E\hat{F}D = E\hat{C}D$ [proved in 8.2.1] E, F, C and D are concyclic EFCD is a cyclic quad [converse ∠s in the same segment/ *omgekeerde ∠e in dies segment*] $\therefore D\hat{F}C = D\hat{E}C$ [∠s in the same segment/*∠e in dies segment*]	✓S ✓R ✓ R (3)
		[16]

QUESTION/*VRAAG* 9

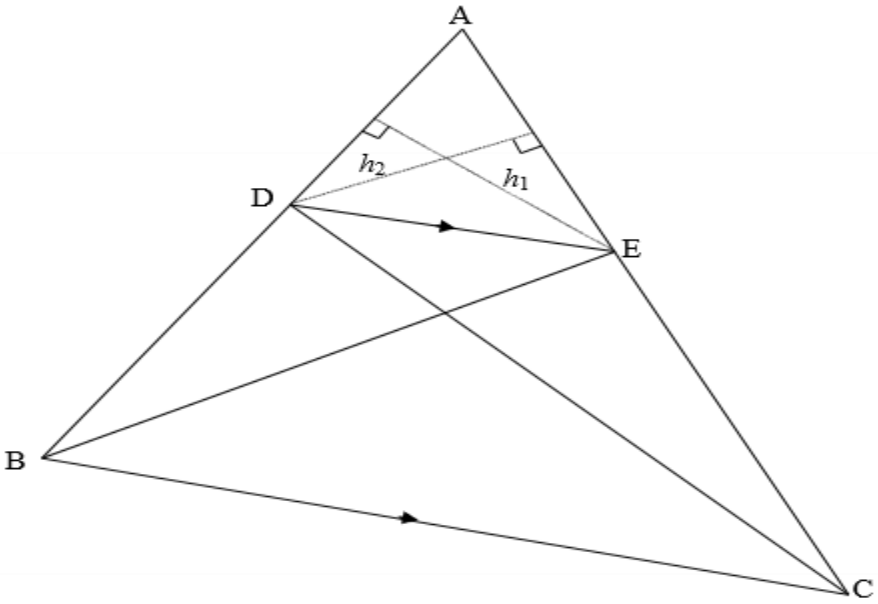


	$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om 'n punt*]	✓ $\hat{O}_2 = 360° - x$
	$\therefore \hat{M} = 180° - \frac{1}{2}x$ [∠ at centre = 2 ×∠ at circumference/ *middelpunts∠ = 2 × omtreks∠*]	✓ $\hat{M} = 180° - \frac{1}{2}x$ ✓R
	$\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of ∠s in Δ/*som ∠e van Δ*]	✓ $\hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$
	$\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	✓ $\hat{P}_1 = \frac{1}{4}x$ ✓R
	$\therefore S\hat{T}M = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]	✓R (7)
	OR/*OF*	
	$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om 'n punt*]	✓ $\hat{O}_2 = 360° - x$
	$\therefore \hat{M} = \frac{1}{2}\hat{O}_2$ [∠ at centre = 2 ×∠ at circumference]	✓ S ✓R
	$\therefore \hat{T}_2 + \hat{P}_1 = 180° - \hat{M}$ [sum of ∠s in Δ/*som ∠e van Δ*]	✓S
	$\therefore \hat{T}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	✓R
	$= \frac{180° - \hat{M}}{2} = \frac{180° - \frac{1}{2}\hat{O}_2}{2} = \frac{180° - \frac{1}{2}(360° - x)}{2} = \frac{1}{4}x$	✓ S
	$\therefore S\hat{T}M = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]	✓R (7)
		[7]

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QUESTION/*VRAAG* 10

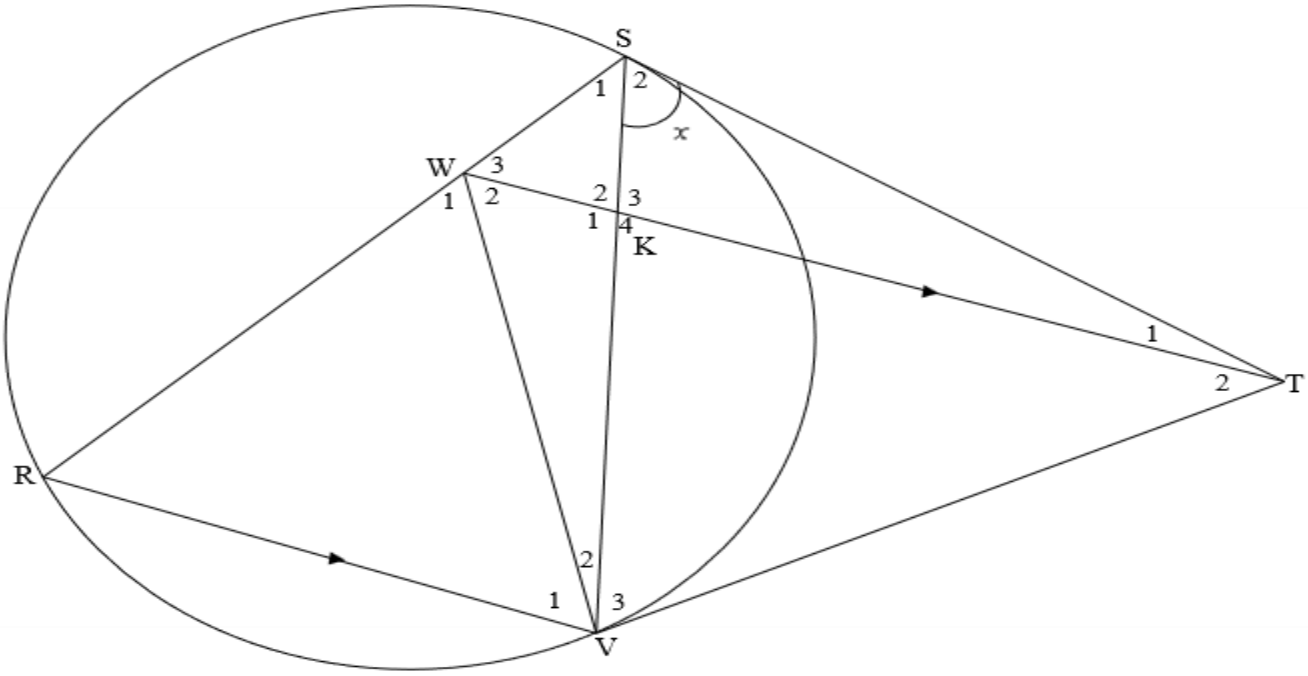
10.1



10.1	Constr: Draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE *Konstr: Trek* h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE Proof/*Bewys*: $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$ But area ΔBDE = area ΔDEC [same base & height or DE $\parallel$ BC/ *dies basis & hoogte*; of DE $\parallel$ BC] $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ constr/*konstr* **OR** **reason: common vertex or same height** ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1}$ ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{AE}{EC}$ ✓ S ✓R ✓ S (6)

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10.2



10.2.1	$\hat{V}_3 = x$ [Tans from same point/*raaklyne vanaf dieselfde pt*] $\hat{R} = x$ [tan chord theorem/*raaklyn koordstelling*] $\hat{W}_3 = x$ [corresp ∠s/*ooreenkomstige ∠e*; WT ∥ RV]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R (6)
10.2.2(a)	$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1] W, S, T and V are concyclic/*is konsiklies* WSTV is a cyclic quad [converse ∠s in the same segment/ *Omgekeerde ∠e in dieselfde segment*]	✓ S ✓ R (2)
10.2.2(b)	$\hat{W}_2 = \hat{S}_2 = x$ [∠s in the same segment/∠e *in dies segment*] $\hat{V}_1 = \hat{W}_2 = x$ [alt ∠s/*verwiss ∠e* ; WT ∥ RV] But $\hat{R} = x$ [proved in 10.2.1] $\therefore \hat{R} = \hat{V}_1 = x$ $\therefore$ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*] ΔWRV is isosceles/*is gelykbenig* **OR/*OF***	✓ S ✓R ✓ S/ R ✓ S (4)

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	$\hat{S}_2 = \hat{W}_2 = x$ [∠s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext ∠ of Δ] $\therefore\ \hat{V}_1 = x = \hat{R}$ $\therefore$ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*] ΔWRV is isosceles/*is gelykbenig*	✓ S ✓R ✓ S/ R ✓ S (4)
10.2.2(c)	In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore$ ΔWRV ||| ΔTSV [∠, ∠, ∠] **OR/*OF*** In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = S\hat{T}V = x$ [sum of ∠s in Δ/*∠e van* Δ] $\therefore$ ΔWRV ||| ΔTSV	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [ΔWRV ||| ΔTSV] $\therefore$ WR × SV = RV × TS $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/*eweredighst*; WT ‖ RV] $\therefore$ WR × SV = KV × SR $\therefore$ RV × TS = KV × SR $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ **OR/*OF*** In ΔRVS and/*en* ΔVKT $S\hat{V}R = \hat{K}_4$ [alt ∠s, WT ‖ RV] $S\hat{R}V = \hat{V}_3$ [proven] ΔRVS ||| ΔVKT [∠, ∠, ∠] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but VT = ST [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$	✓ correct ratios ✓ $\frac{WR}{SR} = \frac{KV}{SV}$ ✓ R ✓ equating WR × SV (4) ✓ identifying correct Δs ✓ proving ||| ✓ correct ratio ✓ S (4)
		[25]

TOTAL/*TOTAAL*: 150

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Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/ NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2020

MARKS: 150

TIME: 3 hours

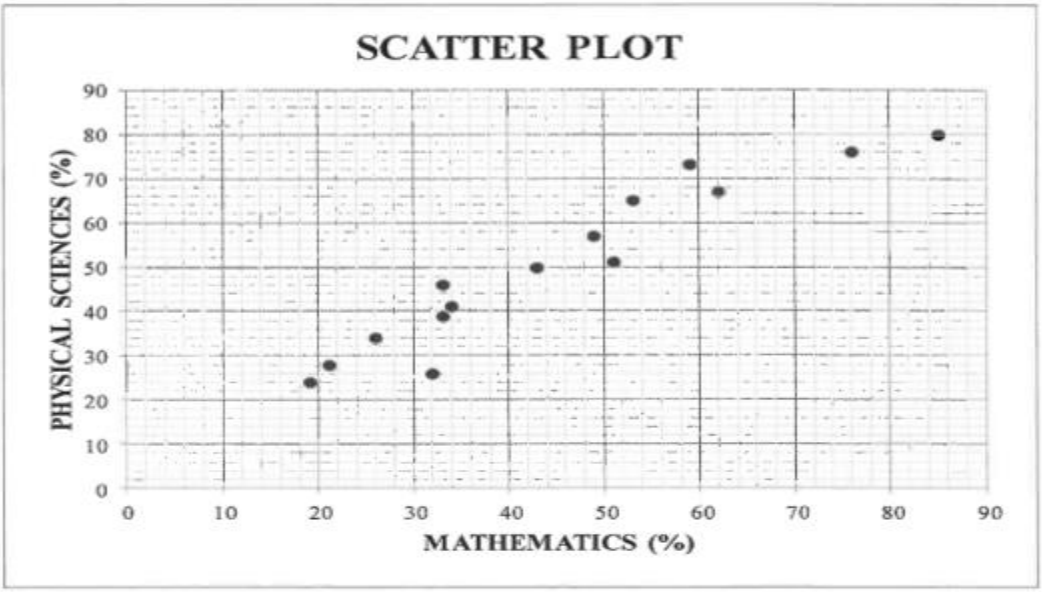
This question paper consists of 14 pages, 1 information sheet and an answer book of 24 pages.

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QUESTION 1

A Mathematics teacher was curious to establish if her learners' Mathematics marks influenced their Physical Sciences marks. In the table below, the Mathematics and Physical Sciences marks of 15 learners in her class are given as percentages (%).

MATHEMATICS (AS %)	26	62	21	33	53	76	32	59	43	33	49	51	19	34	85
PHYSICAL SCIENCES (AS %)	34	67	28	46	65	76	26	73	50	39	57	51	24	41	80



1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 Draw the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)

1.3 Predict the Physical Sciences mark of a learner who achieved 69% for Mathematics. (2)

1.4 Write down the correlation coefficient between the Mathematics and Physical Sciences marks for the data. (1)

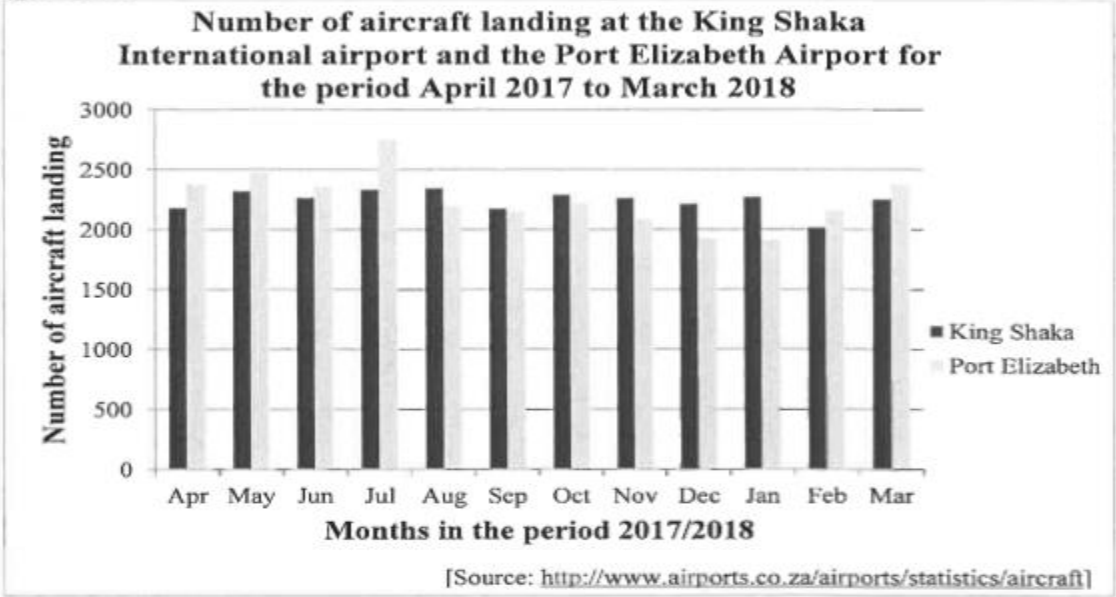
1.5 Comment on the strength of the correlation between the Mathematics and Physical Sciences marks for the data. (1)

1.6 What trend did the teacher observe between the results of the two subjects? (1)
[10]

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QUESTION 2

The number of aircraft landing at the King Shaka International Airport and the Port Elizabeth Airport for the period starting in April 2017 and ending in March 2018, is shown in the double bar graph below.



[Source: http://www.airports.co.za/airports/statistics/aircraft]

2.1 The number of aircraft landing at the Port Elizabeth Airport exceeds the number of aircraft landing at the King Shaka International Airport during some months of the given period. During which month is this difference the greatest? (1)

2.2 The number of aircraft landing at the King Shaka International Airport during these months are:

2 182	2 323	2 267	2 334	2 346	2 175
2 293	2 263	2 215	2 271	2 018	2 254

Calculate the mean for the data. (2)

2.3 Calculate the standard deviation for the number of aircraft landing at the King Shaka International Airport for the given period. (2)

2.4 Determine the number of months in which the number of aircraft landing at the King Shaka International Airport were within one standard deviation of the mean. (3)

2.5 Which ONE of the following statements is CORRECT?

A. During December and January, there were more landings at the Port Elizabeth Airport than at the King Shaka International Airport.

B. There was a greater variation in the number of aircraft landing at the King Shaka International Airport than at the Port Elizabeth Airport for the given period.

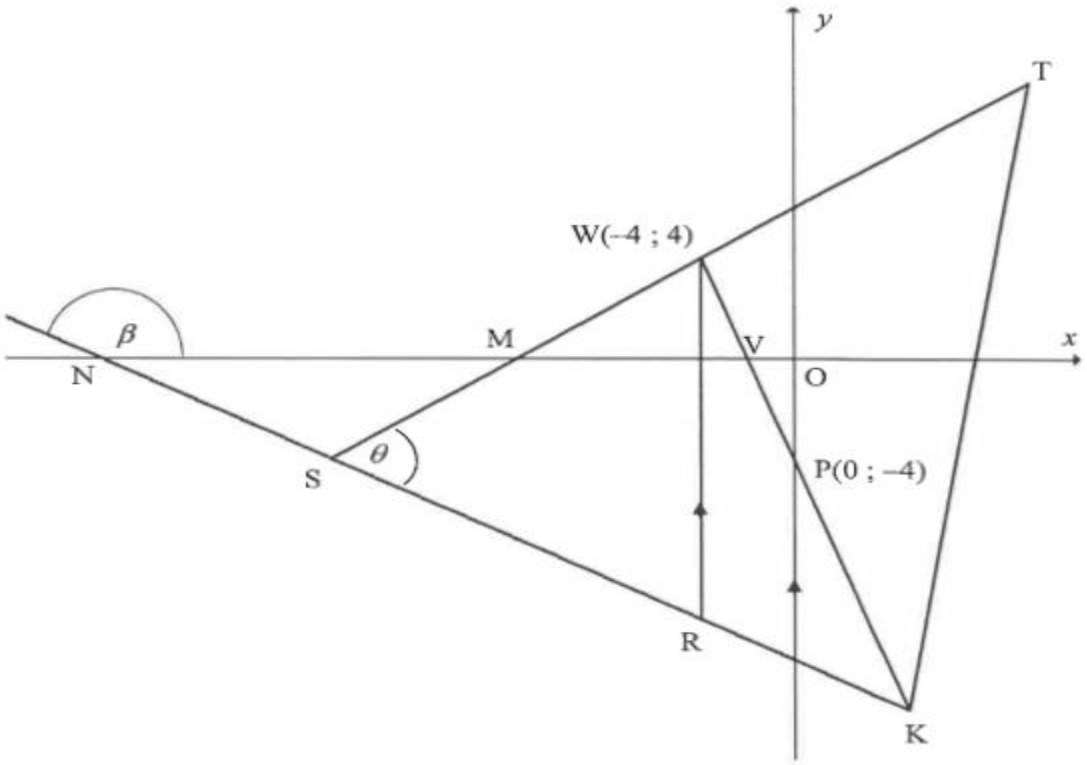
C. The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of landings at the King Shaka International Airport. (1)

[9]

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QUESTION 3

ΔTSK is drawn. The equation of ST is $y = \frac{1}{2}x + 6$ and ST cuts the x-axis at M. W(–4 ; 4) lies on ST and R lies on SK such that WR is parallel to the y-axis. WK cuts the x-axis at V and the y-axis at P(0 ; –4). KS produced cuts the x-axis at N. $T\hat{S}K = \theta$.



3.1 Calculate the gradient of WP. (2)

3.2 Show that WP ⊥ ST. (2)

3.3 If the equation of SK is given as $5y + 2x + 60 = 0$, calculate the coordinates of S. (4)

3.4 Calculate the length of WR. (4)

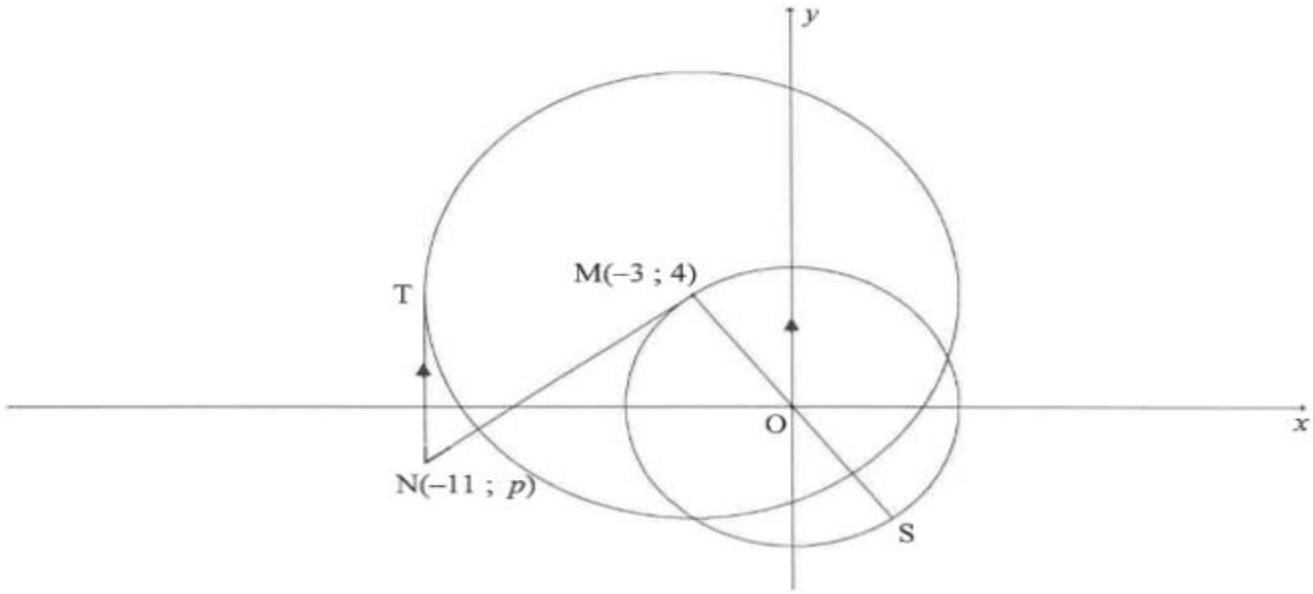
3.5 Calculate the size of θ. (5)

3.6 Let L be a point in the third quadrant such that SWRL, in that order, forms a parallelogram. Calculate the area of SWRL. (4)
[21]

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QUESTION 4

M(–3 ; 4) is the centre of the large circle and a point on the small circle having centre O(0; 0). From N(–11 ; p), a tangent is drawn to touch the large circle at T with NT is parallel to the y-axis. NM is a tangent to the smaller circle at M with MOS a diameter.



4.1 Determine the equation of the small circle. (2)

4.2 Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.3 Determine the equation of NM in the form $y=mx+c$ (4)

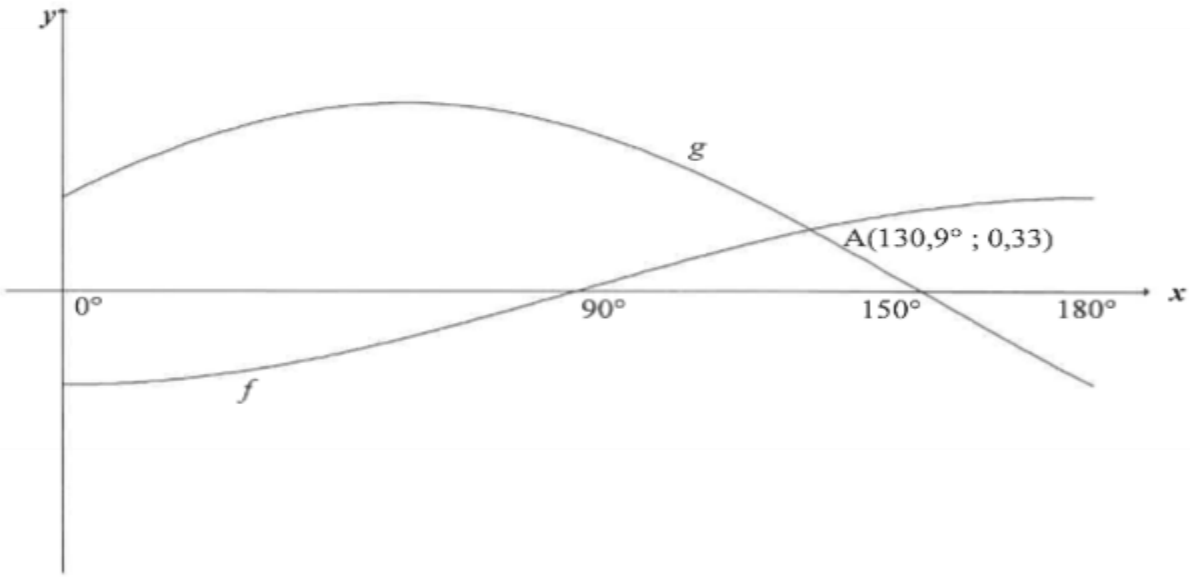
4.4 Calculate the length of SN. (5)

4.5 If another circle with centre B(–2 ; 5) and radius k touches the circle centred at M, determine the value(s) of k, correct to ONE decimal place. (5)
[19]

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QUESTION 5

The graphs of $f(x)=-\frac{1}{2}\cos x$ and $g(x)=\sin(x+30°)$, for the interval $x\in[0°;180°]$, are drawn below. A(130,9° ; 0,33) is the approximate point of intersection of the two graphs.



5.1 Write down the period of g. (1)

5.2 Write down the amplitude of f (1)

5.3 Determine the value of $f(180°)-g(180°)$ (1)

5.4 Use the graphs to determine the values of x, in the interval $x\in[0°\ ;\ 180°]$, for which:

5.4.1 $f(x-10°)=g(x-10°)$ (1)

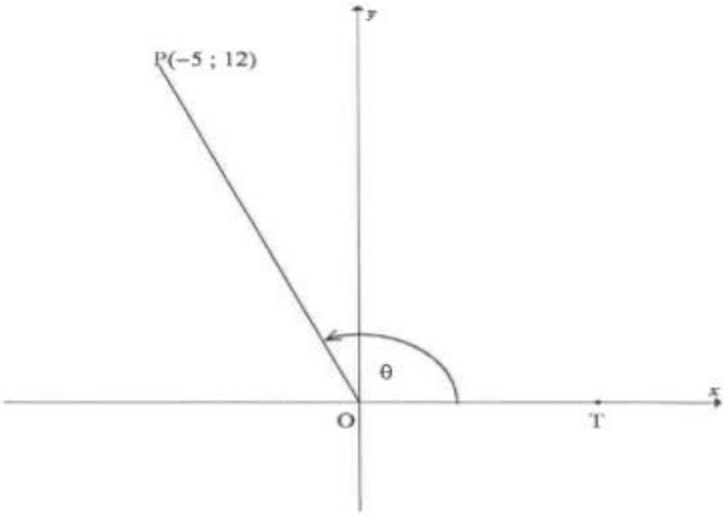
5.4.2 $\sqrt{3}\sin x+\cos x\geq 1$ (4)

[8]

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QUESTION 6

6.1 In the diagram, P(–5 ; 12) and T lies on the positive x-axis. $P\hat{O}T = \theta$



Answer the following **without using a calculator**:

6.1.1 Write down the value of $\tan\theta$ (1)

6.1.2 Calculate the value of $\cos\theta$ (3)

6.1.3 S(a ; b) is a point in the third quadrant such that $T\hat{O}S = \theta + 90°$ and OS = 6,5 units. Calculate the value of b. (4)

6.2 Determine, **without using a calculator**, the value of the following trigonometric expression:

$$\frac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$$ (5)

6.3 Determine the general solution of the following equation:

$$6\sin^2 x + 7\cos x - 3 = 0$$ (6)

6.4 Given: $x + \frac{1}{x} = 3\cos A$ and $x^2 + \frac{1}{x^2} = 2$

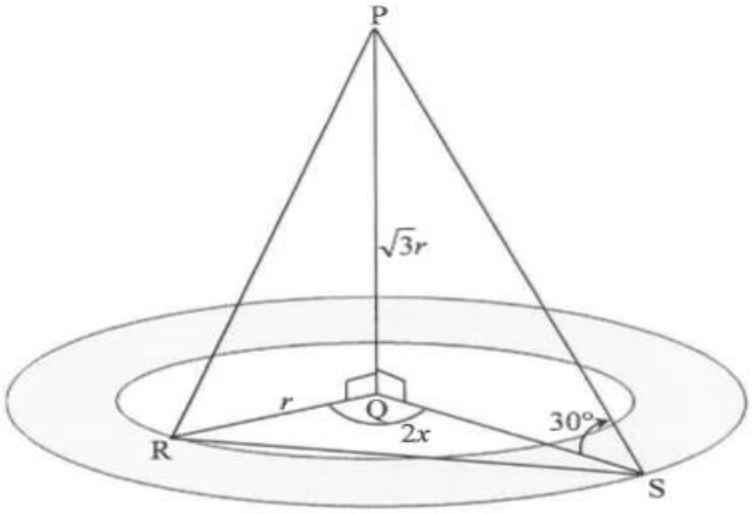
Determine the value of $\cos 2A$ **without using a calculator**. (5)
[24]

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QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ. R is a point on the inner circle and S is a point on the outer circle. R, Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

- The radius of the inner circle is r units and the radius of the outer circle is QS.
- The angle of elevation from S to P is 30°.
- $R\hat{Q}S = 2x$ and $PQ = \sqrt{3}r$



7.1 Show that $QS = 3r$ (3)

7.2 Determine, in terms of r, the area of the flower garden. (2)

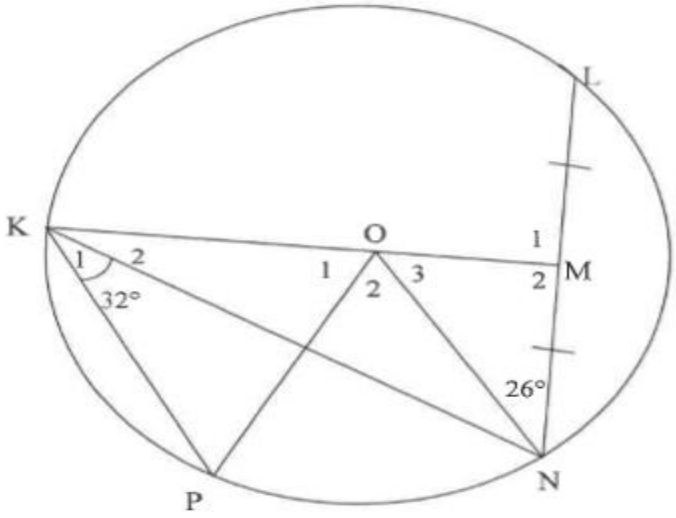
7.3 Show that $RS = r\sqrt{10 - 6\cos 2x}$ (3)

7.4 If $r = 10$ metres and $x = 56°$, calculate RS. (2)
[10]

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QUESTION 8

8.1 O is the centre of the circle.. KOM bisects chord LN and $M\hat{N}O = 26°$. K and P are points on the circle with $N\hat{K}P = 32°$. OP is drawn.



8.1.1 Determine, giving reasons, the size of:

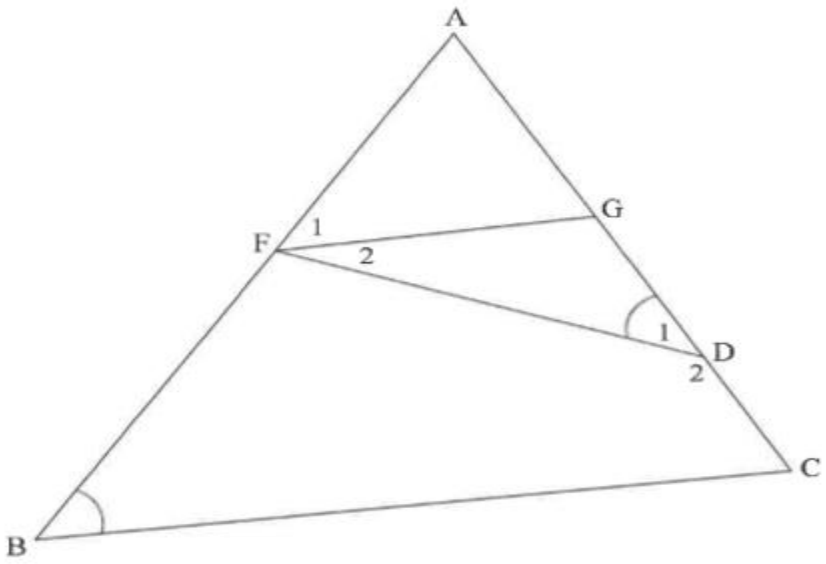
(a) $\hat{O}_2$ (2)

(b) $\hat{O}_1$ (4)

8.1.2 Prove, giving reasons, that KN bisects $O\hat{K}P$. (3)

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8.2 In ΔABC, F and G are points on sides AB and AC respectively. D is a point on GC such that $\hat{D}_1 = \hat{B}$.



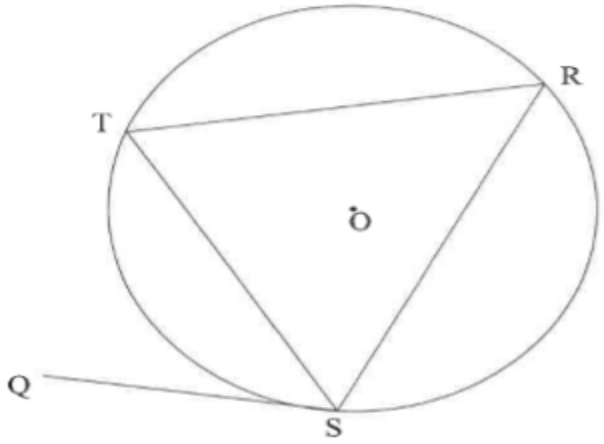
8.2.1 If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that FG || BC. (4)

8.2.2 If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x. (4)
[17]

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QUESTION 9

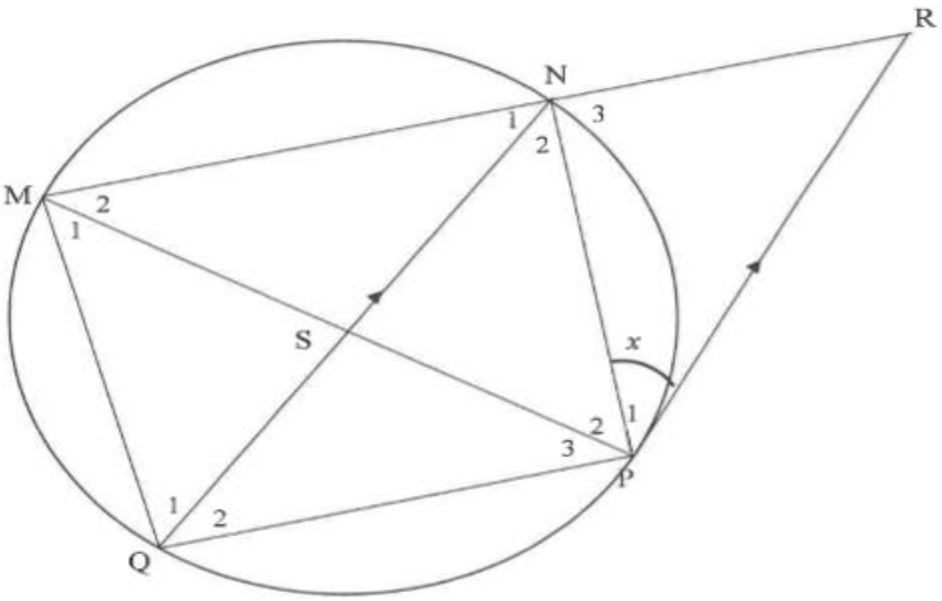
9.1 In the diagram, O is the centre of the circle. Points S, T and R lie on the circle. Chords ST, SR and TR are drawn in the circle. QS is a tangent to the circle at S.



Use the diagram to prove the theorem which states that $Q\hat{S}T = \hat{R}$. (5)

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9.2 Chord QN bisects $M\hat{N}P$ and intersects chord MP at S. The tangent at P meets MN produced at R such that QN || PR. Let $\hat{P}_1 = x$.



9.2.1 Determine the following angles in terms of x. Give reasons

(a) $\hat{N}_2$ (2)

(b) $\hat{Q}_2$ (2)

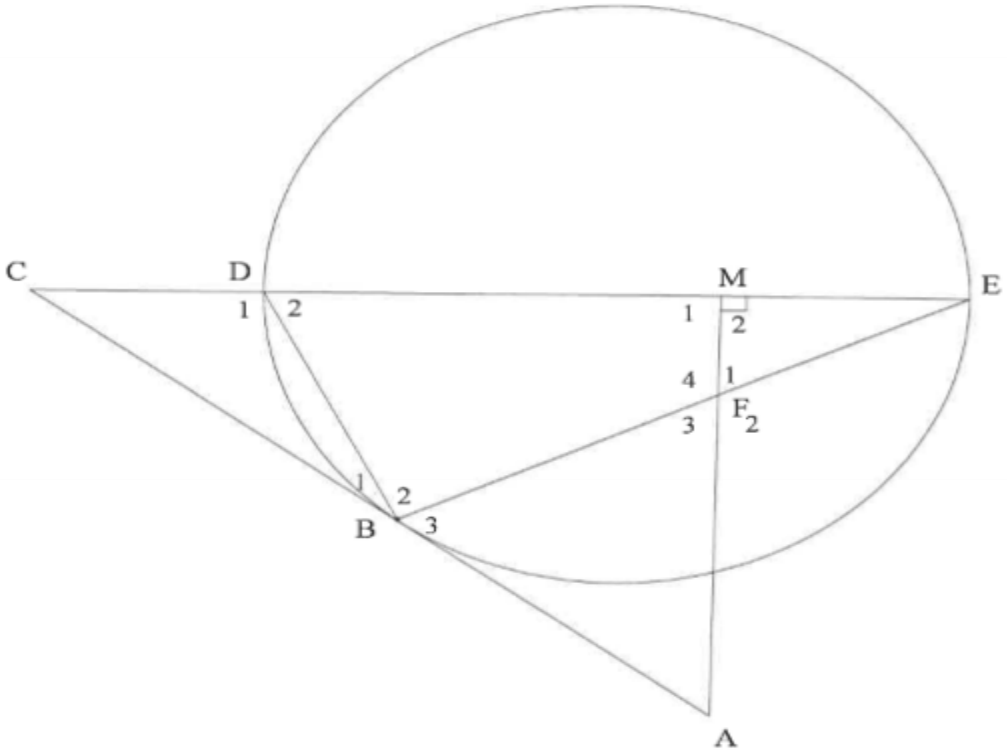
9.2.2 Prove, giving reasons, that $\frac{MN}{NR} = \frac{MS}{SQ}$ (6)

[15]

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QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that AM ⊥ DE. AM and chord BE intersect at F.



10.1 Prove, giving reasons, that:

10.1.1 FBDM is a cyclic quadrilateral (3)

10.1.2 $\hat{B}_3 = \hat{F}_1$ (4)

10.1.3 $\Delta CDB \,|||\, \Delta CBE$ (3)

10.2 If it is further given that CD = 2 units and DE = 6 units, calculate the length of:

10.2.1 BC (3)

10.2.2 DB (4)
[17]

TOTAL: 150

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GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2020

MARKING GUIDELINES/*NASIENRIGLYNE*

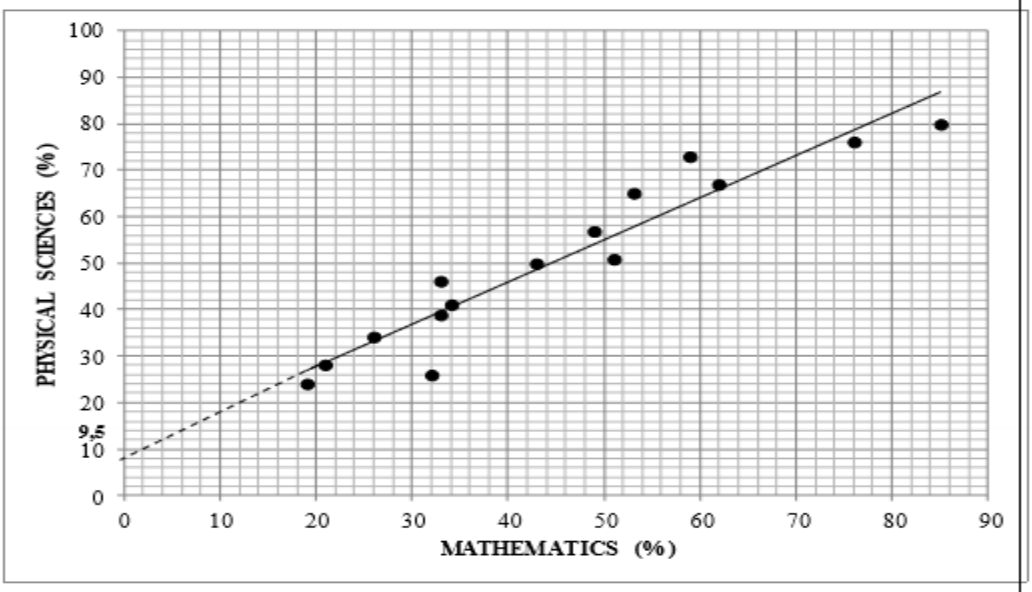
MARKS/*PUNTE*: 150

These marking guidelines consist of 27 pages.
Hierdie nasienriglyne bestaan uit 27 bladsye.

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QUESTION/*VRAAG* 1

1.1	$a = 9{,}5$ $b = 0{,}909.. = 0{,}91$ $\hat{y} = 9{,}5 + 0{,}91x$	✓ $a = 9{,}5$ ✓ $b = 0{,}91$ ✓ equation (3)
1.2		✓✓ correct slope going through 2 points: (50 ; 55) or (40 ; 46) or (60 ; 64) or (0 ; 9,5) or (45 ; 50) (2)
1.3	Final exam mark ≈ 72,22% (calculator) **OR** $\hat{y} = 9{,}5 + 0{,}91(69)$ ≈ 72,29%	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.4	$r = 0{,}95$	✓ answer(A) (1)
1.5	There is a **very strong positive** correlation between the Mathematics and Physical Sciences mark. *Daar is 'n baie sterk positiewe korrelasie tussen die Wiskunde en Fisiese Wetenskappunte.*	✓ **strong**/ ***sterk*** (1)
1.6	The teacher concludes that the higher the learners' Mathematics marks, the higher the learners' Physical Sciences marks. *Die onderwyser het waargeneem dat hoe hoër die wiskunde punte is, hoe hoër is die Fisiese Wetenskappunte.*	✓ answer (1)
		[10]

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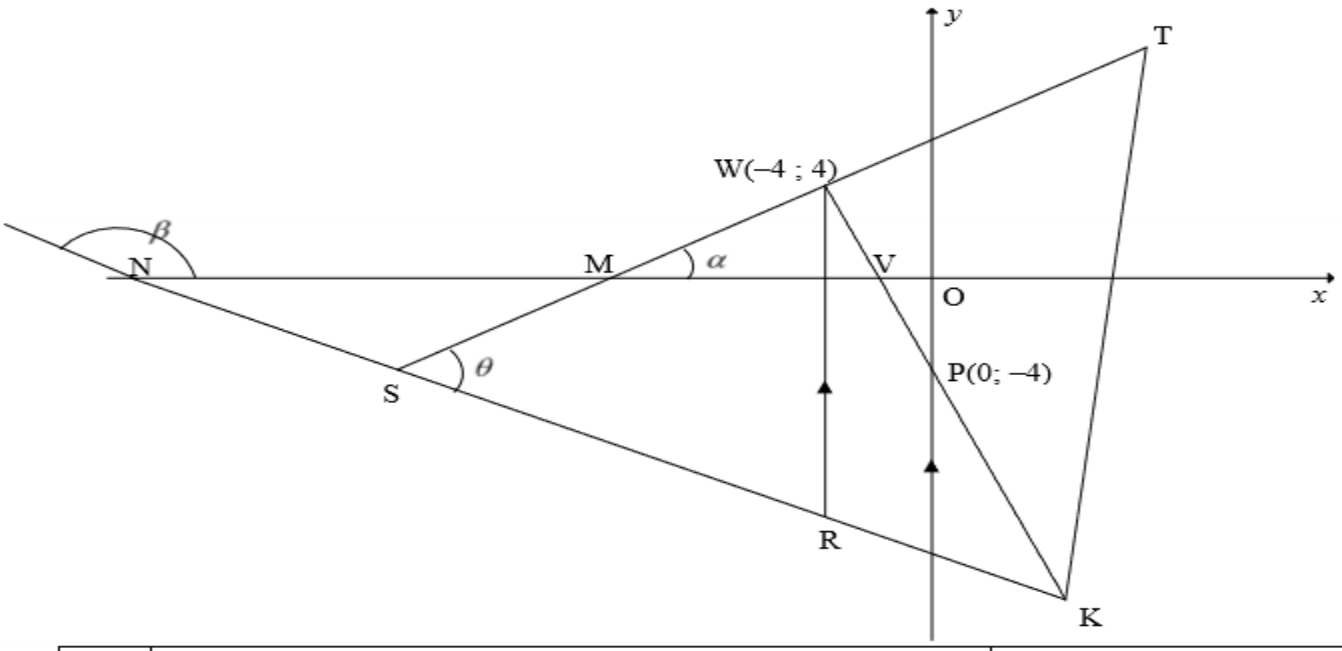
QUESTION/*VRAAG* 2

2 018	**2 175**	**2 182**	**2 215**	**2 254**	**2 263**	**2 267**	**2 271**	**2 293**	**2 323**	**2 334**	**2 346**

2.1	July / *Julie*	✓ answer (1)
2.2	$\bar{x} = \frac{26\,941}{12}$ Answer only: Full marks $= 2\,245{,}083.. \approx 2\,245{,}08$ aircraft landings	✓ 26 941 ✓ answer (2)
2.3	Standard deviation for landings at the King Shaka International airport: $\sigma = 86{,}30$	✓✓ answer (2)
2.4	$(\bar{x} - \sigma\,; \bar{x} + \sigma) = (2\,245{,}08 - 86{,}30\,;\, 2\,245{,}08 + 86{,}30)$ limit = (2 158,78 ; 2 331,38) There were **9** months when the aircraft arrivals at the King Shaka International airport were within one standard deviation of the mean.	✓ $\bar{x} - \sigma$ ✓ $\bar{x} + \sigma$ ✓ answer (3)
2.5	The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of arrivals at the King Shaka International Airport **OR C.**	✓ answer (1)
		[9]

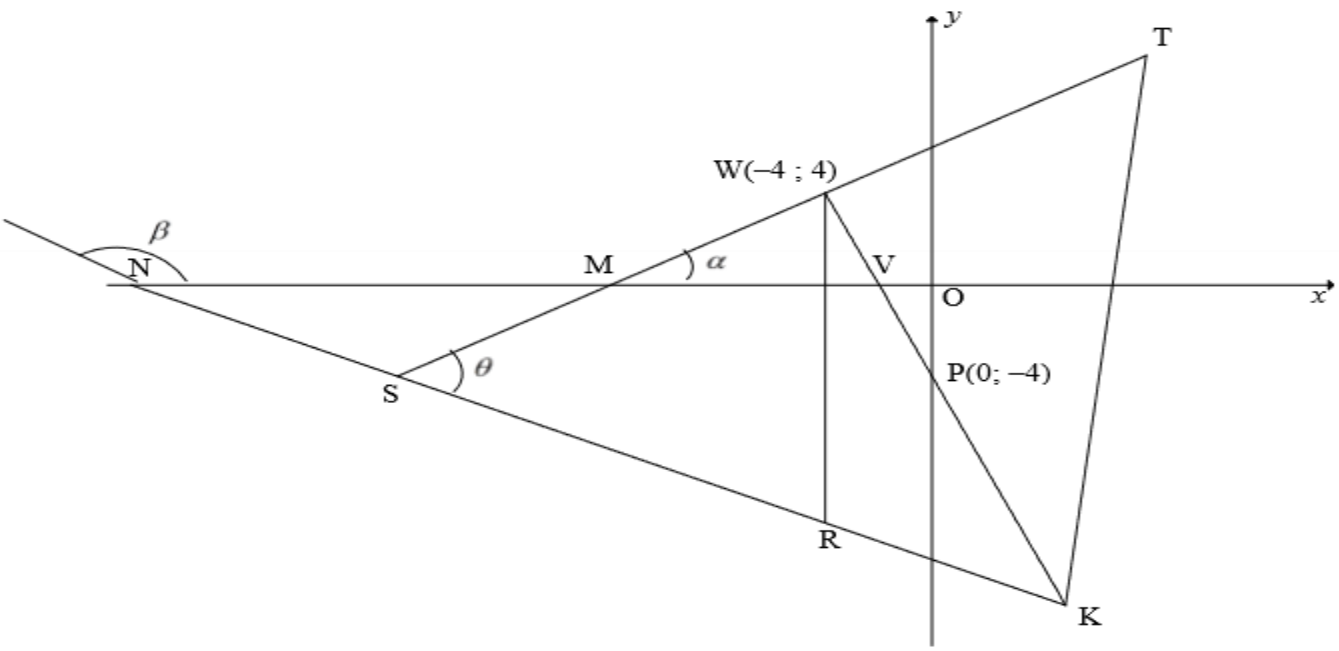
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QUESTION/*VRAAG* 3



3.1	$m_{WP}=\frac{4-(-4)}{-4-0}=\frac{8}{-4}$ $m_{WP}=-2$	✓ substitution of W and P ✓ m_{WP} (2)
3.2	$m_{ST}=\frac{1}{2}$ (given) $(m_{WP})(m_{ST})=(-2)\left(\frac{1}{2}\right)$ $=-1$ $\therefore$ ST ⊥ WP	✓ $(m_{WP})(m_{ST})$ ✓ $(m_{WP})(m_{ST})=-1$ (2)
3.3	$5y+2x+60=0$ $\therefore y=-\frac{2}{5}x-12$ $-\frac{2}{5}x-12=\frac{1}{2}x+6$ $-4x-120=5x+60$ $9x=-180$ $x=-20$ $\therefore y=-\frac{2}{5}(-20)-12$ $\therefore y=-4$ $\therefore$ S(–20; –4) **OR**	✓ equating ✓ x value ✓ substitution ✓ y value (4)

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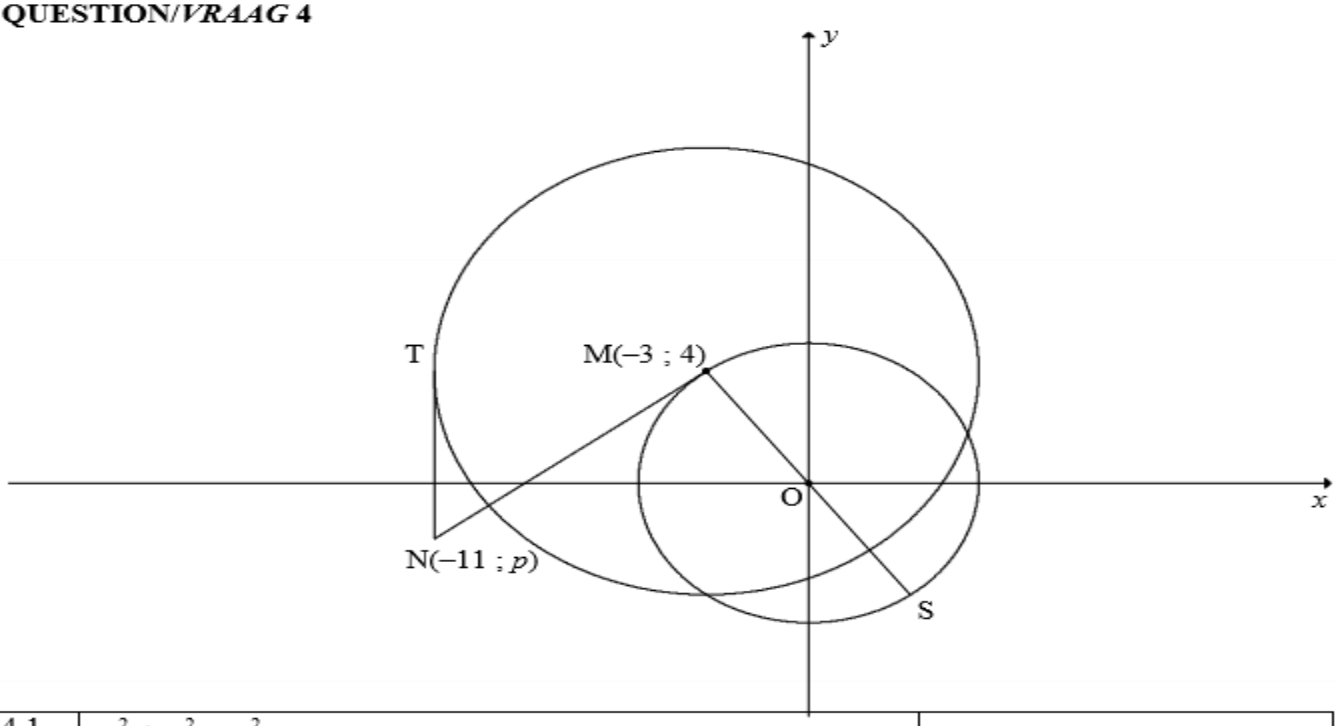
	$5y+2x+60=0$ $5\left(\frac{1}{2}x+6\right)+2x+60=0$ $\frac{5}{2}x+30+2x+60=0$ $\frac{9}{2}x=-90 \qquad \therefore x=-20$ $\therefore y=-\frac{2}{5}(-20)-12$ $\therefore y=-4$ $\therefore$ S(–20; –4)	✓ substitution ✓ x value ✓ substitution ✓ y value (4)
	OR $5y+2x=-60$ .....(1) $2y-x=12$ .....(2) $(1)+2(2): 9y=-36$ $y=-4$ $2(-4)-x=12$ $x=-20$	✓ adding ✓ y value ✓ substitution ✓ x value (4)

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3.4	$y=-\frac{2}{5}(-4)-12$ OR $5y+2(-4)+60=0$ $y=-\frac{52}{5}$ $\therefore \text{R}\left(-4;-\frac{52}{5}\right)$ OR R(–4 ; –10,4) $\therefore \text{WR}=4-\left(-\frac{52}{5}\right)$ OR $\text{WR}=\sqrt{(-4-(-4))^2+(4-\left(-\frac{52}{5}\right))^2}$ $\therefore \text{WR}=\frac{72}{5}$ units or $\text{WR}=14\frac{2}{5}$ units **OR** $\text{WR}=\text{ST}-\text{SK}$ $=\frac{1}{2}x+6-\left(-\frac{2}{5}x-12\right)$ $=\frac{9}{10}x+18$ $=\frac{9}{10}(-4)+18$ $=14,4$ units	✓ substitution ✓ y value ✓ method or subst into distance formula ✓ answer (4) ✓ substitution ✓ simplification ✓ subst $x=-4$ ✓ answer (4)
3.5	$m_{\text{SK}}=-\frac{2}{5}$ $\beta=158,19...°$ (Ref. $\angle$ =21, 801…°) $\text{M}\hat{\text{N}}\text{S}=21,80...°$ $m_{\text{ST}}=\frac{1}{2}$ $\text{N}\hat{\text{M}}\text{S}=26,56...°$ $\theta=21,80...°+26,56...°$ [ext $\angle$ of Δ] $\theta=48,366...°=48,37°$	✓ m_{SK} ✓ size of β ✓ size of $\text{N}\hat{\text{M}}\text{S}$ ✓ method ✓ answer (5)
3.6	In ΔSRW: $\perp h=-4-(-20)$ $\perp h=16$units Area $\Delta\text{SRW}=\frac{1}{2}(\perp h)(\text{WR})$ $=\frac{1}{2}(16)\left(\frac{72}{5}\right)$ $=115,2$ square units Area SWRL = 2Area ΔSRW =2(115,2) = 230,4square units **OR**	✓ $\perp h$ ✓ substitution ✓ area Δ ✓ answer (4)

	In ΔSRW: $\perp h = -4-(-20)$ $\perp h = 16$units Area SWRL $= 16\times\frac{72}{5}$ $= 230{,}40$ square units **OR** $SW=\sqrt{(-20+4)^2+(-4-4)^2}=8\sqrt{5}=17{,}89$ $SR=\sqrt{(-20+4)^2+\left(-4+10\frac{2}{5}\right)^2}=\frac{16\sqrt{29}}{5}=17{,}23$ Area SWRL $= 2\times$ Area ΔSRW $=2\left(\frac{1}{2}SW\times SR\sin\theta\right)$ $=2\left(\frac{1}{2}8\sqrt{5}\times\frac{16\sqrt{29}}{5}\sin 48{,}37°\right)$ $=230{,}41$square units	✓ $\perp h$ ✓ ✓ substitution ✓ answer (4) ✓$SW=8\sqrt{5}$ ✓$SR=\frac{16\sqrt{29}}{5}$ ✓substitution ✓answer (4)
		[21]

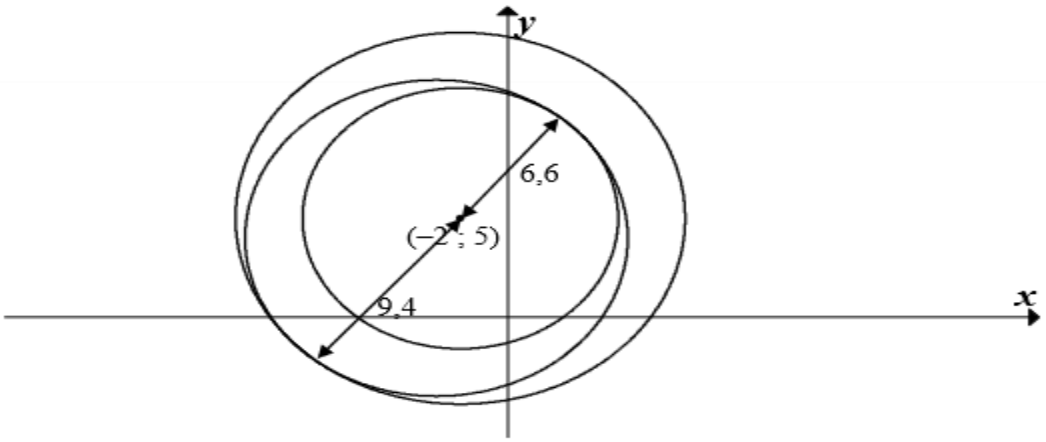
QUESTION/*VRAAG* 4



4.1	$x^2+y^2=r^2$ $\therefore r^2=(-3)^2+(4)^2=25$ $x^2+y^2=25$	✓ substitution ✓ answer (2)
4.2	TM ⊥ TN [tangent ⊥ radius] T(–11 ; 4) $r=-3-(-11)=8$ $(x+3)^2+(y-4)^2=64$	✓ $x_T=-11$ ✓ LHS ✓ RHS (3)
4.3	O (0 ; 0) and M(–3; 4) $m_{OM}=\frac{4-0}{-3-0}=-\frac{4}{3}$ OR $\frac{0-4}{0-(-3)}=-\frac{4}{3}$ $m_{NM}=\frac{3}{4}$ $y-4=\frac{3}{4}(x-(-3))$ **OR** $y=\frac{3}{4}x+c$ $y-4=\frac{3}{4}x+\frac{9}{4}$ $4=\frac{3}{4}(-3)+c$ $\therefore y=\frac{3}{4}x+\frac{25}{4}$ $c=\frac{25}{4}$ $y=\frac{3}{4}x+\frac{25}{4}$	✓ $m_{OM}=-\frac{4}{3}$ ✓ $m_{NM}=\frac{3}{4}$ ✓ substitution of m and M ✓ equation (4)

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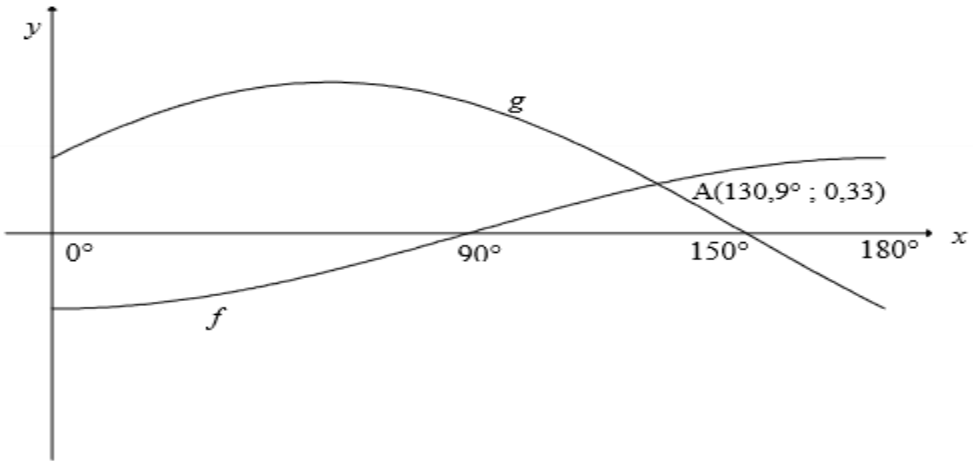
4.4	N(–11 ; p) $y=\frac{3}{4}x+\frac{25}{4}$ $p=\frac{3}{4}(-11)+\frac{25}{4}$ OR $\frac{4-p}{-3-(-11)}=\frac{3}{4}$ $p=-2$ $\therefore$ N(–11;–2) $\frac{-3+x_S}{2}=0$ and $\frac{4+y_S}{2}=0$ $\therefore$ S(3;–4) $SN=\sqrt{(-11-3)^2+(-2-(-4))^2}$ $=10\sqrt{2}$ units or 14,14 units	✓ subst $x=-11$ into eq or gradient ✓ $p=-2$ ✓ x_S ✓ y_S ✓ answer (CA) (5)
4.5	B(– 2; 5) BM = $\sqrt{2}$ units Radius of circle centred at M = 8 units $k=8-\sqrt{2}$ or $k=8+\sqrt{2}$ = 6,59 units = 9,41 units = 6,6 units = 9,4 units	✓ $\sqrt{2}$ ✓✓ $k=6,6$ ✓✓ $k=9,4$ (5)
		[19]



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QUESTION/*VRAAG* 5

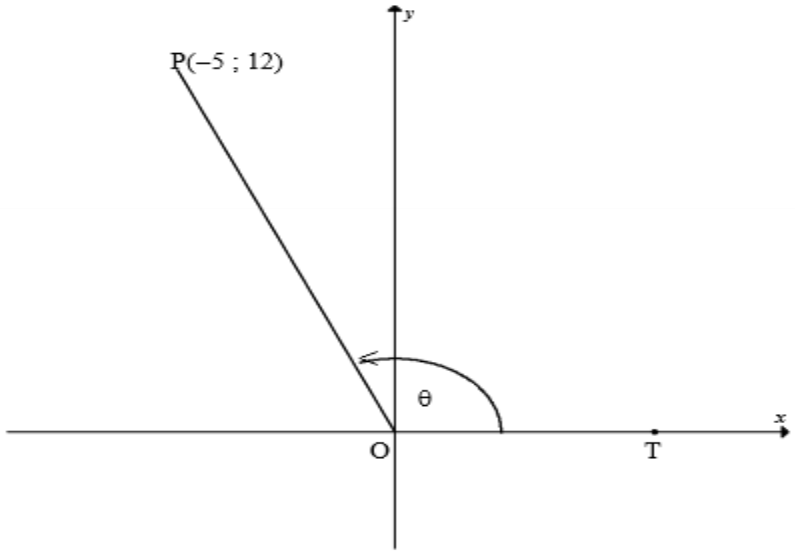


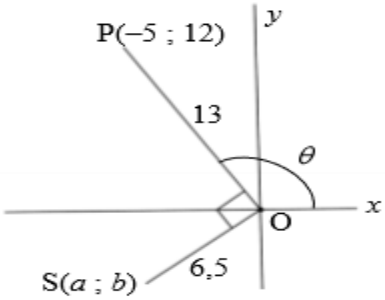
5.1	Period of $g = 360°$	✓ answer (1)
5.2	Amplitude of $f = \frac{1}{2}$	✓ answer (A) (1)
5.3	$f(180°) - g(180°)$ $= \frac{1}{2} - \left(-\frac{1}{2}\right)$ $= 1$	✓ 1 (1)
5.4.1	$x = 140,9°$	✓ $x = 140,9°$ (1)
5.4.2	$\sqrt{3}\sin x + \cos x \geq 1$ $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x \geq \frac{1}{2}$ $\sin x \cos 30° + \cos x \sin 30° \geq \frac{1}{2}$ $\sin(x + 30°) \geq \frac{1}{2}$ $\sin(x + 30°) = \frac{1}{2}$ at $x = 0°$ or $x = 120°$ $\therefore x \in [0°; 120°]$ **OR** $0° \leq x \leq 120°$	✓ dividing by 2 ✓ $\cos 30°; \sin 30°$ ✓ $\sin(x + 30°) \geq \frac{1}{2}$ ✓ interval (4)
		[8]

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QUESTION/*VRAAG 6*



6.1.1	$\tan\theta = -\frac{12}{5}$ or $-2\frac{2}{5}$	✓answer (1)
6.1.2	$(OP)^2 = (-5)^2 + (12)^2$ $OP = 13$ $\cos\theta = -\frac{5}{13}$	✓ Pythagoras ✓ OP ✓ answer (3)
6.1.3	$\sin(\theta + 90°) = \frac{b}{6,5}$ $\cos\theta = \frac{b}{6,5}$ $\frac{-5}{13} = \frac{b}{6,5}$ $b = -\frac{5}{2}$ **OR** $\cos(90° + \theta) = \frac{a}{6,5}$ $-\sin\theta = \frac{a}{6,5}$ $-\frac{12}{13} = \frac{a}{6,5} \quad \therefore a = -6$ $b = \sqrt{(6,5)^2 - (-6)^2} = -\frac{5}{2}$	 ✓ $\sin(\theta + 90°) = \frac{b}{6,5}$ ✓ $\cos\theta$ ✓ $\frac{-5}{13} = \frac{b}{6,5}$ ✓ value of b (4) ✓ $\cos(\theta + 90°) = \frac{a}{6,5}$ ✓ $-\sin\theta$ ✓ value of a ✓ value of b (4)

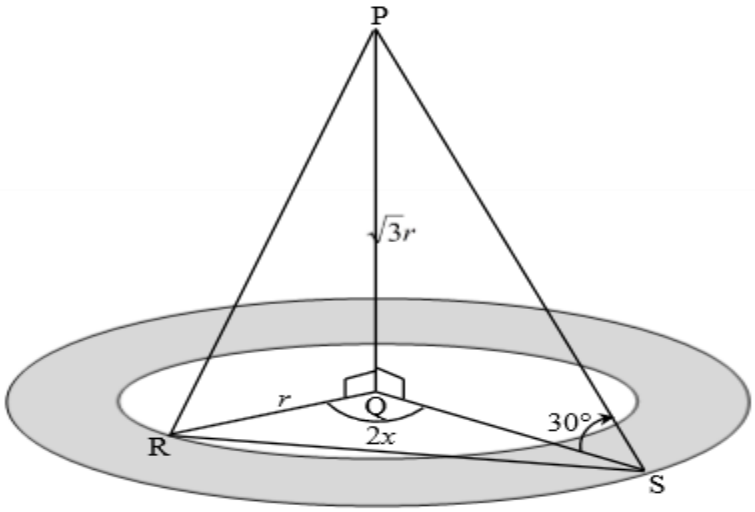
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6.2	$\frac{\sin 2x.\cos(-x)+\cos 2x.\sin(360°-x)}{\sin(180°+x)}$ $=\frac{\sin 2x\ \cos x\ +\ \cos 2x(-\sin x)}{-\sin x}$ $=\frac{\sin(2x-x)}{-\sin x}$ $=\frac{\sin x}{-\sin x}$ $=-1$	✓ $\cos(-x)=\cos x$ ✓ $\sin(360°-x)=-\sin x$ ✓ $\sin(180°+x)=-\sin x$ ✓ numerator = $\sin x$ ✓ answer (5)
6.3	$6\sin^2 x+7\cos x-3=0$ $6(1-\cos^2 x)+7\cos x-3=0$ $6-6\cos^2 x+7\cos x-3=0$ $6\cos^2 x-7\cos x-3=0$ $(3\cos x+1)(2\cos x-3)=0$ $\cos x=-\frac{1}{3}$ or $\cos x=\frac{3}{2}$ (N/A) $\therefore x=109{,}47°+k.360°;\ k\in Z$ or $x=250{,}53°+k.360°;\ k\in Z$	✓ identity ✓ standard form ✓ factors ✓ both solutions of $\cos x$ ✓ $x=109{,}47°\ \&\ 250{,}53°$ ✓ $+k.360°;\ k\in Z$ (6)
6.4	$x+\frac{1}{x}=3\cos A$ $(3\cos A)^2=\left(x+\frac{1}{x}\right)^2$ $9\cos^2 A=x^2+\frac{1}{x^2}+2$ $9\cos^2 A=2+2$ $\cos^2 A=\frac{4}{9}$ $\cos 2A=2\cos^2 A-1$ $=2\left(\frac{4}{9}\right)-1$ $=-\frac{1}{9}$ **OR**	✓squaring both sides ✓$9\cos^2 A=x^2+\frac{1}{x^2}+2$ ✓$\cos^2 A=\frac{4}{9}$ ✓ $\cos 2A=2\cos^2 A-1$ ✓ answer (5)

	$x^2-2+\frac{1}{x^2}=0$ $\left(x-\frac{1}{x}\right)^2=0$ $x^2=1$ $x=\pm 1$ $3\cos A=2 \quad \text{or} \quad 3\cos A=-2$ $\cos A=\frac{2}{3} \quad \text{or} \quad \cos A=-\frac{2}{3}$ $\cos 2A=2\cos^2 A-1$ $=2\left(\pm\frac{2}{3}\right)^2-1$ $=-\frac{1}{9}$	✓ $x=\pm 1$ ✓ $\cos A=\frac{2}{3}$ ✓ $\cos A=-\frac{2}{3}$ ✓ double angle identity ✓ answer (5)
		[24]

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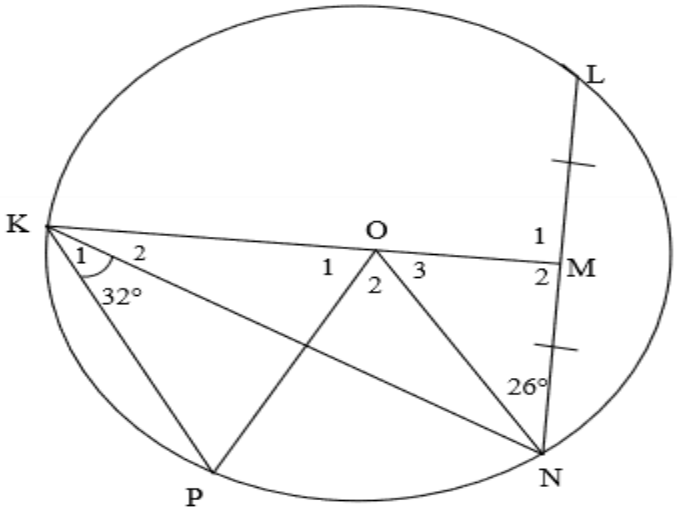
QUESTION/*VRAAG* 7



7.1	$\tan 30° = \frac{\sqrt{3}r}{QS}$ **OR** $\tan 60° = \frac{QS}{\sqrt{3}r}$ $QS = \frac{\sqrt{3}r}{\tan 30°}$ $\sqrt{3} = \frac{QS}{\sqrt{3}r}$ $= \frac{\sqrt{3}r}{\frac{1}{\sqrt{3}}}$ *or* $\frac{\sqrt{3}r}{\frac{\sqrt{3}}{3}}$ $QS = 3r$ $= 3r$	✓✓ trig ratio ✓ QS subject (3)
7.2	Area of flower garden $= \pi(3r)^2 - \pi r^2$ $= 9\pi r^2 - \pi r^2$ $= 8\pi r^2$	✓ substitution into difference of areas ✓ answer (2)
7.3	$RS^2 = r^2 + (3r)^2 - 2(r)(3r)\cos 2x$ $= r^2 + 9r^2 - 6r^2\cos 2x$ $= 10r^2 - 6r^2\cos 2x$ $= r^2(10 - 6\cos 2x)$ $RS = r\sqrt{10 - 6\cos 2x}$	✓substitution into cosine rule correctly ✓ $10r^2 - 6r^2\cos 2x$ ✓ $r^2(10 - 6\cos 2x)$ (3)
7.4	$RS = 10\sqrt{10 - 6\cos 2(56)}$ $= 34{,}9966...$ ≈ 35 m	✓substitution ✓ answer (2)
		[10]

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QUESTION/*VRAAG* 8

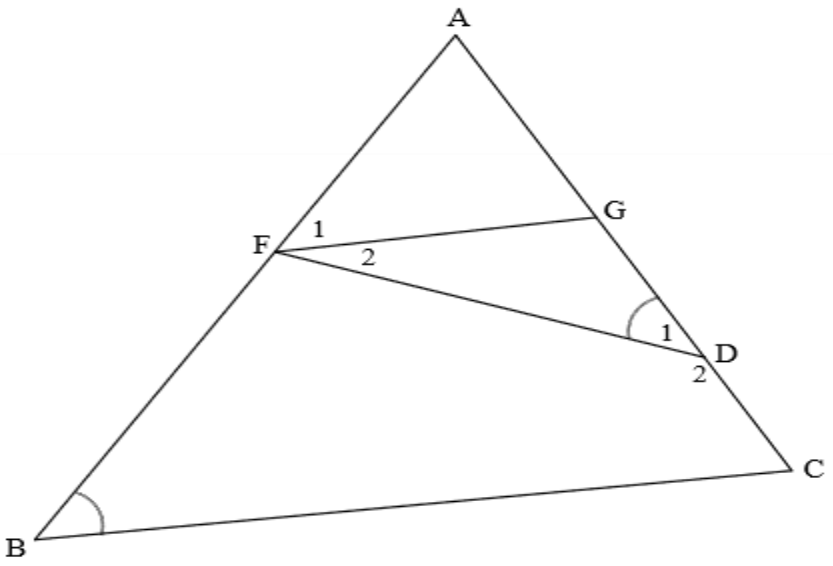


8.1.1(a)	$\hat{O}_2 = 64°$ [∠ at centre = 2 ×∠ at circumference/ *Middelpts ∠ = 2 ×∠ omtreks∠*]	✓ S ✓ R (2)
8.1.1(b)	$\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $K\hat{O}N = 90° + 26° = 116°$ [ext ∠ of Δ/*buite ∠ van* Δ] $\hat{O}_1 = 116° - 64° = 52°$ **OR** $\hat{M}_2 = 90°$ [Line from centre to midpt of chord/*lyn v midpt na midpt v koord*] $\hat{O}_3 = 64°$ [sum of ∠s in Δ] $\hat{O}_1 = 52°$ [∠s on straight line/*op 'n reguitlyn*]	✓ S ✓ R ✓ S ✓ answer (4) ✓ S ✓ R ✓ S ✓ answer (4)
8.1.2	$P\hat{K}O + \hat{P} = 128°$ [sum of ∠s in Δ/*som ∠e van* Δ] $P\hat{K}O = \hat{P}$ [∠s opp = sides/*∠e teenoor = sye*] $= 64°$ $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$ **OR** $\hat{K}_2 = K\hat{N}O$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{K}_2 + K\hat{N}O = 64°$ [sum of ∠s in Δ/*som ∠e van* Δ] $\therefore \hat{K}_2 = 32°$ *or* $\hat{K}_2 = \hat{K}_1$ $\therefore$ KN bisects/*halveer* $O\hat{K}P$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

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8.2



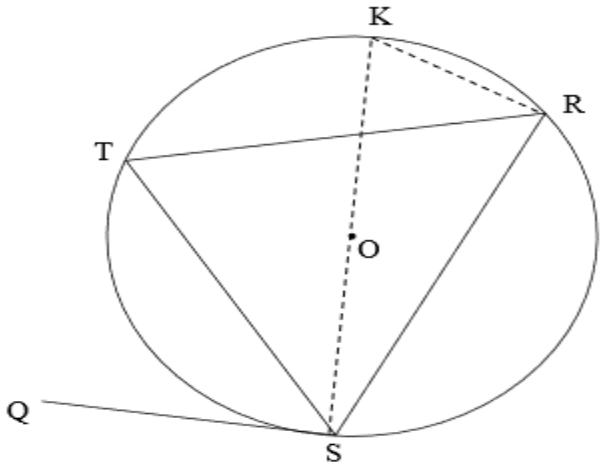
8.2.1	$\hat{F}_1 = \hat{D}_1$ [tan chord theorem/*raaklyn koordst*] $\hat{D}_1 = \hat{B}$ [Given/*Gegee*] $\therefore \hat{F}_1 = \hat{B}$ $\therefore FG \parallel BC$ [corresp ∠s =/*Ooreenkomstige∠e* =]	✓ S ✓ R ✓ $\hat{F}_1 = \hat{B}$ ✓ R (4)
8.2.2	$\frac{GC}{AC} = \frac{FB}{AB}$ [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] $\frac{x+9}{2x-6} = \frac{5}{7}$ $7x + 63 = 10x - 30$ $3x = 93$ $x = 31$ **OR** $AG = 2x - 6 - (x+9) = x - 15$ $\frac{AG}{GC} = \frac{AF}{FB}$ [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] $\frac{x-15}{x+9} = \frac{2}{5}$ $5x - 75 = 2x + 18$ $3x = 93$ $x = 31$ **OR**	✓ S ✓ R ✓ substitution ✓ answer (4) ✓ S ✓ R ✓ substitution ✓ answer (4)

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	$\frac{AF}{AB}=\frac{AG}{AC}$ [line ∥ one side of Δ /lyn ∥ een sy vΔ] $\frac{2}{7}=\frac{x-15}{2x-6}$ $7x-105=4x-12$ $3x=93$ $x=31$	✓ S ✓ R ✓ substitution ✓ answer (4)
		[17]

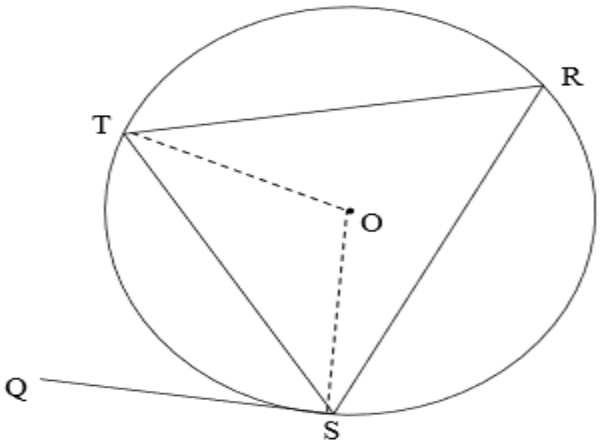
QUESTION/*VRAAG* 9

9.1



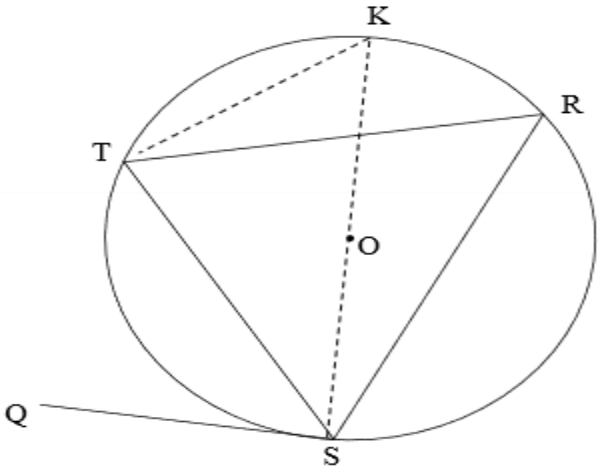
9.1	Construction: Draw diameter KS and draw KR *Konstruksie: Trek middellyn KS en verbind KR* $Q\hat{S}T = 90° - T\hat{S}K$ [radius ⊥ tangent/*raaklyn*] $S\hat{R}K = 90°$ [∠ in semi circle/*halfsirkel*] $\therefore S\hat{R}T = 90° - K\hat{R}T$ $T\hat{S}K = T\hat{R}K$ [∠s same segment/*∠e dieselfde segment*] $\therefore Q\hat{S}T = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



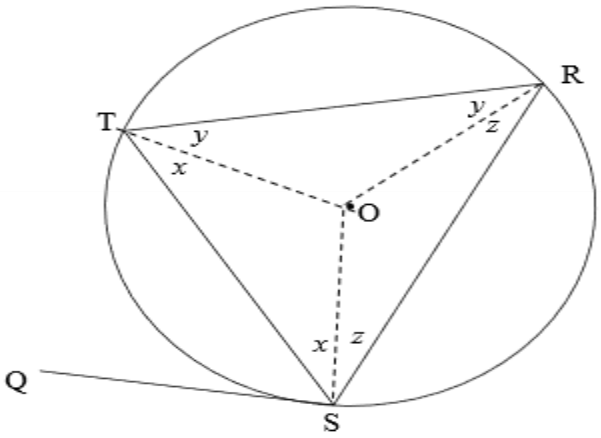
9.1	Construction: Draw radii OS and OT *Konstruksie: Trek radii OS en OT* $\hat{QST} = 90° - \hat{OST}$ [radius ⊥ tangent/*raaklyn*] $\hat{OST} = \hat{STO}$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore \hat{SOT} = 180° - 2\hat{OST}$ [∠s of Δ/*∠e van Δ*] $\hat{R} = 90° - \hat{OST}$ [∠ at centre = 2×∠ circumf/*midpts∠ = 2× omtreks∠*] $\therefore \hat{QST} = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



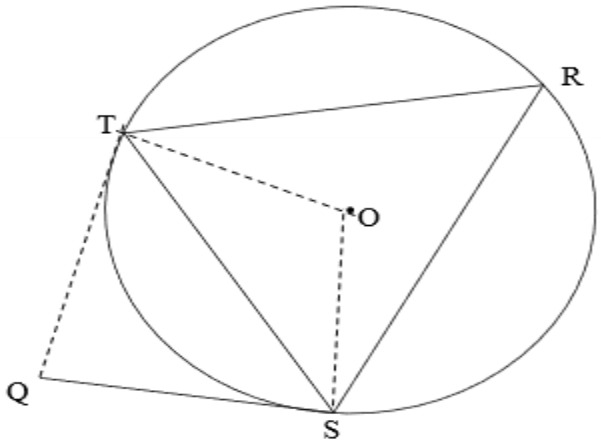
9.1	Construction: Draw diameter KS and join K to T. *Konstruksie: Trek middellyn KS en verbind K tot T* K $Q\hat{S}T = 90° - T\hat{S}K$ [radius ⊥ tangent/*raaklyn*] $S\hat{T}K = 90°$ [∠ in semi circle/*halfsirkel*] $\therefore \hat{K} = 90° - T\hat{S}K$ $\therefore Q\hat{S}T = \hat{K}$ but $\hat{R} = \hat{K}$ [∠s same segment/*∠e dieselfde segment*] $\therefore Q\hat{S}T = \hat{R}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

OR



9.1	Construction: Draw radii OT, OR and OS *Konstruksie: Trek radiuse OT, OR en OS* $O\hat{S}T = O\hat{T}S$ [∠s opp = radii/*∠e teenoor = radiuse*] Also: $O\hat{T}R = O\hat{R}T$ and $O\hat{R}S = O\hat{S}R$ $2x + 2y + 2z = 180°$ [∠s of Δ] $x + y + z = 90°$ $y + z = 90° - x$ $O\hat{S}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore T\hat{S}Q = 90° - x$ $\therefore T\hat{S}Q = y + z = \hat{R}$	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S (5)

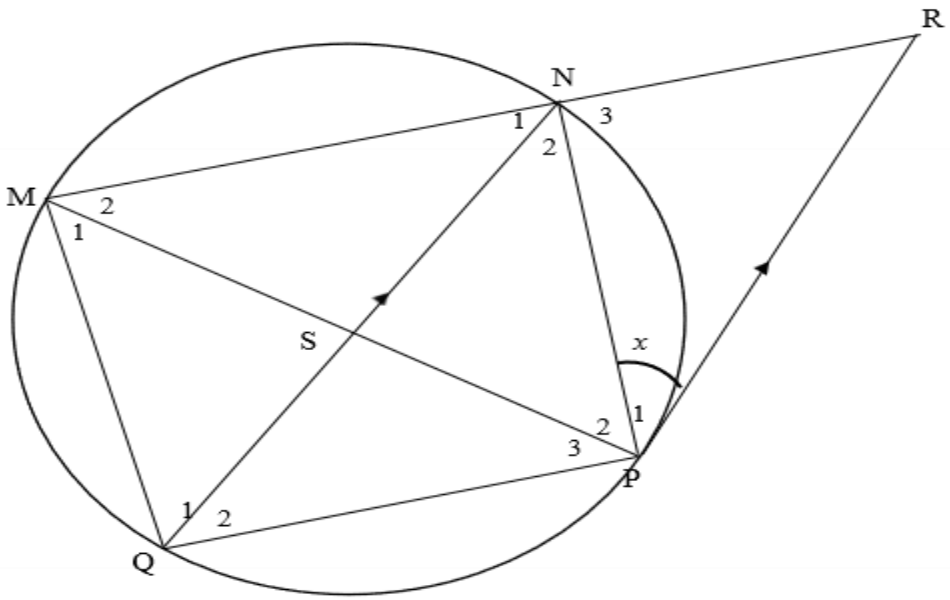
OR



9.1	Construction: Draw radii OT and OS, tangent QT *Konstruksie: Trek radiuse OT en OS, raaklyn QT* $O\hat{S}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore T\hat{S}Q = 90° - T\hat{S}O$ $\therefore T\hat{S}O = S\hat{T}O$ [∠s opp = radii/*∠e teenoor = radiuse*] $T\hat{O}S = 180° - 2T\hat{S}O$ [∠s of Δ] $\hat{R} = 90° - T\hat{S}O$ [∠ at centre = 2×∠ circumf/*midpts∠ = 2× omtreks∠*] $\therefore T\hat{S}Q = \hat{R}$	✓ construction ✓ S/R ✓ S ✓ S ✓ S/R (5)

KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.

9.2

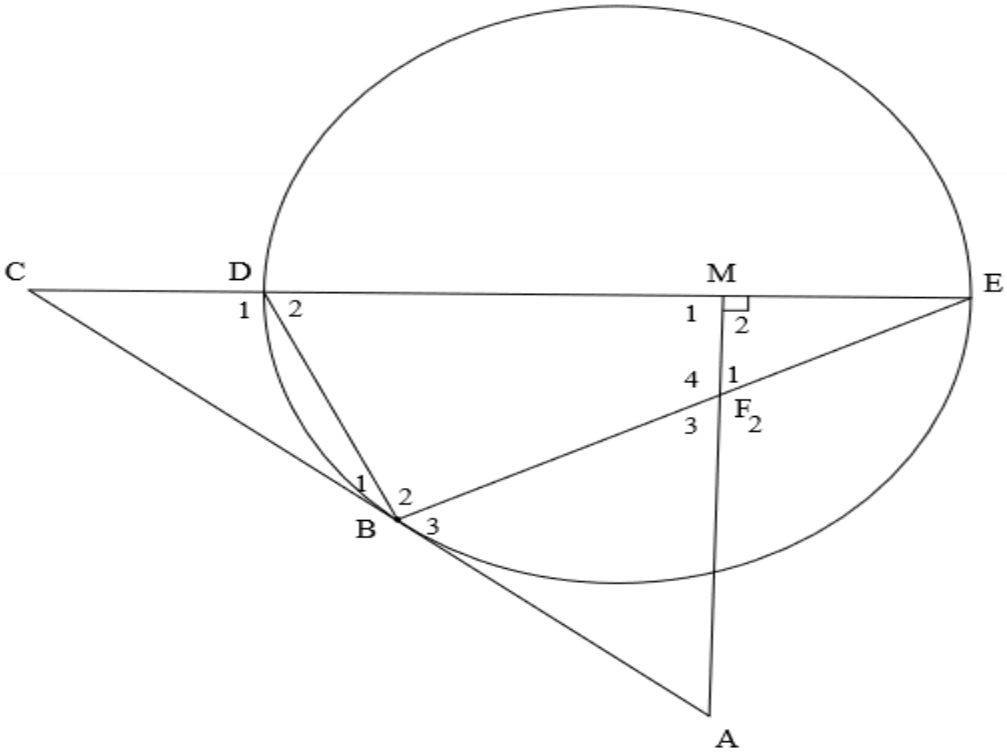


9.2.1(a)	$\hat{N}_2 = x$	[alt ∠s; PR ∥ NQ/*verw. ∠e;* PR ∥ NQ]	✓ S ✓ R (2)
9.2.1(b)	$\hat{Q}_2 = x$ OR $M_2 = x$ $\hat{Q}_2 = x$	[tan chord theorem/*raaklyn koordstelling*] [tan chord theorem/*raaklyn koordstelling*] [∠s in same segment/*∠e in dieselfde segm*]	✓ S ✓ R (2) ✓ S/R ✓ S/R (2)
9.2.2	$\frac{MN}{NR} = \frac{MS}{SP}$ $\hat{N}_1 = \hat{N}_2 = x$ $\hat{P}_3 = x$ $\hat{P}_3 = \hat{Q}_2$ $SQ = PS$ $\frac{MN}{NR} = \frac{MS}{SQ}$	[QN ∥ PR; Prop Th] [given] [∠s in same segment/*∠e in dieselfde segm*] [= x] [sides opp = ∠/*sye teenoor* = ∠e]	✓ S ✓ R ✓ S ✓ S ✓ R ✓ R (6)
			[15]

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QUESTION/*VRAAG* 10



10.1.1	$D\hat{B}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore D\hat{M}A = 90°$ [AM ⊥ DE] ∴ FBDM is a cyclic quadrilateral/*koordevh* [converse opp ∠s cyclic quad/*omgek teenoorst ∠e kvh*] **OR** $D\hat{B}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{M}_2 = D\hat{B}E = 90°$ ∴ FBDM is a cyclic quadrilateral/*koordevh* [converse ext∠ of cyclic quad/*omgek buite∠van kvh*]	✓ S ✓ R ✓ R (3) ✓ S ✓ R ✓ R (3)

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10.1.2	$\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = \hat{D}_2$ [ext ∠ cyc quad/*buite ∠ koordevh*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{F}_1 = 90° - x$ [∠ sum in Δ/*∠ van Δ*] $\hat{D}_2 = 90° - x$ [∠ sum in Δ/*∠ van Δ*] $\therefore \hat{F}_1 = D_2$ $\hat{B}_3 = \hat{D}_2$ [tangent chord th/*raaklyn koordst*] $\therefore \hat{B}_3 = \hat{F}_1$ **OR** $\hat{B}_1 = \hat{E} = x$ [tangent chord th/*raaklyn koordst*] $\hat{B}_3 = 90° - x$ [straight line/*reguitlyn*] $\hat{F}_1 = 90° - x$ [sum of ∠s Δ/*som van ∠e van Δ*] $\therefore \hat{B}_3 = \hat{F}_1$	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ $\hat{F}_1 = 90° - x$ $= \hat{D}_2$ ✓ R (4) ✓ S ✓ R ✓ S ✓ S (4)
10.1.3	In ΔCDB and ΔCBE $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike ∠*] $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $C\hat{D}B = C\hat{B}E$ [∠ sum in Δ/*∠ van Δ*] ΔCDB ||| ΔCBE **OR** In ΔCDB and ΔCBE $C\hat{B}D = C\hat{E}B$ [tangent chord th/*raaklyn koordst*] $\hat{C} = \hat{C}$ [common ∠/*gemeenskaplike ∠*] ΔCDB ||| ΔCBE [∠, ∠, ∠]	✓ S ✓ S/R ✓ R (3) ✓ S/R ✓ S ✓ R (3)
10.2.1	$\frac{BC}{EC} = \frac{DC}{BC}$ [||| Δs] $BC^2 = EC \times DC$ $= 8 \times 2$ $= 16$ $BC = 4$	✓ ratio ✓ substitution ✓answer (3)

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10.2.2	$\frac{BC}{EC} = \frac{DB}{BE}$ [‖ Δs] $\frac{DB}{BE} = \frac{4}{8} = \frac{1}{2}$ $BE = 2DB$ $DB^2 + BE^2 = DE^2$ [Pyth theorem] $DB^2 + (2DB)^2 = 36$ $5DB^2 = 36$ $DB^2 = \frac{36}{5}$ $DB = \frac{6}{\sqrt{5}} = 2{,}68$ units	✓ $BE = 2DB$ ✓ substitution into Pyth theorem ✓ $DB^2 = \frac{36}{5}$ ✓answer (4)
		[17]

TOTAL/*TOTAAL*: 150

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GRADE 12

MATHEMATICS P2

NOVEMBER 2021

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.

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QUESTION 1

A bakery kept a record of the number of loaves of bread a tuck-shop ordered daily over the last 18 days. The information is shown in the table below.

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1 Calculate the:

1.1.1 Mean number of loaves of bread ordered daily (2)

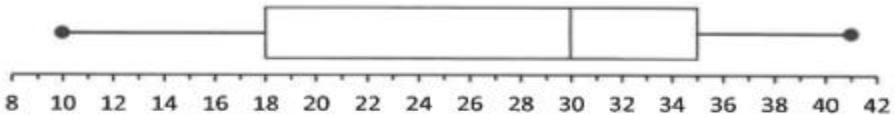
1.1.2 Standard deviation of the data (1)

1.1.3 Number of days on which the number of loaves of bread ordered was more than one standard deviation above the mean (2)

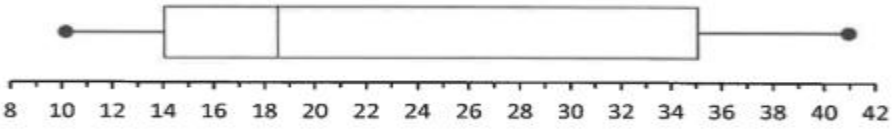
1.2 The tuck-shop owner was not able to sell all the loaves of bread delivered daily. He calculated the mean number of loaves sold over the 18 days to be 20. Calculate the number of loaves of bread which were NOT sold over the 18 days. (2)

1.3 One of the two box and whisker diagrams drawn below represents the data given in the table above.

Graph A:



Graph B:



1.3.1 Which ONE of the two box and whisker diagrams, drawn above, correctly represents the data? Write down a reason for your answer. (2)

1.3.2 Describe the skewness of the data. (1)
[10]

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QUESTION 2

A farm stall sells milk in 5-litre containers to the local community. The price varies according to the availability of milk at the farm stall. The price of milk, in rands per 5-litre container, and the number of 5-litre containers of milk sold, are recorded in the table below.

Price of milk in rands per 5-litre container (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)	48	30	26	44	23	32	39	29	42	33

2.1 On the grid provided in the ANSWER BOOK, draw the scatter plot to represent the data. (3)

2.2 Determine the equation of the least squares regression line for the data. (3)

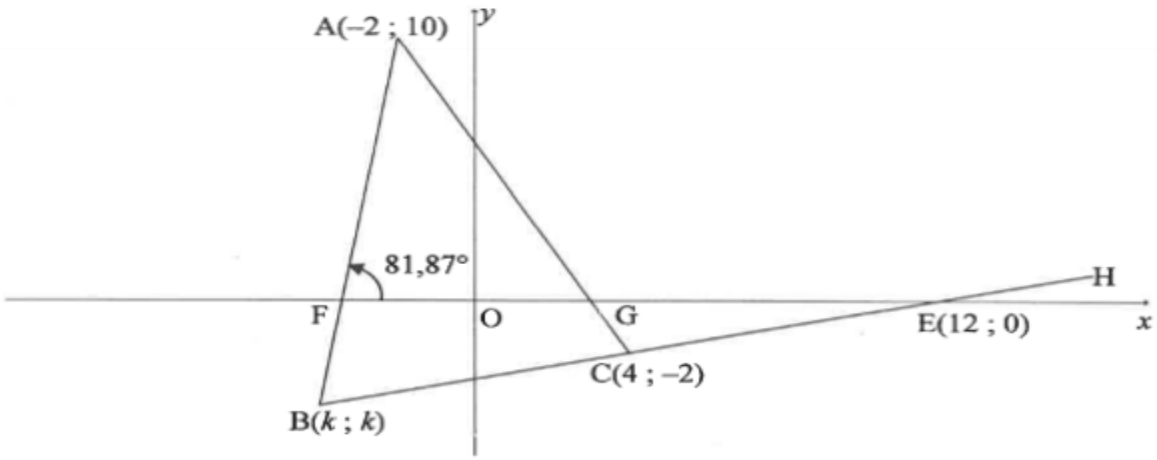
2.3 If the farmer sells a 5-litre container of milk for R38, predict the number of 5-litre containers of milk he will sell. (2)

2.4 Refer to the correlation between the price of 5-litre containers of milk and the number of 5-litre containers of milk sold, and comment on the accuracy of your answer to QUESTION 2.3. (2)
[10]

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QUESTION 3

In the diagram, A(–2 ; 10), B(k ; k) and C(4 ; –2) are the vertices of ΔABC. Line BC is produced to H and cuts the x-axis at E(12 ; 0). AB and AC intersects the x-axis at F and G respectively. The angle of inclination of line AB is 81,87°.



3.1 Calculate the gradient of:

3.1.1 BE (2)

3.1.2 AB (2)

3.2 Determine the equation of BE in the form $y = mx + c$ (2)

3.3 Calculate the:

3.3.1 Coordinates of B, where $k < 0$ (2)

3.3.2 Size of $\hat{A}$ (4)

3.3.3 Coordinates of the point of intersection of the diagonals of parallelogram ACES, where S is a point in the first quadrant (2)

3.4 Another point T(p ; p), where $p > 0$, is plotted such that ET = BE = $4\sqrt{17}$ units.

3.4.1 Calculate the coordinates of T. (5)

3.4.2 Determine the equation of the:

(a) Circle with centre at E and passing through B and T in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

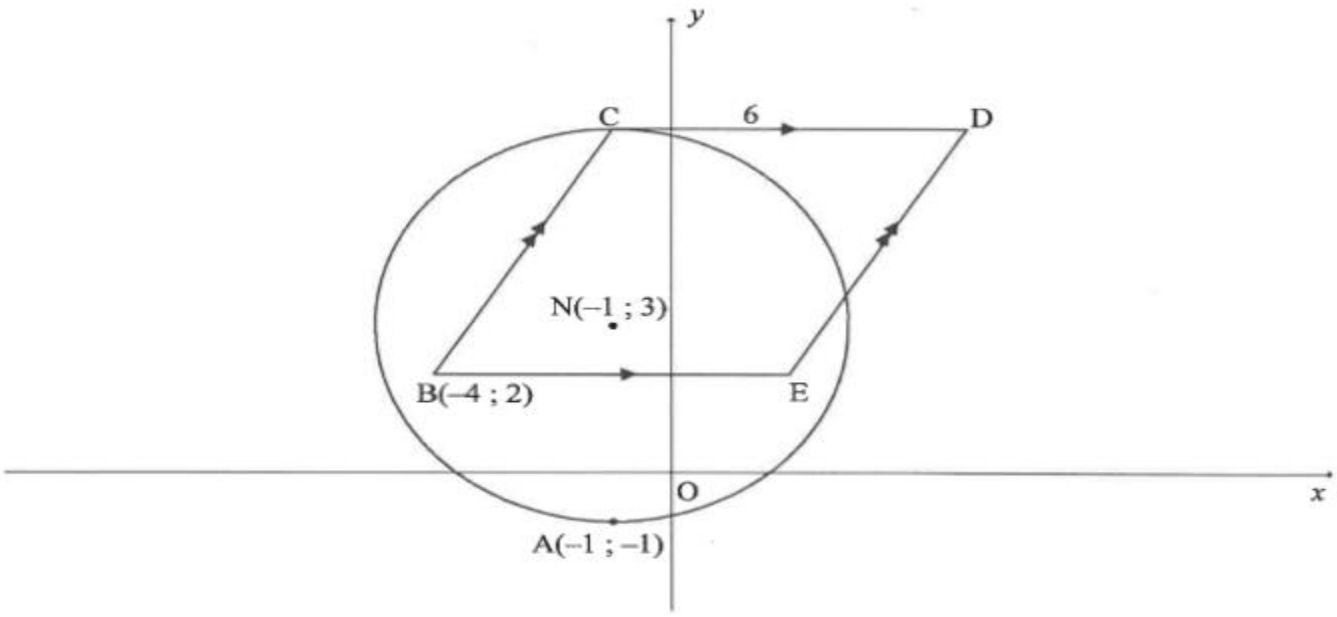
(b) Tangent to the circle at point B(k ; k) (3)

[24]

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QUESTION 4

In the diagram, the circle centred at N(–1 ; 3) passes through A(–1 ; –1) and C. B(–4 ; 2), C, D and E are joined to form a parallelogram such that BE is parallel to the x-axis. CD is a tangent to the circle at C and CD = 6 units.



4.1 Write down the length of the radius of the circle. (1)

4.2 Calculate the:

4.2.1 Coordinates of C (2)

4.2.2 Coordinates of D (2)

4.2.3 Area of ΔBCD (3)

4.3 The circle, centred at N, is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F.

Calculate the:

4.3.1 Length of NM (3)

4.3.2 Midpoint of AF (4)
[15]

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QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)}$$ (6)

5.2 Prove the identity: $\frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)} = 2\cos x - 1$ (4)

5.3 Given: $\sin 36° = \sqrt{1 - p^2}$

Without using a calculator, determine EACH of the following in terms of p:

5.3.1 $\tan 36°$ (3)

5.3.2 $\cos 108°$ (4)
[17]

QUESTION 6

6.1 Given: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

6.1.1 Use the given identity to derive a formula for $\cos(\alpha + \beta)$ (3)

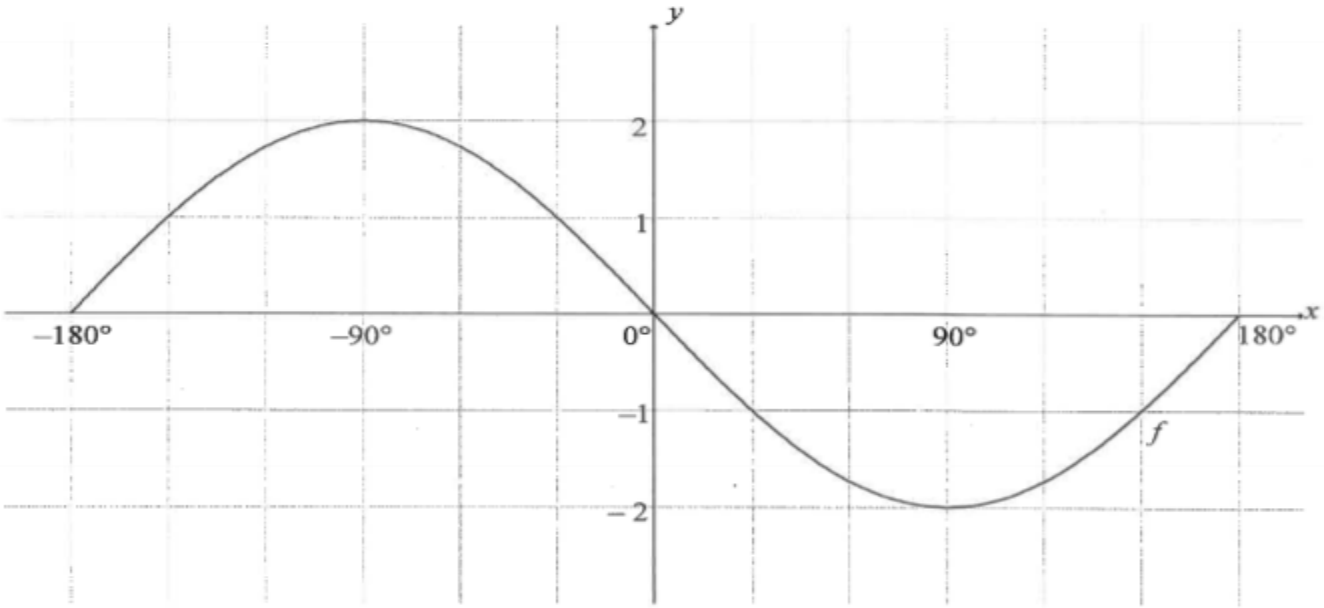
6.1.2 Simplify completely: $2\cos 6x\cos 4x - \cos 10x + 2\sin^2 x$ (5)

6.2 Determine the general solution of $\tan x = 2\sin 2x$ where $\cos x < 0$. (7)
[15]

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QUESTION 7

In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for the interval $x \in [-180° ; 180°]$.



7.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(x - 60°)$ for $x \in [-180° ; 180°]$. Clearly show ALL intercepts with the axes and turning points of the graph. (3)

7.2 Write down the period of $f(3x)$. (2)

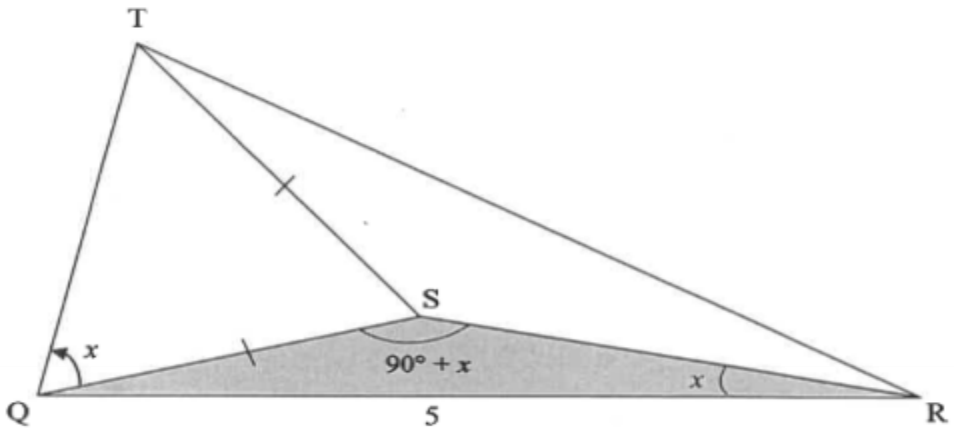
7.3 **Use the graphs** to determine the value of x in the interval $x \in [-180° ; 180°]$ for which $f(x) - g(x) = 1$. (1)

7.4 Write down the range of k, if $k(x) = \frac{1}{2}g(x) + 1$. (2)
[8]

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QUESTION 8

In the diagram below, T is a hook on the ceiling of an art gallery. Points Q, S and R are on the same horizontal plane from where three people are observing the hook T. The angle of elevation from Q to T is x.
$Q\hat{S}R = 90° + x$, $Q\hat{R}S = x$, QR = 5 units and TS = SQ.



8.1 Prove that $QS = 5 \tan x$ (3)

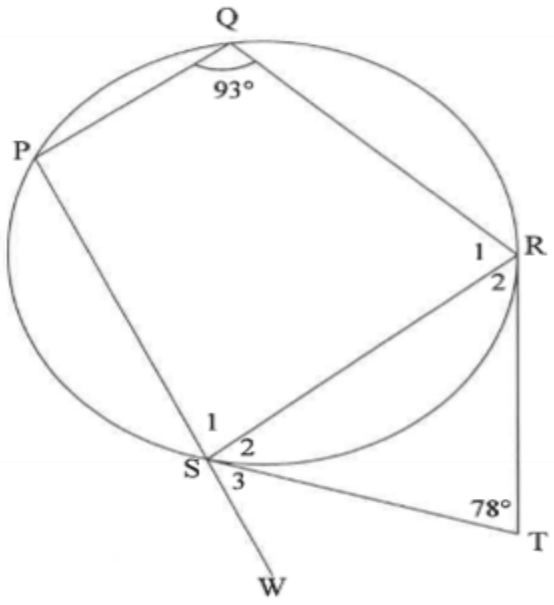
8.2 Prove that the length of $QT = 10 \sin x$ (5)

8.3 Calculate the area of ΔTQR if $T\hat{Q}R = 70°$ and $x = 25°$. (2)
[10]

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QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. PS is produced to W. TR and TS are tangents to the circle at R and S respectively. $\hat{T}=78°$ and $\hat{Q}=93°$.



9.1 Give a reason why ST = TR. (1)

9.2 Calculate, giving reasons, the size of:

9.2.1 $\hat{S}_2$ (2)

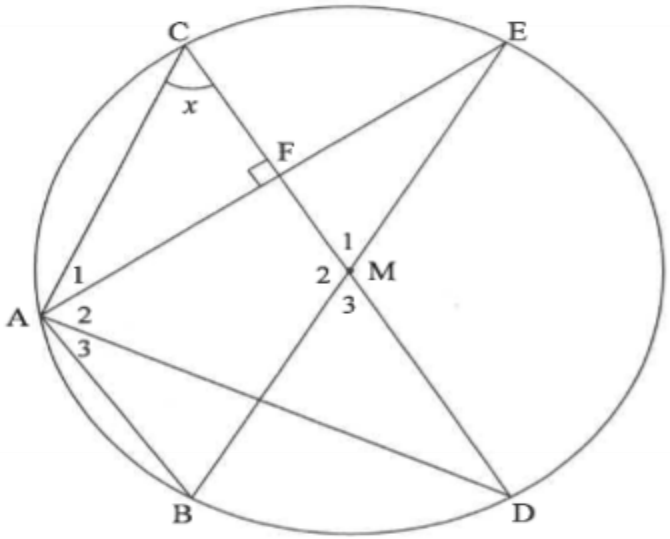
9.2.2 $\hat{S}_3$ (2)

[5]

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QUESTION 10

In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. $AE \perp CD$. Let $\hat{C} = x$.



10.1 Give a reason why AF = FE. (1)

10.2 Determine, giving reasons, the size of $\hat{M}_1$ in terms of x. (3)

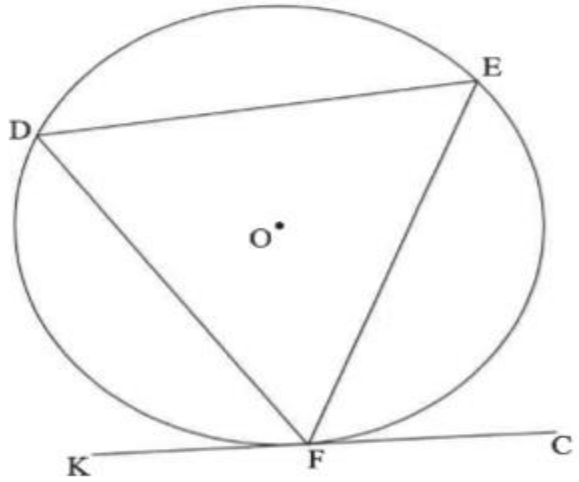
10.3 Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F. (4)

10.4 Given that CF = 6 units and AB = 24 units, calculate, giving reasons, the length of AE. (5)
[13]

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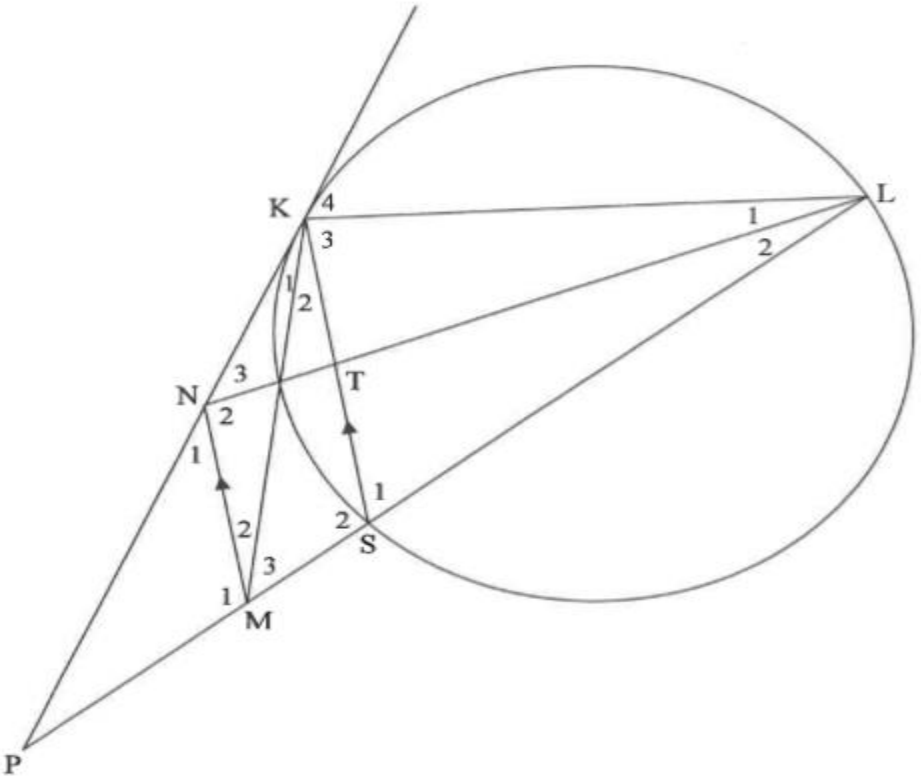
QUESTION 11

11.1 In the diagram, chords DE, EF and DF are drawn in the circle with centre O. KFC is a tangent to the circle at F.



Prove the theorem which states that $D\hat{F}K = \hat{E}$. (5)

11.2 In the diagram, PK is a tangent to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that MN ∥ SK. Chord KS and LN intersect at T.



11.2.1 Prove, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4)

(b) KLMN is a cyclic quadrilateral (1)

11.2.2 Prove, giving reasons, that $\Delta LKN \,|||\, \Delta KSM$. (5)

11.2.3 If LK = 12 units and 3KN = 4SM, determine the length of KS. (4)

11.2.4 If it is further given that NL = 16 units, LS = 13 units and KN = 8 units, determine, with reasons, the length of LT. (4)

[23]

TOTAL: 150

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GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

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Please turn over/*Blaai om asseblief*

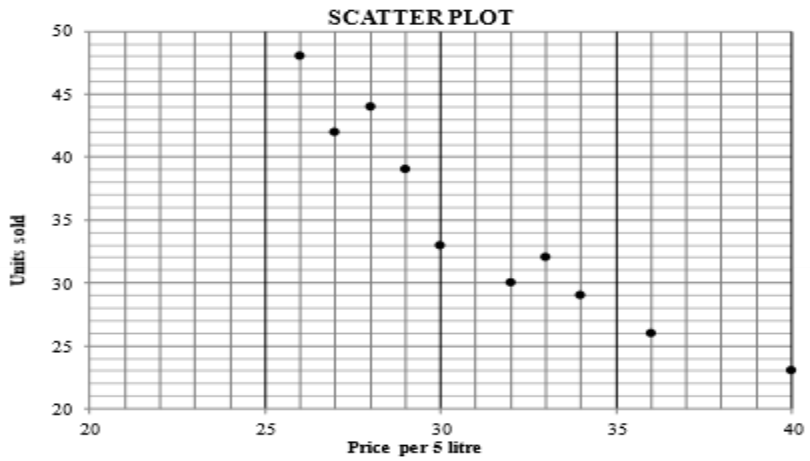
QUESTION/*VRAAG* 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$ Answer only: Full marks *Slegs antw: Volpunte*	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10{,}1707 \approx 10{,}17$	✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore$ 5 days	✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/*bestel* $20 \times 18 = 360$ sold/*verkoop* Total not sold/*Totaal nie verkoop nie*: 36 **OR/*OF*** $22 - 20 = 2$ $2 \times 18 = 36$	✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/*Opsie B* Any one of the following reasons/*Enige een van die vlg redes*: • Median/*Mediaan* = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right *Data is positief skeef/skeef na regs*	✓ answer (1)
		[10]

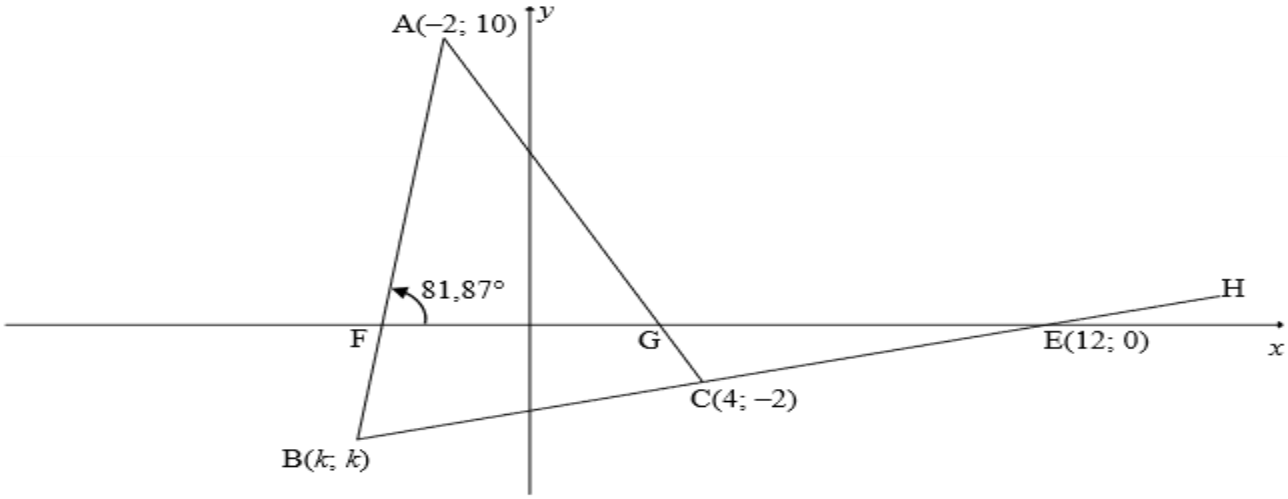
QUESTION/*VRAAG* 2

Price of milk in rands per 5-litre container (x) ***Prys van melk in rand, per 5 liter-houer (x)***	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) ***Aantal 5 liter-houers melk verkoop (y)***	48	30	26	44	23	32	39	29	42	33

2.1		1 mark: 3 to 5 points plotted correctly 2 marks: 6 to 9 points plotted correctly 3 marks: all points plotted correctly (3)
2.2	$a = 90{,}478\ldots \approx 90{,}48$ $b = -1{,}773\ldots \approx -1{,}77$ $\hat{y} = 90{,}48 - 1{,}77x$ Answer only: Full marks *Slegs antw: Volpunte*	✓ a ✓ b ✓ equation (3)
2.3	$y = 23{,}069\ldots \approx 23{,}07$ units/*eenhede* (calculator/*sakrekenaar*) **OR/*OF*** $y = 90{,}48 - 1{,}77(38)$ $y = 23{,}22$ units/*eenhede*	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	$r = -0{,}94$ The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold ∴ there is a good chance of the prediction being accurate./ *Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan ∴ daar is 'n goeie kans dat die voorspelling akkuraat is*	✓ value of r **OR/OF** strong relationship/ *sterk verwantskap* ✓ accurate/*akkuraat* (2)
		[10]

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QUESTION/*VRAAG* 3



3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $= \frac{1}{4}$		✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$	Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $0 = \frac{1}{4}(12) + c$ $c = -3$ $y = \frac{1}{4}x - 3$	$y - y_1 = m(x - x_1)$ or $y - 0 = \frac{1}{4}(x - 12)$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2)
	OR/*OF*		
	$y = mx + c$ $-2 = \frac{1}{4}(4) + c$ $c = -3$ $y = \frac{1}{4}x - 3$	$y - y_1 = m(x - x_1)$ or $y - (-2) = \frac{1}{4}(x - 4)$ $y = \frac{1}{4}x - 3$	✓ substitution of C ✓ answer (2)

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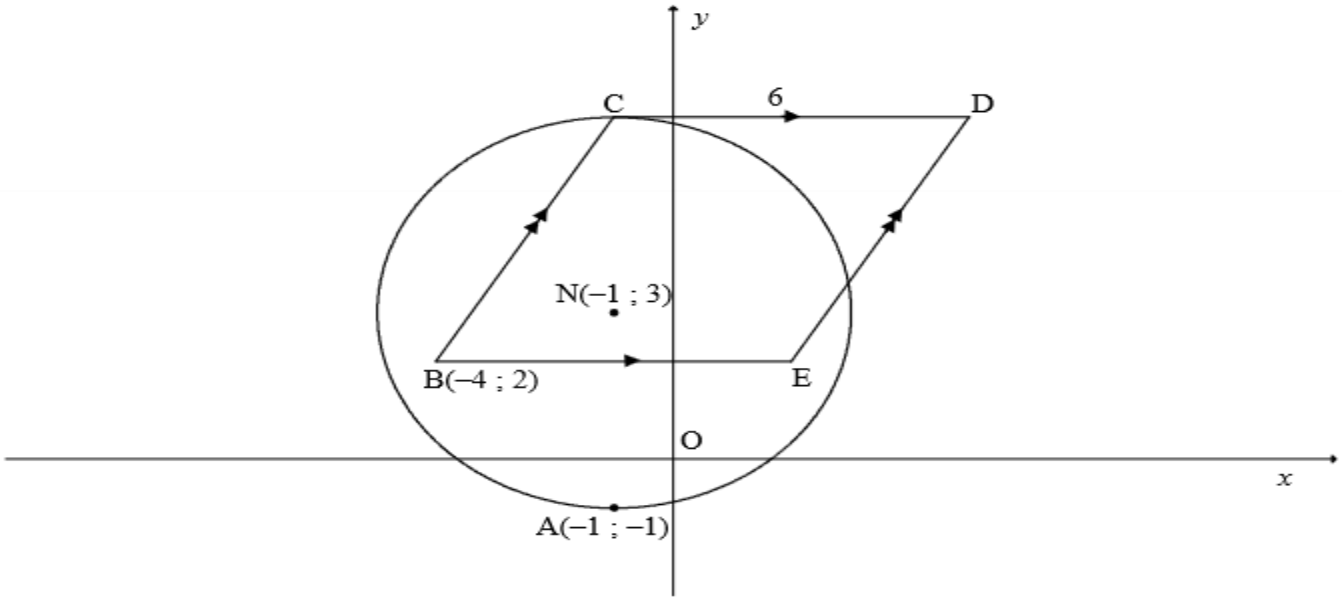
3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore$B$(-4;-4)$ **OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $-4k=12-k$ $k=-4$ $\therefore$B$(-4;-4)$ **OR/*OF*** $m_{AB}=\tan 81{,}87°$ $m_{AB}=7$ $m_{AB}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore$B$(-4;-4)$ **OR/*OF*** EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore$B$(-4;-4)$	**OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{k}{k-12}=\frac{1}{4}$ $4k=k-12$ $k=-4$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2) ✓ equating EB & AB ✓ answer (2)

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3.3.2	In ΔAFG: $m_{AC} = \frac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63,43...°$ $\therefore \theta = 116,57°$ $\therefore \hat{A} = 116,57° - 81,87°$ [ext ∠ of Δ] $\therefore \hat{A} = 34,70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $(2\sqrt{17})^2 = (6\sqrt{5})^2 + (10\sqrt{2})^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \frac{(6\sqrt{5})^2 + (10\sqrt{2})^2 - (2\sqrt{17})^2}{2(6\sqrt{5})(10\sqrt{2})}$ $= 0,822...$ $\therefore A = 34,7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116,57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $(4\sqrt{17})^2 = (\sqrt{(12-p)^2 + (0-p)^2})^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$ T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = (4\sqrt{17})^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \frac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

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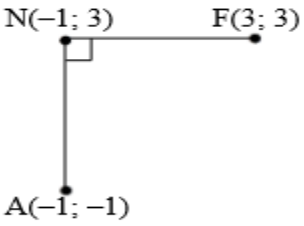
QUESTION/*VRAAG* 4



4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	CD ⊥ CN ∴ C(−1 ; 7)	✓x value ✓ y value (2)
4.2.2	CD = 6 units ∴ D(5 ; 7)	✓x value ✓ y value (2)
4.2.3	$\perp h = 5$ units DC = 6 units Area $\Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units Area $\Delta BCD = \frac{1}{2}$[Area of $\parallel^m$] $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

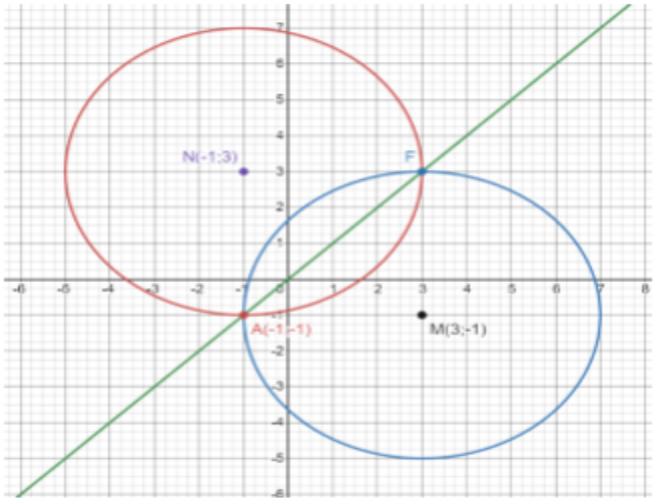
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	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \frac{5}{3}$ $\alpha = 59{,}036...°$ $B\hat{C}D = 180° - \alpha$ $B\hat{C}D = 180° - 59{,}036...°$ $B\hat{C}D = 120{,}96°$ Area $\Delta BCD = \frac{1}{2}(\sqrt{34})(6)\sin 120{,}96°$ $= 15$ units^2	✓ $B\hat{C}D = 120{,}96°$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; −1) [reflection of N(−1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; −1) $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1) **OR/*OF*** N(−1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\frac{-1+3}{2}; \frac{-1+3}{2}\right)$ = (1 ; 1)  	✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4) ✓✓ coordinates of F ✓x value ✓ y value (4)

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OR/*OF*



N(–1; 3) F(3; 3)

A(–1; –1) M(3;–1)

NAMF is a square (NA=NF=AM=MF and NA ⊥ AM)

Midpoint NM = (1 ; 1)

= Midpoint of AF

✓ NAMF = square

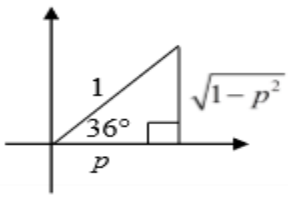
✓x ✓y of midpt NM

✓ midpt AF

(4)

[15]

QUESTION/*VRAAG* 5

5.1	$\frac{\sin 140°.\sin(360°-x)}{\cos 50°.\tan(-x)}$ $=\frac{\sin 40°(-\sin x)}{\sin 40°(-\tan x)}$ $=\frac{-\sin x}{-\frac{\sin x}{\cos x}}$ $=\cos x$	✓ $\sin 40°$ ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ answer (6)
5.2	$\text{LHS}=\frac{-2\sin^2 x+\cos x+1}{1-\cos(540°-x)}$ $\quad$ $\text{RHS}=2\cos x-1$ $\text{LHS}=\frac{-2(1-\cos^2 x)+\cos x+1}{1-(-\cos x)}$ $\text{LHS}=\frac{-2+2\cos^2 x+\cos x+1}{1+\cos x}$ $\text{LHS}=\frac{2\cos^2 x+\cos x-1}{1+\cos x}$ $\text{LHS}=\frac{(2\cos x-1)(\cos x+1)}{1+\cos x}$ $\text{LHS}=2\cos x-1$ $\therefore \text{LHS}=\text{RHS}$	✓ identity i. t. o. $\cos x$ ✓ $\cos(540°-x)=-\cos x$ ✓ standard form ✓ factors (4)
5.3.1	$\sin 36°=\sqrt{1-p^2}$ $\tan 36°=\frac{\sqrt{1-p^2}}{p}$  **OR/*OF*** $\cos^2 36°=1-\sin^2 36°$ $\cos 36°=\sqrt{1-(1-p^2)}$ $=p$ $\tan 36°=\frac{\sin 36°}{\cos 36°}$ $=\frac{\sqrt{1-p^2}}{p}$	✓ method ✓ value of p ✓ answer (3) ✓ method ✓ $\cos 36°=p$ ✓ answer (3)

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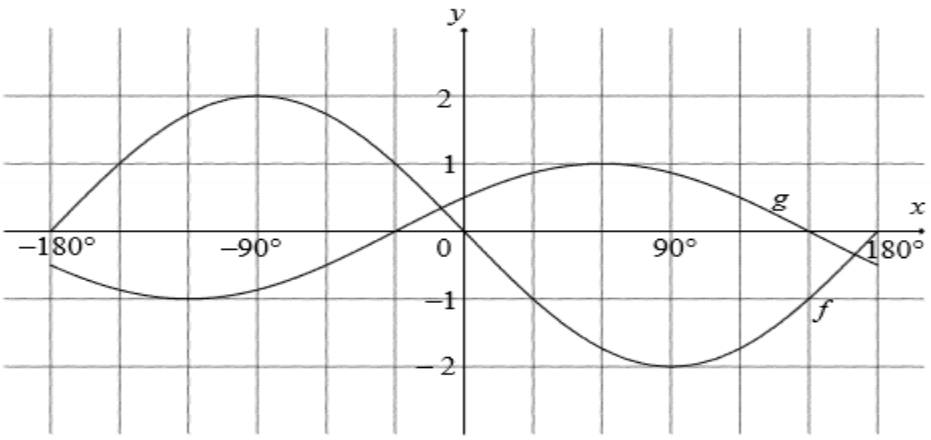
5.3.2	$\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(2\cos^2 36° - 1)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(1 - 2\sin^2 36°)$ $= -1 + 2\left(\sqrt{1-p^2}\right)^2$ $= -1 + 2(1-p^2)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(\cos^2 36° - \sin^2 36°)$ $= -\left(p^2 - \left(\sqrt{1-p^2}\right)^2\right)$ $= -\left(p^2 - (1-p^2)\right)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= \cos(2\times 54°)$ $= 2\cos^2 54° - 1$ $= 2(1-p^2) - 1$ $= 1 - 2p^2$	✓ double angle ✓✓ expansion ✓ answer i. t. o. p
	OR/*OF* $\cos 108° = \cos(72° + 36°)$ $= \cos 72°\cos 36° - \sin 72°\sin 36°$ $= (2\cos^2 36° - 1)\cos 36° - (2\sin 36°\cos 36°)\sin 36°$ $= 2\cos^3 36° - \cos 36° - 2\cos 36°\sin^2 36°$ $= 2p^3 - p - 2p\left(\sqrt{1-p^2}\right)^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$	(4) ✓ expansion ✓ both double angle identities ✓ value of sin 36° ✓ answer i. t. o. p (4)
		[17]

QUESTION/*VRAAG 6*

6.1.1	$\cos(\alpha+\beta)$ $=\cos(\alpha-(-\beta))$ $=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)$ $=\cos\alpha\cos\beta+\sin\alpha(-\sin\beta)$ $=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	✓ $\cos(\alpha-(-\beta))$ ✓ expansion ✓ reduction (3)
6.1.2	$2\cos 6x\cos 4x-\cos 10x+2\sin^2 x$ $=2\cos 6x\cos 4x-\cos(6x+4x)+2\sin^2 x$ $=2\cos 6x\cos 4x-(\cos 6x\cos 4x-\sin 6x\sin 4x)+2\sin^2 x$ $=\cos 6x\cos 4x+\sin 6x\sin 4x+2\sin^2 x$ $=\cos 2x+2\sin^2 x$ $=1-2\sin^2 x+2\sin^2 x$ $=1$	✓ $\cos 10x=\cos(6x+4x)$ ✓ expansion of $\cos(6x+4x)$ ✓ $\cos 2x$ ✓ $1-2\sin^2 x$ ✓ answer (5)
6.2	$\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=2(2\sin x\cos x)$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $\sin x(4\cos^2 x-1)=0$ $\sin x=0$ or $\cos^2 x=\frac{1}{4}$ $\cos x=-\frac{1}{2}$ $x=180°+k.360°;\ k\in Z$ or $x=120°+k.360°;\ k\in Z$ $x=240°+k.360°;\ k\in Z$ **OR/*OF*** $\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=4\sin x\cos x$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $4\sin x(1-\sin^2 x)-\sin x=0$ $3\sin x-4\sin^3 x=0$ $\sin x(3-4\sin^2 x)=0$ $\sin x=0$ or $\sin^2 x=\frac{3}{4}$ $\sin x=\frac{\sqrt{3}}{2}$ or $\sin x=-\frac{\sqrt{3}}{2}$ $x=180°+k.360°, k\in Z$ or $x=120°+k.360°, k\in Z$ or $x=240°+k.360°, k\in Z$	✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm 120°$ ✓ $k.360°;\ k\in Z$ (7) ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm 120°$ ✓ $k.360°;\ k\in Z$ (7)
		[15]

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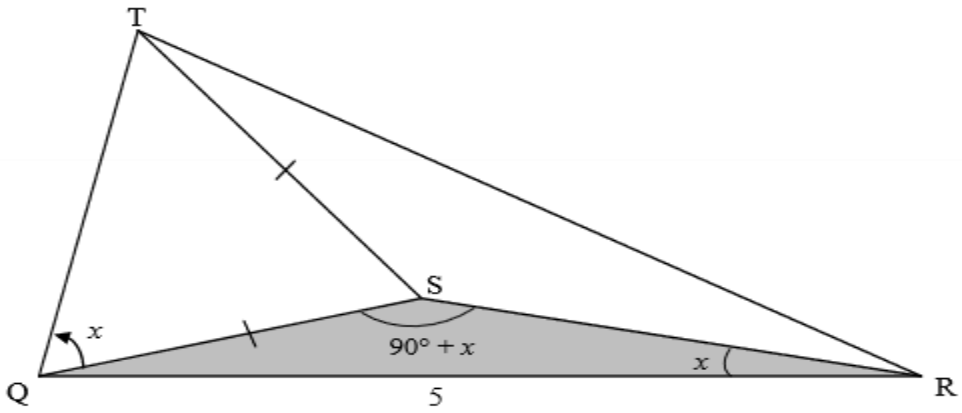
QUESTION/*VRAAG* 7

7.1		✓ both turning points ✓ both x intercepts ($-30°$ & $150°$) ✓ shape (3)
7.2	Period = $120°$	✓✓ answer (2)
7.3	$x = -30°$	✓ answer (1)
7.4	Range of/*waardeversameling van* g: $y \in [-1;1]$ Range of/*Waardeversameling van* $\frac{1}{2}g$: $y \in \left[-\frac{1}{2};\frac{1}{2}\right]$ Range of/*Waardeversameling van* $\frac{1}{2}g+1$: $y \in \left[\frac{1}{2};\frac{3}{2}\right]$ **OR/*OF*** Range of/*Waardeversameling van* $\frac{1}{2}g+1$: $\frac{1}{2} \le y \le \frac{3}{2}$	✓ critical values ✓ correct notation (2) ✓ critical values ✓ correct notation (2)
		[8]

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QUESTION/*VRAAG* 8



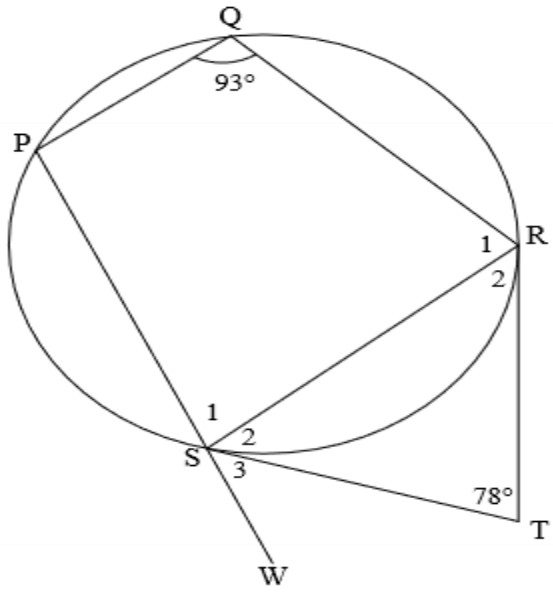
8.1	In ΔSQR: $\frac{QS}{\sin x}=\frac{QR}{\sin(90°+x)}$ $\frac{QS}{\sin x}=\frac{5}{\cos x}$ $QS=\frac{5\sin x}{\cos x}$ $QS=5\tan x$	✓ correct use of sine rule ✓ $\sin(90°+x)=\cos x$ ✓ $QS=\frac{5\sin x}{\cos x}$ (3)
8.2	$\frac{QT}{\sin(180°-2x)}=\frac{TS}{\sin x}$ $\frac{QT}{\sin 2x}=\frac{5\tan x}{\sin x}$ $QT=\frac{5\tan x\sin 2x}{\sin x}$ $QT=\frac{5\left(\frac{\sin x}{\cos x}\right)(2\sin x\cos x)}{\sin x}$ $QT=\frac{5\sin x(2\sin x)}{\sin x}$ $QT=10\sin x$	✓ correct use of sine rule ✓ TS = QS = 5tanx ✓ $QT=\frac{5\tan x\sin 2x}{\sin x}$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $\sin 2x=2\sin x\cos x$ (5)

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	OR/*OF*	
	$QT^2 = QS^2 + TS^2 - 2QS.TS\cos Q\hat{S}T$ $QT^2 = (5\tan x)^2 + (5\tan x)^2 - 2(5\tan x).(5\tan x)\cos(180° - 2x)$ $QT^2 = 50\tan^2 x - 50\tan^2 x(-\cos 2x)$ $QT^2 = 50\tan^2 x(1 + \cos 2x)$ $QT^2 = 50\tan^2 x(1 + 2\cos^2 x - 1)$ $QT^2 = 50\tan^2 x(2\cos^2 x)$ $QT^2 = 100\frac{\sin^2 x}{\cos^2 x}(\cos^2 x)$ $QT^2 = 100\sin^2 x$ $QT = 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan*x* ✓ $\cos 2x = 2\cos^2 x - 1$ & reduction ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $QT^2 = 100\sin^2 x$ (5)
	OR/*OF*	
	$TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5\tan x)^2 = (5\tan x)^2 + TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ[TQ - 10\tan x.\cos x]$ $TQ = 10\tan x.\cos x \quad (TQ \neq 0)$ $= 10\frac{\sin x}{\cos x}.\cos x$ $= 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan*x* ✓ quadratic equation ito TQ ✓ $TQ = 10\tan x \cdot \cos x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ (5)
8.3	$\text{Area of } \Delta TQR = \frac{1}{2}.TQ.QR\sin T\hat{Q}R$ $= \frac{1}{2}(10\sin 25°)(5)(\sin 70°)$ $= 9{,}93\text{ unit}^2$	✓ correct substitution into the area rule ✓ answer (2)
		[10]

QUESTION/VRAAG 9

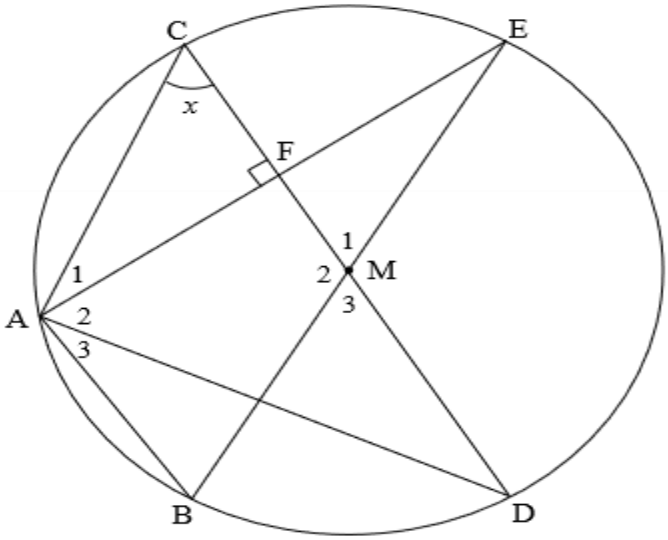


9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*	✓ R (1)
9.2.1	$\hat{S}_2 = S\hat{R}T$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of ∠s in Δ/*som van ∠e in Δ*]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp ∠s of cyclic quad/*teenoorst ∠e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [∠s on a str line/*∠e op reguitlyn*]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
		[5]

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QUESTION/*VRAAG* 10

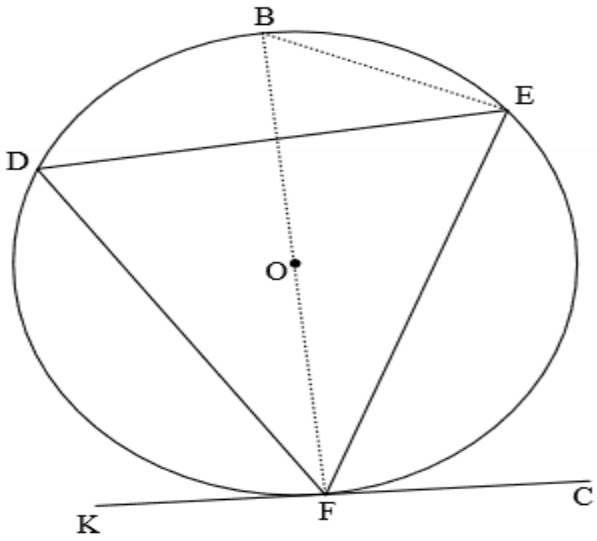


10.1	line from centre ⊥ to chord/*lyn vanaf middelpunt ⊥ op koord*	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\therefore \hat{M}_1 = 180° - 2x$ [∠at centre=2× at circumf/*midpts∠*= 2×omtreks∠]	✓ S ✓ S ✓ R (3)
10.3	$C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $\hat{A}_2 = 90° - (90° - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $E\hat{M}D = 2x$ [adj suppl ∠s/*aanligg suppl ∠e*] $\therefore \hat{A}_2 = x$ [∠at centre= 2×∠ at circumf/*midpts∠*= 2 ×omtreks∠] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $\hat{M}_3 = 180° - 2x$ [vert. opp/ *regoorstaande ∠e*] $\therefore \hat{A}_3 = 90° - x$ [∠at centre= 2×∠ at circumf/*midpts∠*= 2 ×omtreks∠] $B\hat{A}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF***	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ S ✓ R ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

	CD ∥ AB [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90^\circ$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.∠s; CD∥AB/*verwiss ∠e*] $\therefore \hat{A}_2 = x =$ C ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] AC = diameter [converse ∠ in semi circle/*omgek ∠ in halfsirkel*] ∴ AD is a tangent [converse radius ⊥ tangent/*omgek radius⊥rkl*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)
10.4	AF = FE and BM = ME [given & radii] $\therefore$ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelptstelling*] EM = MB = CM = 18 units [radii] ∴ EB = 36 units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] AE = $12\sqrt{5}$ or 26,83 units **OR/*OF*** AF = FE and BM = ME [given & radii] $\therefore$ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelpuntstelling*] EM = MB = CM = 18 units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] FE = $6\sqrt{5}$ AE = $12\sqrt{5}$ or 26,83 units	✓ FM = 12 ✓ R ✓ EB = 36 ✓ using Pyth correctly ✓ answer (5) ✓ FM = 12 ✓ R ✓ EM = 18 ✓ using Pyth correctly ✓ answer (5)
		[13]

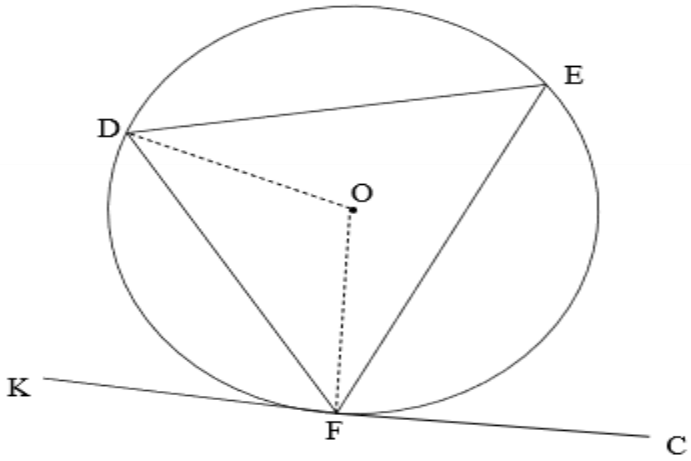
QUESTION/*VRAAG* 11

11.1



	Construction: Draw diameter BF and draw BE *Konstruksie: Trek middellyn* BF *en verbind* BE	✓Constr
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$B\hat{E}F = 90°$ [∠ in semi-circle/*semi-sirkel*]	✓ S
	$\therefore D\hat{E}F = 90° - B\hat{E}D$ $= 90° - B\hat{F}D$ [∠s same segment/*∠e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	(5)

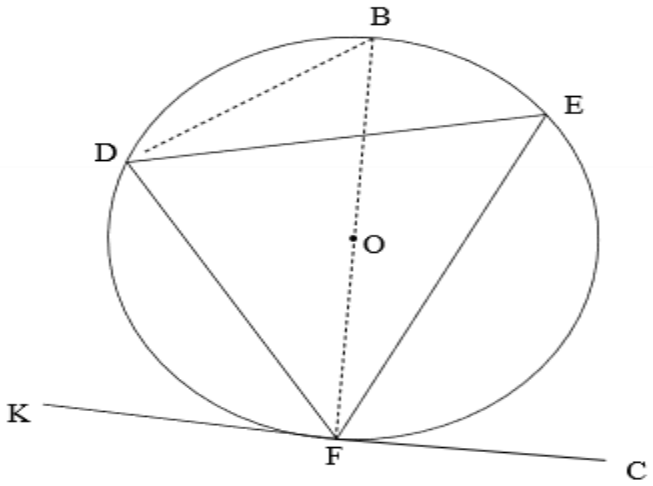
OR/*OF*



	Construction: Draw radii DO and OF *Konstruksie: Trek radii DO en OF* $O\hat{F}K = 90°$ or $D\hat{F}K = 90° - O\hat{F}D$ radius⊥ tangent/*raaklyn*] $O\hat{D}F = O\hat{F}D$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore D\hat{O}F = 180° - 2O\hat{F}D$ [∠s of Δ/*∠e van Δ*] $D\hat{E}F = 90° - O\hat{F}D$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore D\hat{F}K = D\hat{E}F$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

OR/OF

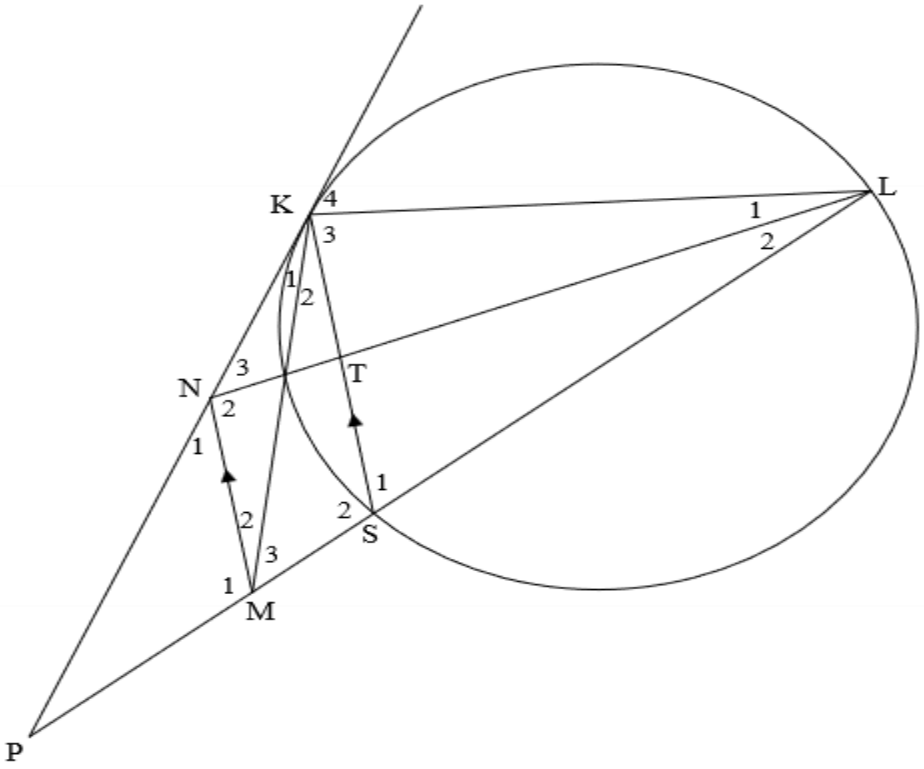
KEEP WORKING HARD, YOU WILL BE GREATFUL FOR YOUR EFFORT AT THE END.



	Construction: Draw diameter BF and join BD. *Konstruksie: Trek middellyn BF en verbind BD.* $B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius⊥ tangent/*raaklyn*] $F\hat{D}B = 90°$ [∠ in half circle/*semi-sirkel*] $\hat{B} = 90° - B\hat{F}D$ $\therefore D\hat{F}K = \hat{B}$ but $\hat{B} = \hat{E}$ [∠s same segment/*∠e dieselfde segment*] $\therefore D\hat{F}K = \hat{E}$	✓ construction ✓ S ✓/R ✓ S ✓ S/R (5)

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11.2



11.2.1(a)	$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp ∠s; / *ooreenk ∠s;* MN ∥ KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$	✓ S ✓ R ✓ S ✓ R (4)
11.2.1(b)	$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$ ∴ KLMNis a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF** $N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp ∠s; / *ooreenk ∠s;* MN ∥ KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*] $\hat{N}_1 = K\hat{L}S$ ∴ KLMNis a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF**	✓ R (1) ✓ R (1)

	$\hat{NKL} = 180^\circ - K_4$ [adj. suppl.] $\therefore \hat{NKL} = 180^\circ - \hat{NML}$ [proved] $\therefore$ KLMN is a cyclic quad [opp.$\angle$s supplementary]	✓ R (1)
11.2.2	In $\Delta LKN \| \| \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $\hat{L}_1 = \hat{M}_2$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $= \hat{K}_2$ [alt $\angle$s; / *verw $\angle$e;* MN||KS] $\hat{NKL} = \hat{MSK}$ [$\angle$s of Δ / *$\angle$e van Δ*] $\Delta LKN \| \| \Delta KSM$ **OR/OF** In $\Delta LKN \| \| \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $\hat{NKL} = \hat{M}_1$ [ext $\angle$ of cyclic quad/*buite $\angle$ van koordevh*] $= \hat{S}_2$ [corresp $\angle$s/*ooreenk $\angle$e*; KS || NM] $\Delta LKN \| \| \Delta KSM$ [$\angle,\angle,\angle$] **OR/OF** In $\Delta LKN \| \| \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $\hat{K}_4 + \hat{NKL} = \hat{S}_1 + \hat{S}_2$ [$\angle$s on straight line/*$\angle$e op reguitlyn*] $\therefore \hat{NKL} = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\Delta LKN \| \| \Delta KSM$ [$\angle,\angle,\angle$]	✓ S ✓ R ✓ S ✓ S/R ✓ S (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\Delta LKN \| \| \Delta KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	$4SM = 3KN$ $SM = \frac{3(8)}{4}$ $SM = 6$ $\frac{LT}{NL} = \frac{LS}{ML}$ [line || one side of Δ / *lyn || een sy v Δ*] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10{,}95$	✓ SM = 6 ✓ S ✓ R ✓ answer (4)
		[23]

TOTAL/*TOTAAL*: 150

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GRADE 12

MATHEMATICS P2

NOVEMBER 2022

MARKS: 150

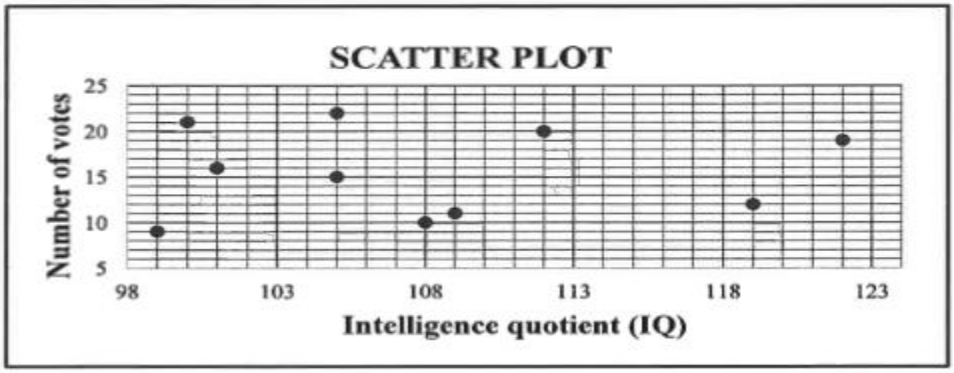
TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.

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QUESTION 1

The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligence quotient) of the 10 learners who received the most votes and the number of votes that they received.



Before the election, the popularity of each of these ten learners was established and a popularity score (out of a 100) was assigned to each. The popularity scores and the number of votes of the same 10 learners who received the most votes are shown in the table below.

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

1.1 Calculate the:

1.1.1 Mean number of votes that these 10 learners received (2)

1.1.2 Standard deviation of the number of votes that these 10 learners received (1)

1.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited? (2)

1.3 Determine the equation of the least squares regression line for the data given in the table. (3)

1.4 Predict the number of votes that a learner with a popularity score of 72 will receive. (2)

1.5 Using the scatter plot and table above, provide a reason why:

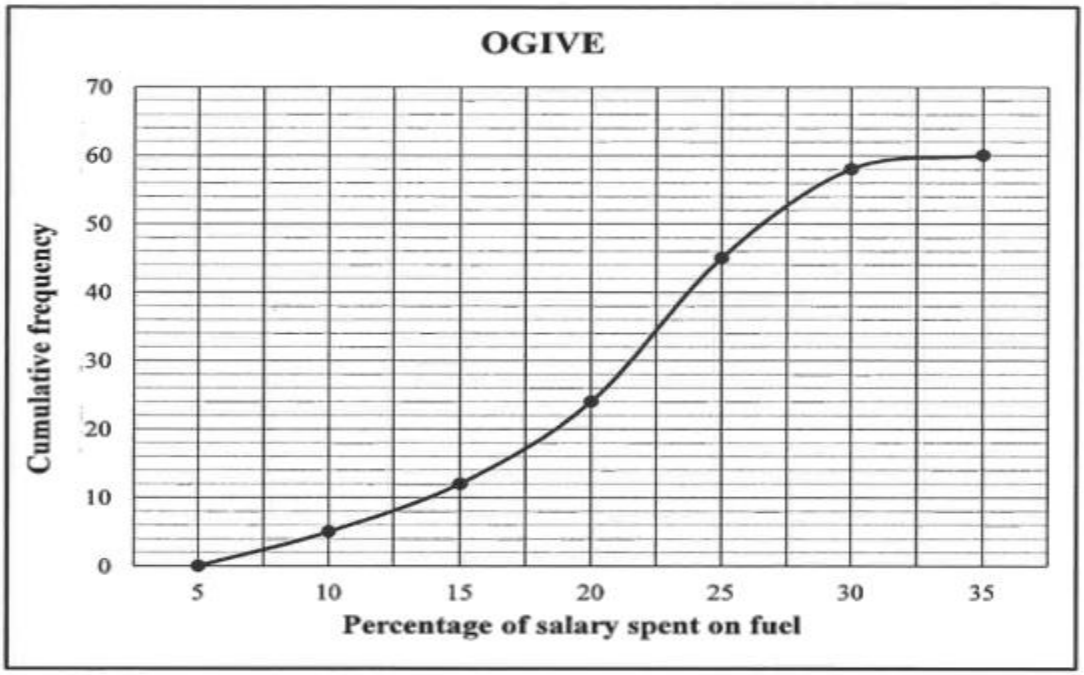
1.5.1 IQ is not a good indicator of the number of votes that a learner could receive (1)

1.5.2 The prediction in QUESTION 1.4 is reliable (1)
[12]

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QUESTION 2

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.1 How many people are employed at this company? (1)

2.2 Write down the modal class of the data. (1)

2.3 How many employees spent more than 22,5% of their monthly salary on fuel? (2)

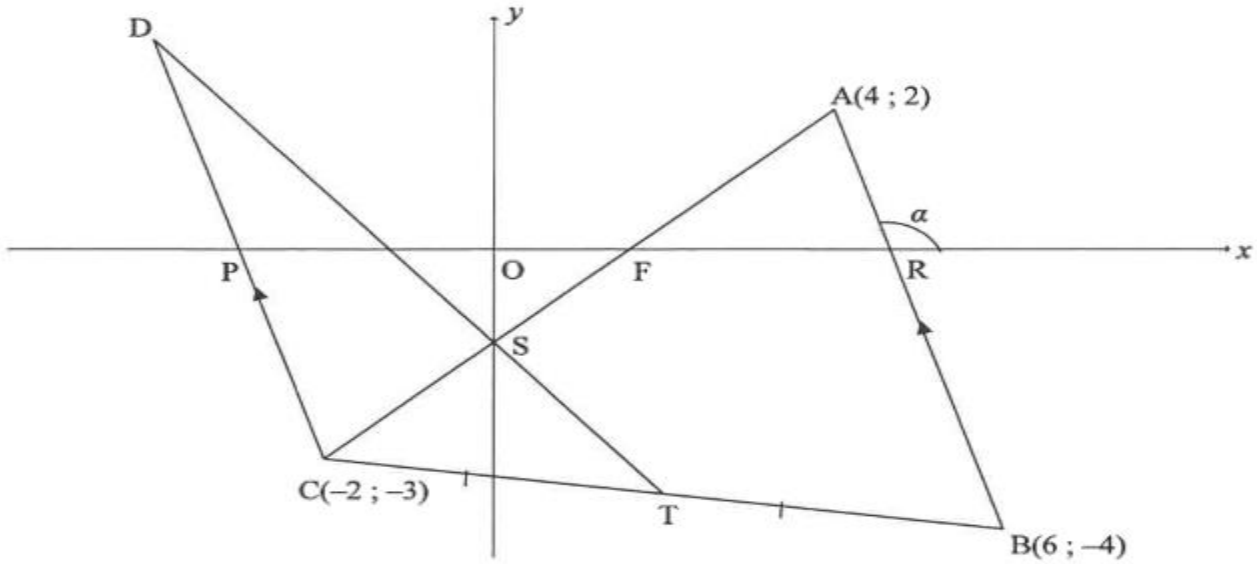
2.4 An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel. (2)

2.5 The monthly salaries of these employees remains constant and the number of litres of fuel used in each month also remains constant. If the fuel price increases from R21,43 per litre to R22,79 per litre at the beginning of the next month, how will the above ogive change? (2)
[8]

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QUESTION 3

In the diagram, A(4 ; 2), B(6 ; –4) and C(–2 ; –3) are vertices of ΔABC. T is the midpoint of CB. The equation of line AC is $5x-6y=8$. The angle of inclination of AB is α. ΔDCT is drawn such that CD ∥ BA. The lines AC and DT intersect at S, the y-intercept of AC. P, F and R are the x-intercepts of DC, AC and AB respectively.



3.1 Calculate the:

3.1.1 Gradient of AB (2)

3.1.2 Size of α (2)

3.1.3 Coordinates of T (2)

3.1.4 Coordinates of S (2)

3.2 Determine the equation of CD in the form $y=mx+c$. (3)

3.3 Calculate the:

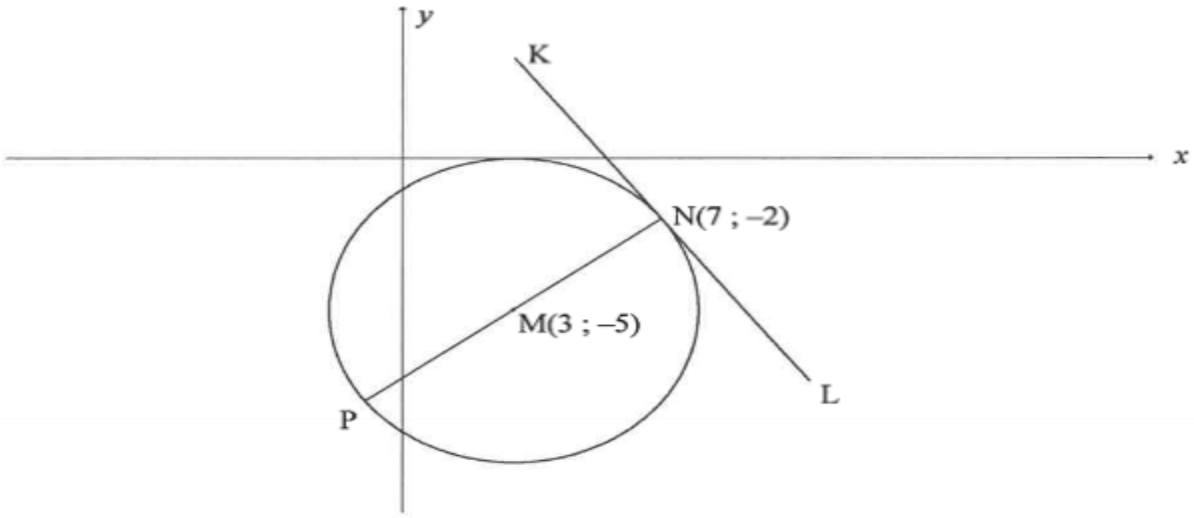
3.3.1 Size of $D\hat{C}A$ (4)

3.3.2 Area of POSC (5)
[20]

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QUESTION 4

In the diagram, M(3 ; –5) is the centre of the circle having PN as its diameter. KL is a tangent to the circle at N(7 ; –2).



4.1 Calculate the coordinates of P. (2)

4.2 Determine the equation of:

4.2.1 The circle in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2.2 KL in the form $y=mx+c$ (5)

4.3 For which values of k will $y=-\frac{4}{3}x+k$ be a secant to the circle? (4)

4.4 Points A(t ; t) and B are not shown on the diagram.

From point A, another tangent is drawn to touch the circle with centre M at B.

4.4.1 Show that the length of tangent AB is given by $\sqrt{2t^2+4t+9}$. (2)

4.4.2 Determine the minimum length of AB. (4)
[20]

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QUESTION 5

5.1 Given that $\sqrt{13}\sin x+3=0$, where $x\in(90°;270°)$.

Without using a calculator, determine the value of:

5.1.1 $\sin(360°+x)$ (2)

5.1.2 $\tan x$ (3)

5.1.3 $\cos(180°+x)$ (2)

5.2 Determine the value of the following expression, **without using a calculator**:

$$\frac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$$ (5)

5.3 Determine the general solution of the following equation:

$(\cos x+2\sin x)(3\sin 2x-1)=0$ (6)

5.4 Given the identity: $\cos(x+y).\cos(x-y)=1-\sin^2 x-\sin^2 y$

5.4.1 Prove the identity. (4)

5.4.2 Hence, determine the value of $1-\sin^2 45°-\sin^2 15°$, **without using a calculator**. (3)

5.5 Consider the trigonometric expression: $16\sin x.\cos^3 x-8\sin x.\cos x$

5.5.1 Rewrite the expression as a single trigonometric ratio. (4)

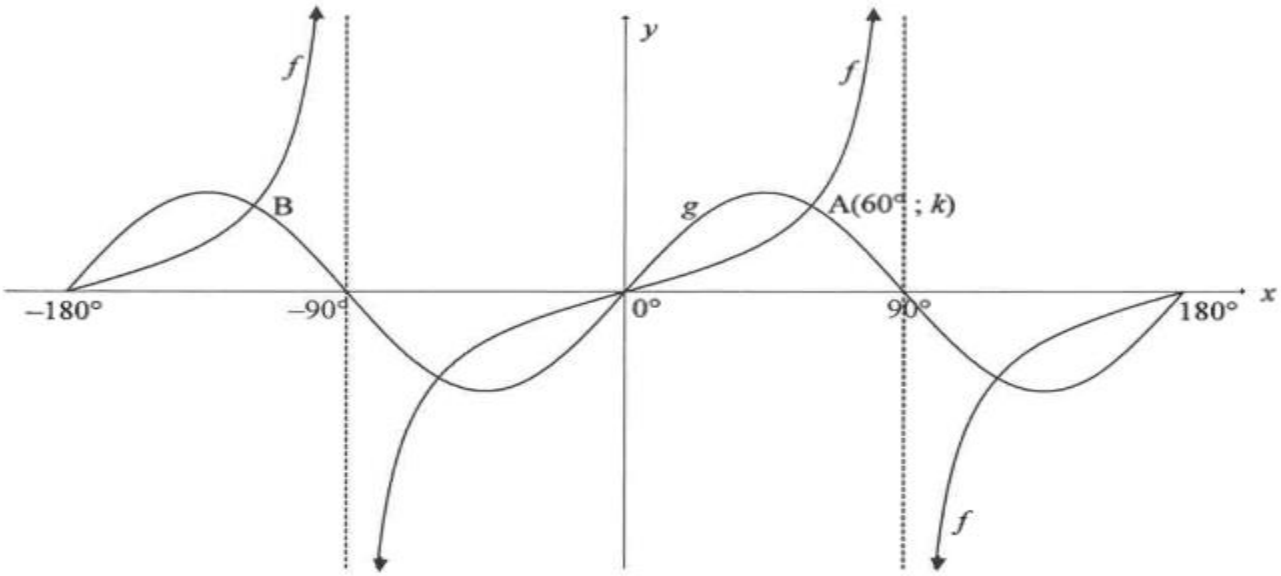
5.5.2 For which value of x in the interval $x\in[0°;90°]$ will $16\sin x.\cos^3 x-8\sin x.\cos x$ have its minimum value? (1)

[30]

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QUESTION 6

In the diagram below, the graphs of $f(x)=\tan x$ and $g(x)=2\sin 2x$ are drawn for the interval $x\in[-180°;180°]$ A$(60°;k)$ and B are two points of intersection of f and g.



6.1 Write down the period of g. (1)

6.2 Calculate the:

6.2.1 Value of k (1)

6.2.2 Coordinates of B (1)

6.3 Write down the range of $2g(x)$. (2)

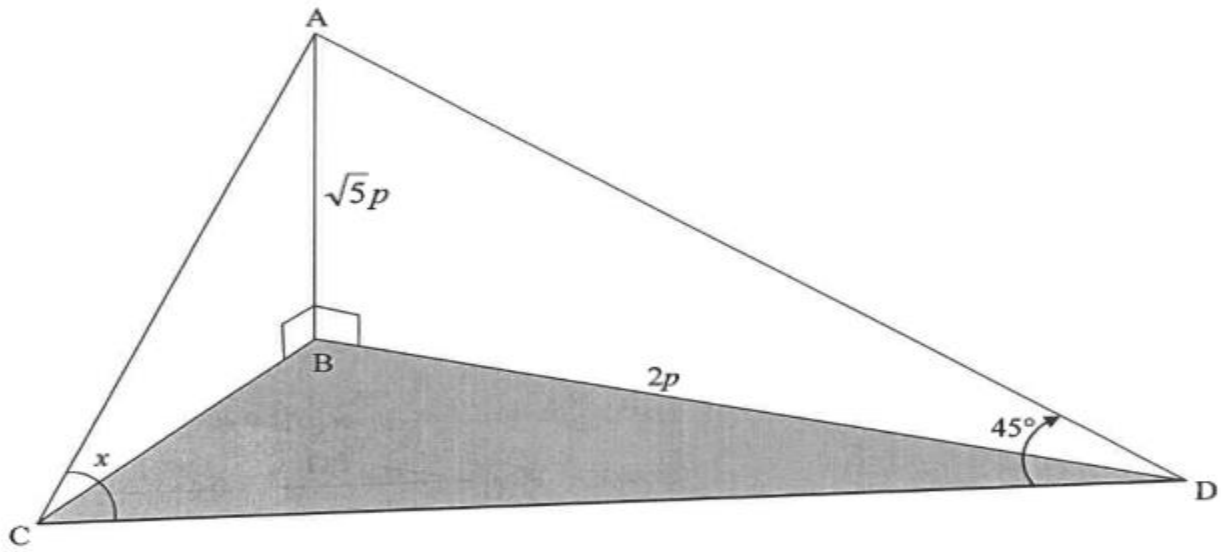
6.4 For which values of x will $g(x+5°)-f(x+5°)\leq 0$ in the interval $x\in[-90°;0°]$? (2)

6.5 Determine the values of p for which $\sin x.\cos x=p$ will have exactly two real roots in the interval $x\in[-180°;180°]$. (3)
[10]

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QUESTION 7

AB is a vertical flagpole that is $\sqrt{5}p$ metres long. AC and AD are two cables anchoring the flagpole. B, C and D are in the same horizontal plane.
$BD = 2p$ metres, $A\hat{C}D = x$ and $A\hat{D}C = 45°$.



7.1 Determine the length of AD in terms of p. (2)

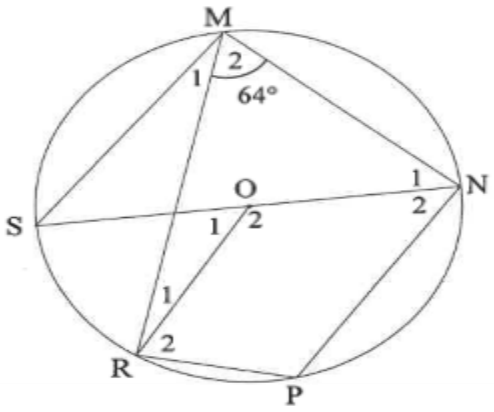
7.2 Show that the length of $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$. (5)

7.3 If it is further given that $p = 10$ and $x = 110°$, calculate the area of ΔADC. (3)
[10]

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QUESTION 8

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle. Chord MS and radius OR are drawn. $\hat{M}_2 = 64°$.



Determine, giving reasons, the size of the following angles:

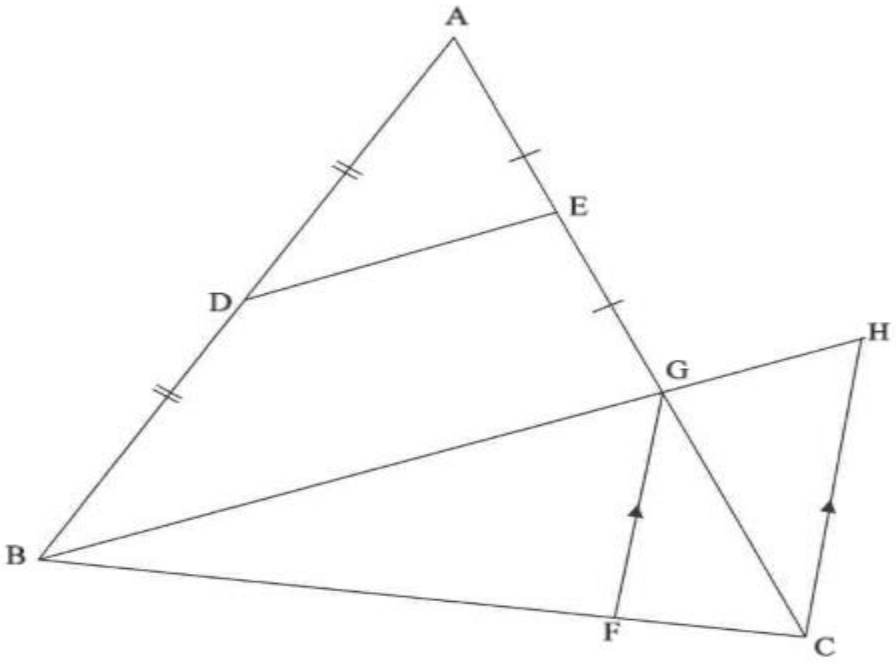
8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

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8.2 In the diagram, ΔABG is drawn. D and E are midpoints of AB and AG respectively. AG and BG are produced to C and H respectively. F is a point on BC such that FG || CH.



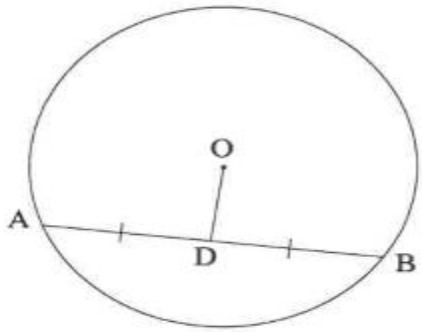
8.2.1 Give a reason why DE || BH. (1)

8.2.2 If it is further given that $\frac{FC}{BF}=\frac{1}{4}$, $DE = 3x - 1$ and $GH = x + 1$, calculate, giving reasons, the value of x. (6)
[13]

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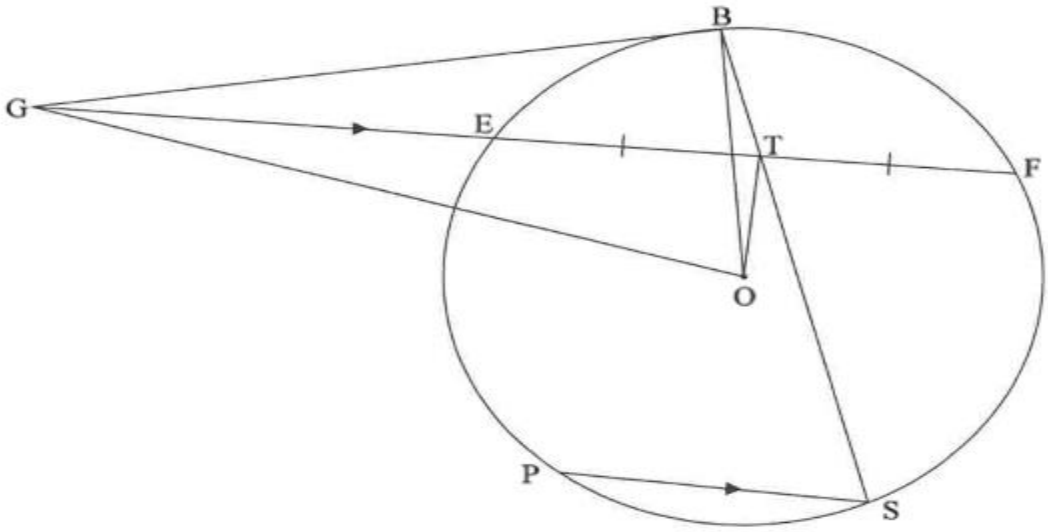
QUESTION 9

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.



Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. OD ⊥ AB. (5)

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. PS || GF.



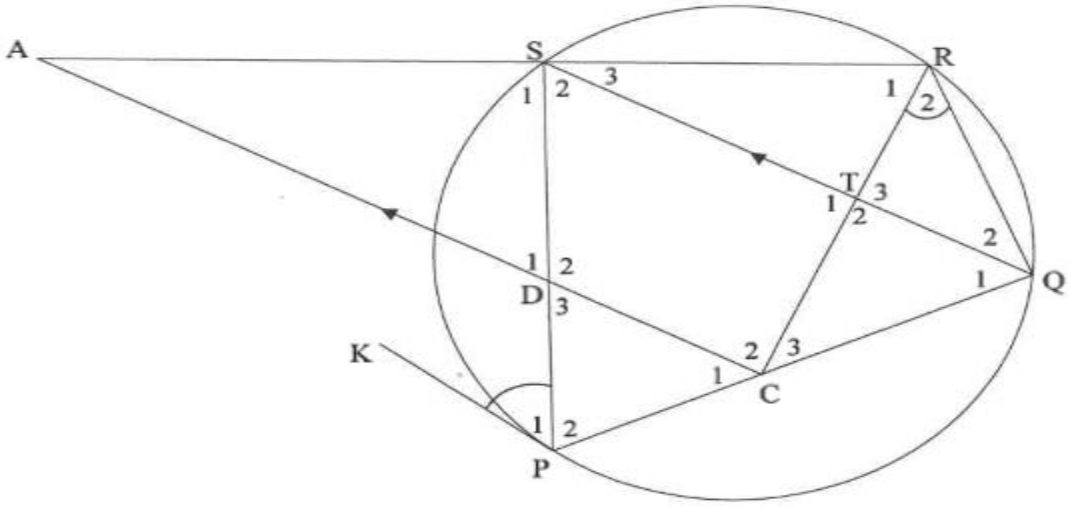
Prove, giving reasons, that:

9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)
[14]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A. CA ∥ QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\frac{AD}{AR} = \frac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)
[13]

TOTAL: 150

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GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2022

MARKING GUIDELINES/*NASIENRIGLYNE*

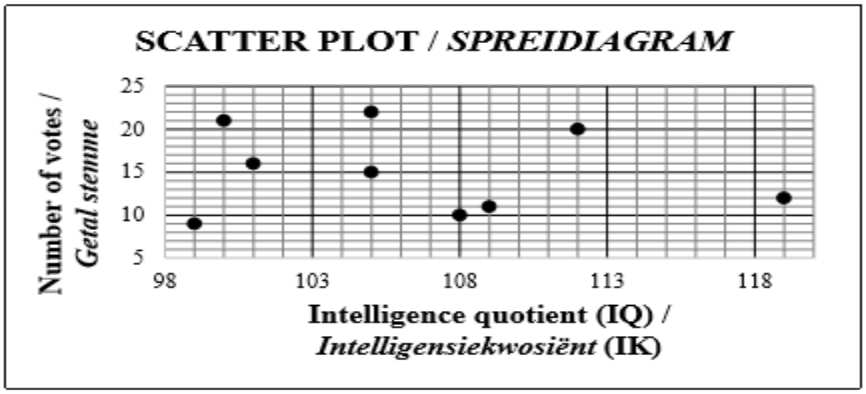
MARKS/*PUNTE*: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

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Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 1



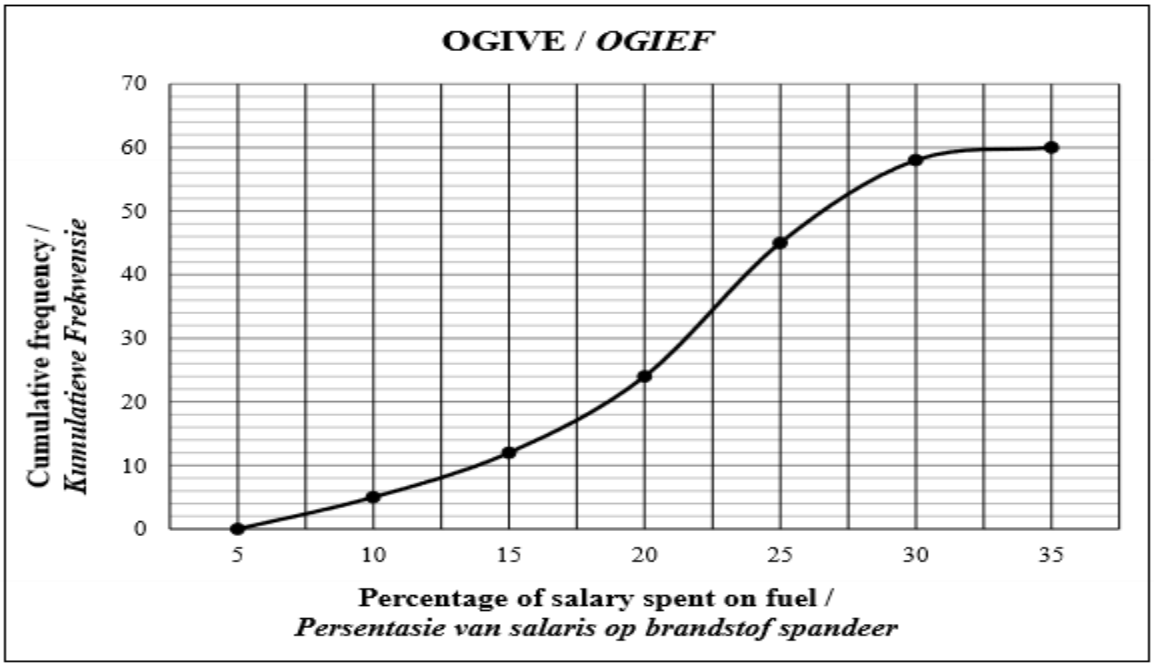
Popularity score (x) ***Gewildheidspunt*** **(x)**	32	89	35	82	50	59	81	40	79	65
Number of votes (y) ***Getal stemme*** **(y)**	9	22	10	21	11	15	20	12	19	16

1.1.1	$\bar{y} = \frac{155}{10}$ $= 15,5$ ANSWER ONLY: Full marks	✓155 ✓answer (2)
1.1.2	SD = 4,59	✓ answer (1)
1.2	$\bar{y} - \text{SD}$ $= 15,5 - 4,59$ $= 10,91$ $\therefore$ 10 − 2 = 8 learners	✓ value of $\bar{y} - \text{SD}$ ✓ answer (2)
1.3	$a = 1,7709\ldots$ $b = 0,2243\ldots$ $\hat{y} = 1,77 + 0,22x$	✓ a ✓ b ✓ equation (3)
1.4	$\hat{y} = 1,77 + 0,22(72)$ $= 17,61$ ≈ 18 votes **OR/*OF*** $\hat{y} = 17,92 \approx 18$ votes	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.5.1	Points are all scattered therefore low correlation and unrealistic prediction./*Punte is versprei daarom 'n lae korrelasie en onrealistiese voorspelling.*	✓ R (1)
1.5.2	$r = 0,98$/correlation very strong/*korrelasie baie sterk* $\therefore$ a reliable prediction/*'n betroubare voorspelling*	✓ S (1)
		[12]

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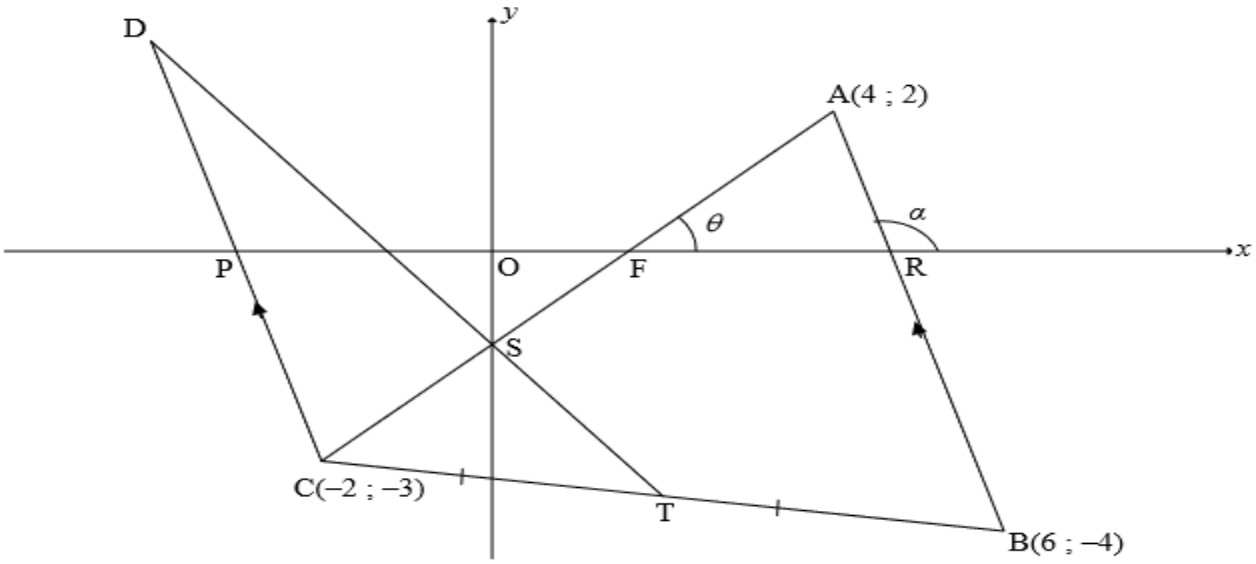
QUESTION/*VRAAG* 2



2.1	60 employees	✓ answer (A) (1)
2.2	$20 < x \leq 25$	✓ answer (1)
2.3	$60 - 34$ $= 26$ employees ANSWER ONLY: Full marks	✓ 34 ✓ answer (2)
2.4	Salary $= \frac{100}{7} \times 2400$ Salary = R34 285,71 ANSWER ONLY: Full marks	✓ method ✓ answer (2)
2.5	∴ Ogive/Cumulative frequency graph will shift to the right/will become steeper. ∴ *Ogief/Kumulatiewe frekwensie grafiek sal na regs skuif/sal steiler wees.*	✓✓ answer (2)
		[8]

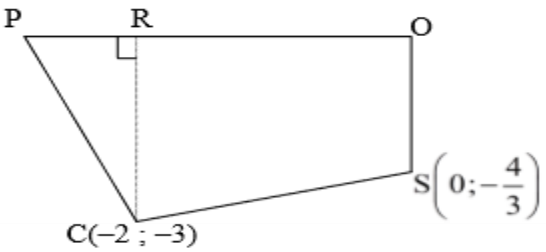
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QUESTION/*VRAAG* 3



3.1.1	$m_{AB}=\frac{2-(-4)}{4-6}$ **OR** $m_{AB}=\frac{-4-2}{6-4}$ $m_{AB}=-3$ ANSWER ONLY: Full marks	✓ substitution ✓ answer (2)
3.1.2	$\tan\alpha=m_{AB}=-3$ $\alpha=108,43°$ ANSWER ONLY: Full marks	✓ $\tan\alpha=m_{AB}=-3$ ✓ answer (2)
3.1.3	$T\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $T\left(\frac{-2+6}{2};\frac{-3-4}{2}\right)$ $T\left(2;\frac{-7}{2}\right)$	✓ $x_T=2$ ✓ $y_T=\frac{-7}{2}$ (2)
3.1.4	$5(0)-6y=8$ $y=-\frac{4}{3}$ $S\left(0;\frac{-4}{3}\right)$	✓ $x_S=0$ ✓ $y_S=\frac{-4}{3}$ (2)
3.2	$m_{CD}=m_{AB}=-3$ $-3=-3(-2)+c$ OR $y-(-3)=-3(x-(-2))$ $c=-9$ $y=-3x-9$ $y=-3x-9$	✓ gradient ✓ substitution of C(–2 ; –3) ✓ equation (3)

3.3.1	$5x-6y=8$ $y=\frac{5}{6}x-\frac{8}{6}$ $\tan\theta=m_{AC}=\frac{5}{6}$ $\theta=39{,}81°$ $\hat{A}=108{,}43°-39{,}81°$ $=68{,}62°$ $D\hat{C}A=68{,}62°$ [alt ∠s ; DC||AB]	✓ $\tan\theta=m_{AC}=\frac{5}{6}$ ✓ $\theta=39{,}81°$ ✓ $\hat{A}=68{,}62°$ ✓ answer (4)
3.3.2	$P(-3;0)$ and $F(1{,}6\ ;\ 0)$ Area POSC = Area ΔFPC – Area ΔOFS $=\frac{1}{2}(4{,}6)(3)-\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ $=6{,}9-1{,}07$ $=5{,}83$ units^2 **OR/*OF*** $P(-3;0)$ $FC=\sqrt{\left(-2-\frac{8}{5}\right)^2+(-3-0)^2}=\frac{3\sqrt{61}}{5}$ Area $\Delta PFC=\frac{1}{2}(PF)(FC)\sin O\hat{F}S$ $=\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ $=6{,}90$ Area $\Delta OFS=\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ $=1{,}07$ Area POSC $=6{,}90-1{,}07$ $=5{,}83$ units^2 **OR/*OF***	✓ $P(-3;0)$ ✓ method ✓ $\frac{1}{2}(4{,}6)(3)$ ✓ $\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ ✓ answer (5) ✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ ✓ $\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)



P(–3;0)

✓ P(–3;0)

Area of POSC = Area of OSCR + Area of ΔPRC

✓ method

$$=\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2+\frac{1}{2}(1\times 3)$$

✓ $\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2$ ✓ $\frac{1}{2}(1\times 3)$

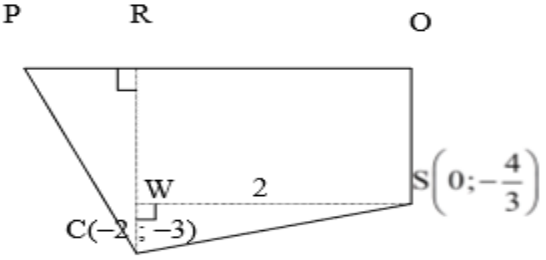
$$=\frac{35}{6}$$

$$=5,83\text{ units}^2$$

✓ answer

(5)

OR/
OF



P(–3;0)

✓ P(–3;0)

Area POSC = Area ROSW + Area ΔPRC + Area ΔWSC

✓ method

$$=\left(\frac{4}{3}\right)(2)+\frac{1}{2}(1)(3)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$$

✓ $\frac{1}{2}(1)(3)$

✓ $\left(\frac{4}{3}\right)(2)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$

$$=\frac{35}{6}$$

$$=5,83\text{units}^2$$

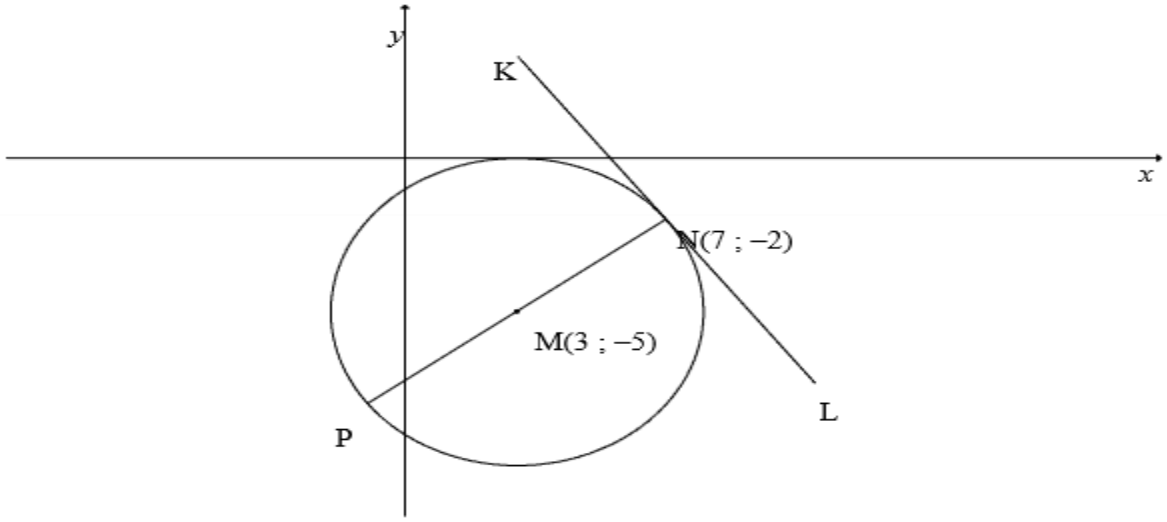
✓ answer

(5)

OR/*OF*

	$P(-3;0)$ Area of $\Delta PSC = \frac{1}{2}(PC)(CS)\sin D\hat{C}A$ $= \frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ $= 3,833..$ Area of $\Delta POS = \frac{1}{2}(PO)(OS)$ $= \frac{1}{2}(3)\left(\frac{4}{3}\right)$ $= 2$ Area POSC $= 3,833\ldots + 2$ $= 5,83 \text{units}^2$	✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ ✓ $\frac{1}{2}(3)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)
		[20]

QUESTION/*VRAAG* 4



4.1	P$(x;y)$; N$(7;-2)$; M$(3;-5)$ $\frac{x+7}{2}=3$ $\quad$ $\frac{y-2}{2}=-5$ $x=-1$ $\quad$ $y=-8$ P$(-1;-8)$	✓ $x_P=-1$ ✓ $y_P=-8$ (2)
4.2.1	$r^2=(7-3)^2+(-2-(-5))^2$ **OR/*OF*** $r^2=(-1-3)^2+(-8-(-5))^2$ $r^2=25$ $(x-3)^2+(y+5)^2=25$	✓ substitution into distance formula ✓ $(x-3)^2+(y+5)^2$ ✓ $r^2=25$ (3)
4.2.2	$m_{\text{radius}}=\frac{-5-(-2)}{3-7}=\frac{3}{4}$ $m_{\text{tangent}}=-\frac{4}{3}$ [radius ⊥ tangent/*raaklyn ⊥ radius*] $-2=-\frac{4}{3}(7)+c$ **OR** $y-(-2)=-\frac{4}{3}(x-7)$ $c=\frac{22}{3}$ $\quad$ $y=-\frac{4}{3}x+\frac{22}{3}$ $y=-\frac{4}{3}x+\frac{22}{3}$	✓ substitution ✓ $m_{\text{radius}}=\frac{-3}{-4}=\frac{3}{4}$ ✓ $m_{\text{tangent}}=-\frac{4}{3}$ ✓ substitution of m and N$(7 ; -2)$ ✓ equation (5)
4.3	$-8=-\frac{4}{3}(-1)+c$ $\therefore\ c=-\frac{28}{3}$ $-\frac{28}{3}<k<\frac{22}{3}$	✓ subst m and P ✓ value of c ✓✓ answer (4)

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4.4.1	$AB^2 = AM^2 - MB^2$ $AB^2 = \left[(t-3)^2 + (t+5)^2\right] - 5^2$ $= t^2 - 6t + 9 + t^2 + 10t + 25 - 25$ $AB = \sqrt{2t^2 + 4t + 9}$	✓ substitution into Pythagoras ✓ simplification (A) (2)
4.4.2	$t = \frac{-4}{2(2)}$ $= -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** $4t + 4 = 0$ $t = -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** Length of $AB = \sqrt{2t^2 + 4t + 9}$ $= \sqrt{2\left(t^2 + 2t + \frac{9}{2}\right)}$ $= \sqrt{2\left[(t+1)^2 + \frac{7}{2}\right]}$ $= \sqrt{2(t+1)^2 + 7}$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$	✓ substitution into correct formula ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ derivative = 0 ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ completing of the square ✓ $t = -1$ ✓ substitution ✓ answer (4)
		[20]

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QUESTION/*VRAAG* 5

5.1.1	$\sin(360°+x)$ $=\sin x$	✓ + ✓ $\sin x$ (2)
5.1.2	$x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(-3)^2}$ $=-2$ $\tan x=\frac{-3}{-2}$ $=\frac{3}{2}$ **OR/*OF*** $x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(3)^2}$ $=2$ $\tan x=\frac{3}{2}$	✓✓ substitution ✓ method (3) ✓✓ substitution ✓ method (3)
5.1.3	$\cos(180°+x)$ $=-\cos x$	✓ – ✓ $\cos x$ (2)
5.2	$\frac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$ $=\frac{-\sin\theta}{\sin(-(180°-\theta))-3\sin\theta}$ $=\frac{-\sin\theta}{-\sin\theta-3\sin\theta}$ $=\frac{-\sin\theta}{-4\sin\theta}$ $=\frac{1}{4}$	✓ $-\sin\theta$ ✓ $-3\sin\theta$ ✓ $-\sin\theta$ ✓ simplification ✓ answer (5)

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5.3	$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ $\cos x + 2\sin x = 0$ or $3\sin 2x - 1 = 0$ $\tan x = -\frac{1}{2}$ $\sin 2x = \frac{1}{3}$ ref $\angle = 26{,}565...°$ ref $\angle = 19{,}471...°$ $x = 153{,}43° + k.180°\ ;\ k \in Z$ $x = 9{,}74° + k.180°;\ k \in Z$ **OR/*OF*** or $x = 153{,}43° + k.360°\ ;\ k \in Z$ $x = 80{,}26° + k.180°;$ $k \in Z$ or $x = 333{,}43° + k.360°\ ;\ k \in Z$	✓ both equations ✓ $\tan x = -\frac{1}{2}$ ✓ $\sin 2x = \frac{1}{3}$ ✓ $x = 153{,}43°$ **OR** $x = 153{,}43°$ &$333{,}43°$ ✓ $x = 9{,}74°$ & $80{,}26°$ ✓ $+ k.180°\ ;\quad k \in Z$ (6)
5.4.1	$\text{LHS} = \cos(x+y).\cos(x-y)$ $= [\cos x.\cos y - \sin x.\sin y][\cos x.\cos y + \sin x.\sin y]$ $= \cos^2 x.\cos^2 y - \sin^2 x.\sin^2 y$ $= (1-\sin^2 x)(1-\sin^2 y) - \sin^2 x.\sin^2 y$ $= 1 + \sin^2 x.\sin^2 y - \sin^2 x - \sin^2 y - \sin^2 x.\sin^2 y$ $= 1 - \sin^2 x - \sin^2 y \quad = \text{RHS}$	✓ expansion ✓ simplification ✓ square identity ✓ product (4)
5.4.2	$1 - \sin^2 45° - \sin^2 15°$ $= \cos(45° + 15°).\cos(45° - 15°)$ $= \cos 60°.\cos 30°$ $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}$ **OR/*OF***	✓ identifying x and y ✓ substitution ✓ answer (3)

	$1-\sin^2 45°-\sin^2 15°$ $=\sin^2 15°+\cos^2 15°-\sin^2 45°-\sin^2 15°$ $=\cos^2 15°-\left(\frac{\sqrt{2}}{2}\right)^2$ $=\cos^2 15°-\frac{1}{2}$ $=\frac{2\cos^2 15°-1}{2}$ $=\frac{\cos 30°}{2}$ $=\frac{\sqrt{3}}{2}\times\frac{1}{2}$ $=\frac{\sqrt{3}}{4}$	✓ identity ✓ substitution ✓ answer (3)
	OR $1-\sin^2 45°-\sin^2 15°$ $=\cos^2 45°-\sin^2(45°-30°)$ $=\left(\frac{1}{\sqrt{2}}\right)^2-(\sin 45°\cos 30°-\cos 45°\sin 30°)^2$ $=\frac{1}{2}-\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{3}{8}-\frac{\sqrt{3}}{4}+\frac{1}{8}\right)$ $=\frac{\sqrt{3}}{4}$	✓ expansion ✓ substitution ✓ answer (3)

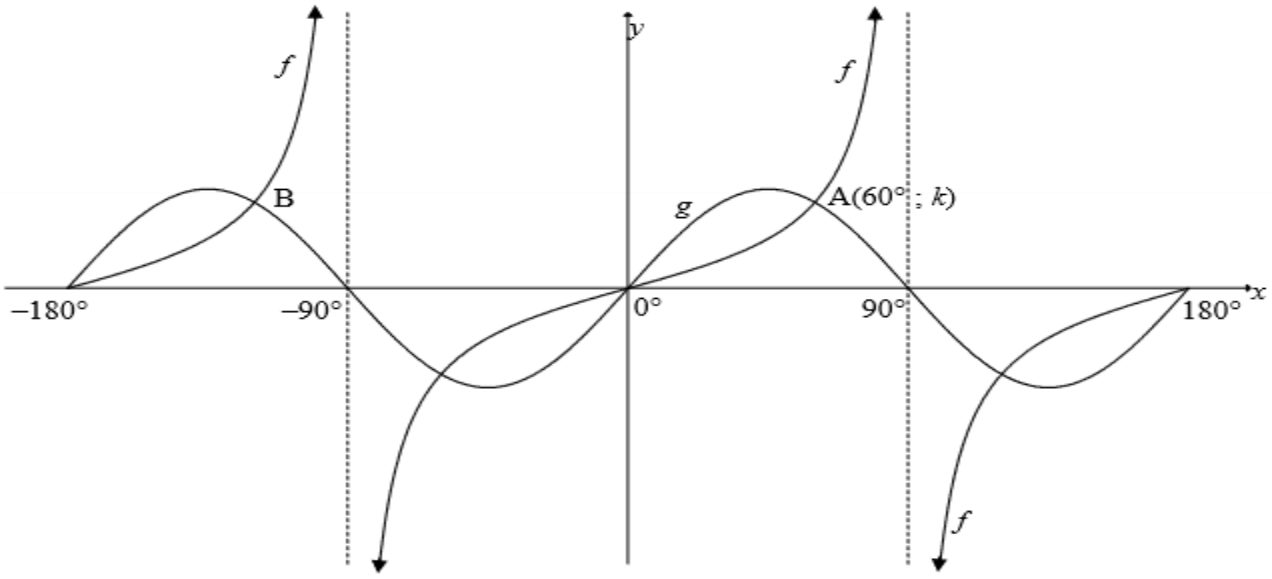
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5.5.1	$16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 8\sin x.\cos x(2\cos^2 x - 1)$ $= 4\sin 2x(\cos 2x)$ $= 2\sin 4x$ **OR/*OF*** $16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 16\cos^2 x\left(\frac{1}{2}\sin 2x\right) - 8\left(\frac{1}{2}\sin 2x\right)$ $= 8\left(2\cos^2 x - 1\right)\left(\frac{1}{2}\sin 2x\right)$ $= 4\sin 2x.\cos 2x$ $= 2\sin 4x$	✓ factorisation ✓ $4\sin 2x$ ✓ $\cos 2x$ ✓ double angle (4) ✓ factorisation ✓ $4\sin 2x$ ✓ $\cos 2x$ ✓ double angle (4)
5.5.2	$16\sin x.\cos^3 x - 8\sin x.\cos x = 2\sin 4x$ Minimum at $x = 67{,}5°$	✓ answer (1)
		[30]

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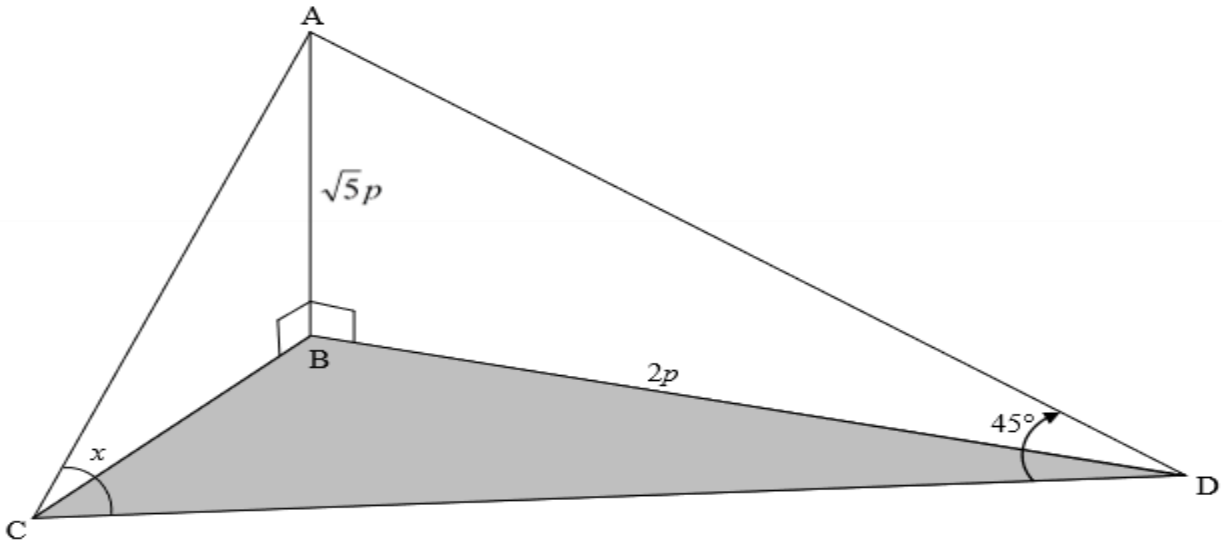
QUESTION/*VRAAG* 6



6.1	180°	✓ answer (1)
6.2.1	$k=\sqrt{3}=1,73$	✓ answer (1)
6.2.2	$B(-120°;\sqrt{3})$	✓ $x=-120°$ (1)
6.3	Range of g: $y\in[-2;2]$ Range of $2g(x)$: $y\in[-4;4]$ **OR/*OF*** Range of g: $-2\le y\le 2$ Range of $2g(x)$: $-4\le y\le 4$ ANSWER ONLY: Full marks	✓ $y\in[-2;2]$ ✓ answer (2) ✓ $-2\le y\le 2$ ✓ answer (2)
6.4	$x\in[-65°;-5°]$ **OR/*OF*** $-65°\le x\le -5°$	✓✓ $x\in[-65°;-5°]$ (2) ✓✓ $-65°\le x\le -5°$ (2)
6.5	$\sin x.\cos x=p$ $4\sin x.\cos x=4p$ $2\sin 2x=4p$ $4p=\pm 2$ $\therefore\ p=-\frac{1}{2}$ or $\frac{1}{2}$ ANSWER ONLY: Full marks	✓ $2\sin 2x=4p$ ✓ $4p=\pm 2$ ✓ answers (3)
		[10]

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QUESTION/*VRAAG* 7



7.1	$AD^2 = AB^2 + BD^2$ $AD^2 = (\sqrt{5}p)^2 + (2p)^2$ $AD^2 = 9p^2$ $AD = 3p$	✓ substitution in Pythagoras ✓ answer (2)
7.2	$\frac{CD}{\sin(135° - x)} = \frac{3p}{\sin x}$ $CD = \frac{3p\sin(135° - x)}{\sin x}$ $CD = \frac{3p(\sin 135°\cos x - \cos 135°\sin x)}{\sin x}$ $CD = \frac{3p(\sin 45°\cos x + \cos 45°\sin x)}{\sin x}$ $CD = \frac{3p\left(\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x\right)}{\sin x}$ $CD = \frac{3p\left(\frac{\sqrt{2}}{2}\right)(\cos x + \sin x)}{\sin x}$ $CD = \frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$	✓ correct use of sine rule ✓ $135° - x$ ✓ compound angle ✓ special values ✓ factorisation (5)

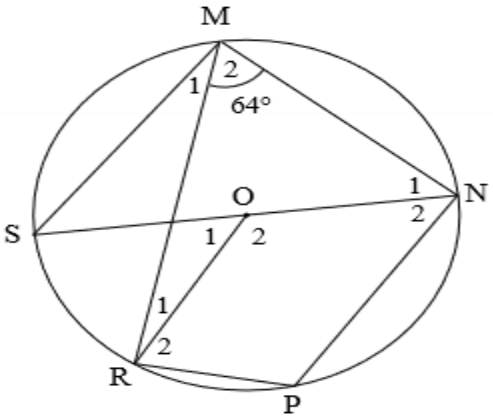
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7.3	$\text{Area } \Delta ADC = \frac{1}{2}(AD)(CD)\sin A\hat{D}C$ $= \frac{1}{2}(3p)\left(\frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}\right)(\sin 45°)$ $= \frac{1}{2}(30)\left(\frac{30(\sin 110° + \cos 110°)}{\sqrt{2}\sin 110°}\right)\sin 45°$ $= 143{,}11 m^2$	✓ correct use of area rule ✓ substitution in area rule ✓ answer (3)
		[10]

QUESTION/*VRAAG* 8

8.1

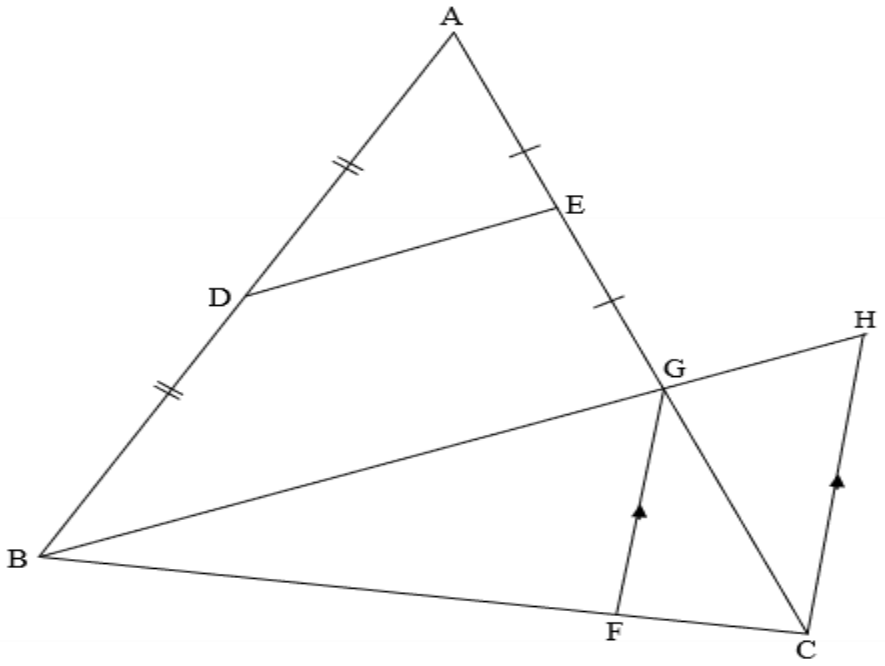


8.1.1	$\hat{P} = 116°$	[opp ∠s of cyclic quad/*teenoorst. ∠e van kvh*]	✓ S ✓ R (2)
8.1.2	$\hat{M}_1 + 64° = 90°$ $\hat{M}_1 = 26°$	[∠ in semi-circle/∠ *in halwe sirkel*]	✓ R ✓ S (2)
8.1.3	$\hat{O}_1 = 52°$	[∠ at centre = 2 x ∠ at circumference/*midpts. ∠ = 2 x omtreks. ∠*]	✓ S ✓R (2)

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8.2



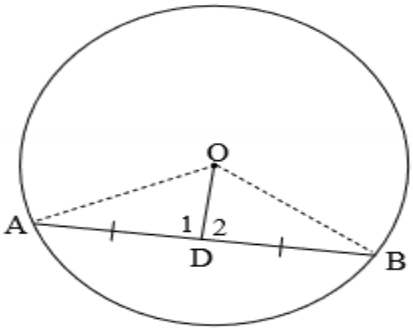
8.2.1	Midpt theorem/*Midpt. Stelling* **OR/*OF*** Converse prop intercept theorem	✓ R (1) ✓ R (1)
8.2.2	BG = 2DE or $6x - 2$ [Midpt theorem/*Midpt. stelling*] BG = $6x - 2$ $\frac{GH}{BG} = \frac{FC}{BF}$ [line ‖ one side of Δ **OR** prop theorem; FG ‖ CH / *lyn* ‖ *een sy v.* Δ] $\frac{x+1}{6x-2} = \frac{1}{4}$ $4x + 4 = 6x - 2$ $2x = 6$ $x = 3$ **OR/*OF***	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)

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	$\frac{BF}{FC} = \frac{BG}{GH}$	[line ‖ one side of Δ **OR** prop theorem; FG ‖ CH / *lyn* ‖ *een sy v.* Δ]	✓S ✓R
	$\frac{AE}{AG} = \frac{DE}{BG}$	[ΔADE ||| ΔABG]	✓S ✓R
	$BG = 4x + 4$		
	$\frac{1}{2} = \frac{3x-1}{4x+4}$		✓ equation into x
	$\therefore 4x + 4 = 6x - 2$		
	$\therefore x = 3$		✓ answer (6)
			[13]

QUESTION/*VRAAG* 9

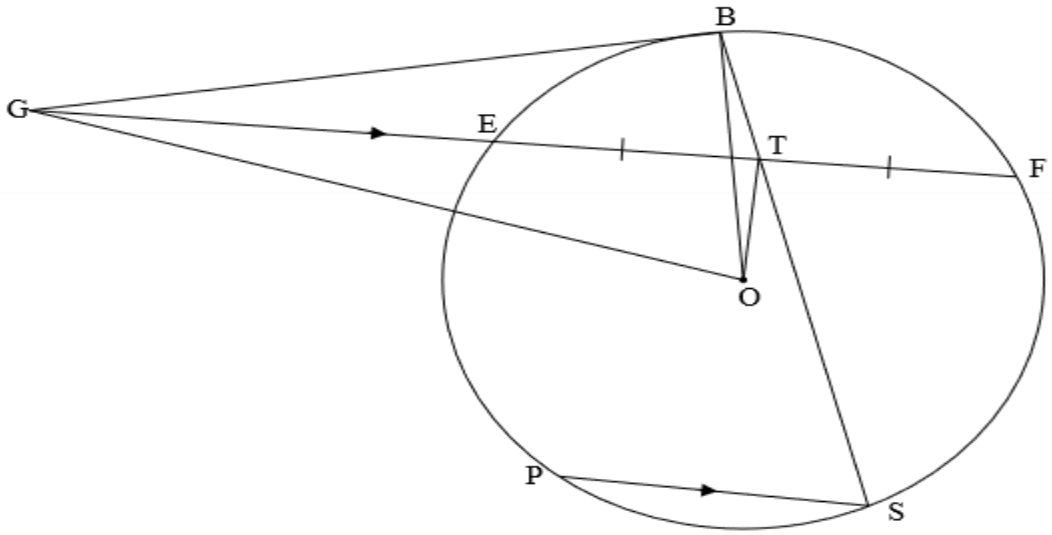
9.1



9.1.1	Construction: Draw OA and OB In ΔADO and ΔBDO OA = OB OD = OD AD = DB ∴ ΔADO ≡ ΔBDO ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ ∴ OD ⊥ AB	 [radii/*radiusse*] [common side/*gemeenskaplike sy*] [given/*gegee*] [S;S;S] ΔADO ≡ ΔBDO [∠s on a str line/*∠e op 'n reguitlyn*]	✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5)
	OR/*OF* Construction: Draw OA and OB In ΔADO and ΔBDO AD = DB $\hat{A} = \hat{B}$ OA = OB ∴ ΔADO ≡ ΔBDO ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ ∴ OD ⊥ AB	 [given/*gegee*] [∠s opp; ∠s sides /*∠e teenoor gelyke sye*] [radii/*radiusse*] [S;∠;S] ΔADO ≡ ΔBDO [∠s on a str line/*∠e op 'n reguitlyn*]	✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5)

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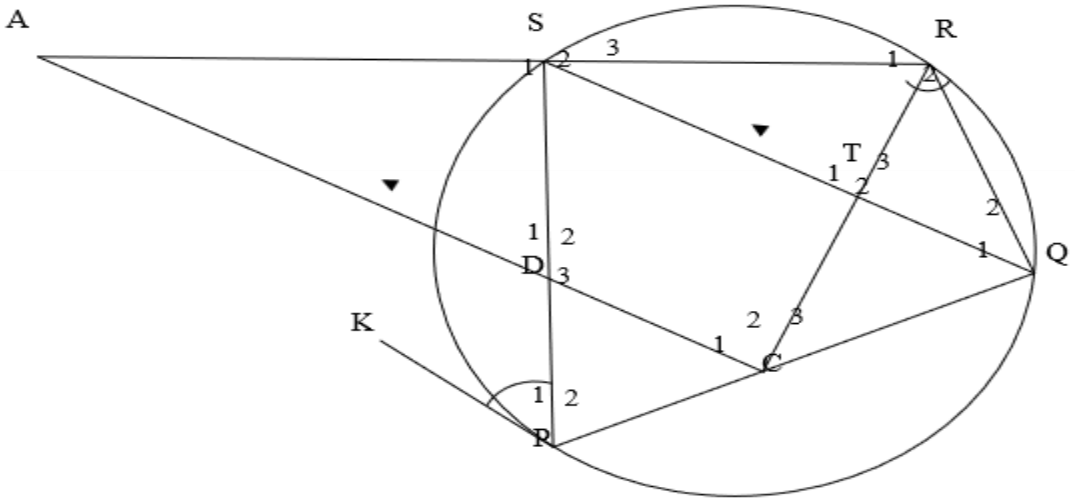
9.2



9.2.1	$O\hat{T}G = 90°$ [line from centre to midpt of chord/ *midpt. sirkel; midpt. koord*] $O\hat{B}G = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*] $\therefore O\hat{T}G = O\hat{B}G = 90°$ $\therefore$ OTBG is a cyclic quadrilateral [line subtends equal ∠s **OR** converse ∠s in the same segment/ *lyn onderspan gelyke ∠e*]	✓S ✓R ✓ S ✓R ✓ R (5)
9.2.2	$\hat{S} = B\hat{T}G$ [corresp ∠s; GF ‖ PS / *ooreenk. ∠s; GF ‖ PS*] But $B\hat{T}G = G\hat{O}B$ [∠s in the same segment/*∠e in dies. sirkelsegment*] $G\hat{O}B = \hat{S}$	✓ S ✓R ✓ S ✓R (4)
		[14]

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QUESTION/*VRAAG* 10



10.1	$\hat{P}_1 = \hat{Q}_1$	[tan-chord theorem/∠ *tussen raaklyn en koord*]	✓ S
	$\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$ $\therefore \hat{S}_1 = \hat{P}_1 + \hat{Q}_2$	[ext ∠ of cyclic quad/*buite ∠ v. kvh*]	✓ S / R
	$\hat{T}_2 = \hat{R}_2 + \hat{Q}_2$	[ext ∠ of Δ/*buite ∠ v. Δ*]	✓ S
	but $\hat{P}_1 = \hat{R}_2$	[given/*gegee*]	
	$\hat{T}_2 = \hat{P}_1 + \hat{Q}_2$ $\therefore \hat{S}_1 = \hat{T}_2 = \hat{P}_1 + \hat{Q}_2$		✓ S (4)
10.2	In Δ ASD and ΔACR		✓ identifying Δ's
	$\hat{A} = \hat{A}$	[common ∠/*gemeenskaplike ∠*]	✓ S
	$\hat{S}_1 = \hat{T}_2$	[proven/*reeds bewys*]	
	$\hat{T}_2 = \hat{C}_2$]	[alt ∠s; QS ‖ CA/verw. ∠*e*; QS ‖ CA	✓ S / R
	$\therefore \hat{S}_1 = \hat{C}_2$		✓ S
	$\hat{D}_1 = \hat{R}_1$	[sum of ∠s in Δ/*∠e v. Δ*]	✓ S
	ΔASD ||| ΔACR $\therefore \frac{AD}{AR} = \frac{AS}{AC}$	[corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]	(5)
	OR/*OF*		

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	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common ∠/*gemeenskaplike ∠*] $\hat{S}_1 = \hat{T}_2$ [proven/*gegee*] $\hat{T}_2 = \hat{C}_2$ [alt ∠s; QS ‖ CA/verw. ∠*e*; QS ‖ CA] $\therefore \hat{S}_1 = \hat{C}_2$ ΔASD ||| ΔACR [∠;∠;∠] $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]	✓ identifying Δ's ✓ S ✓ S/R ✓ S ✓ R (5)
10.3	$\frac{AS}{AC} = \frac{SD}{CR}$ [ΔASD ||| ΔACR] $\therefore AS = \frac{AC \times SD}{CR}$ $\frac{AS}{AR} = \frac{CT}{CR}$ [line ‖ one side of Δ OR prop theorem; TS ‖ CA/*lyn* ‖ *een sy v.* Δ] $\therefore AS = \frac{AR \times CT}{CR}$ $\therefore \frac{AC \times SD}{CR} = \frac{AR \times CT}{CR}$ $\therefore AC \times SD = AR \times CT$	✓ S ✓ S ✓ R ✓ equating (4)
		[13]

TOTAL/*TOTAAL*: 150

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GRADE 12

MATHEMATICS P2

NOVEMBER 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 23 pages.

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QUESTION 1

Truck drivers travel a certain distance and have a rest before travelling further. A driver kept record of the distance he travelled (in km) on 8 trips and the amount of time he rested (in minutes) before he continued his journey. The information is given in the table below.

Distance travelled (in km) (x)	180	200	400	600	170	350	270	300
Amount of rest time (in minutes) (y)	20	25	55	120	15	50	40	45

1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 If a truck driver travelled 550 km, predict the amount of time (in minutes) that he should rest before continuing his journey. (2)

1.3 Write down the correlation coefficient for the data. (1)

1.4 Interpret your answer to QUESTION 1.3. (1)

1.5 At each stop, the truck driver spent money buying food and other refreshments. The amount spent (in rands) is given in the table below.

100	150	130	200	50	180	200	190

1.5.1 Calculate the mean amount of money he spent at each stop. (2)

1.5.2 Calculate the standard deviation for the data. (1)

1.5.3 At how many stops did the driver spend an amount that was less than one standard deviation below the mean? (2)
[12]

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QUESTION 2

At a certain school, the staff committee wanted to determine how many glasses of water the staff members drank during a school day. All teachers present on a specific day were interviewed. The information is shown in the table below.

NUMBER OF GLASSES OF WATER DRANK PER DAY	NUMBER OF STAFF MEMBERS
$0 \leq x < 2$	5
$2 \leq x < 4$	15
$4 \leq x < 6$	13
$6 \leq x < 8$	5
$8 \leq x < 10$	2

2.1 Complete the cumulative frequency column provided in the table in the ANSWER BOOK. (2)

2.2 How many staff members were interviewed? (1)

2.3 How many staff members drank fewer than 6 glasses of water during a school day? (1)

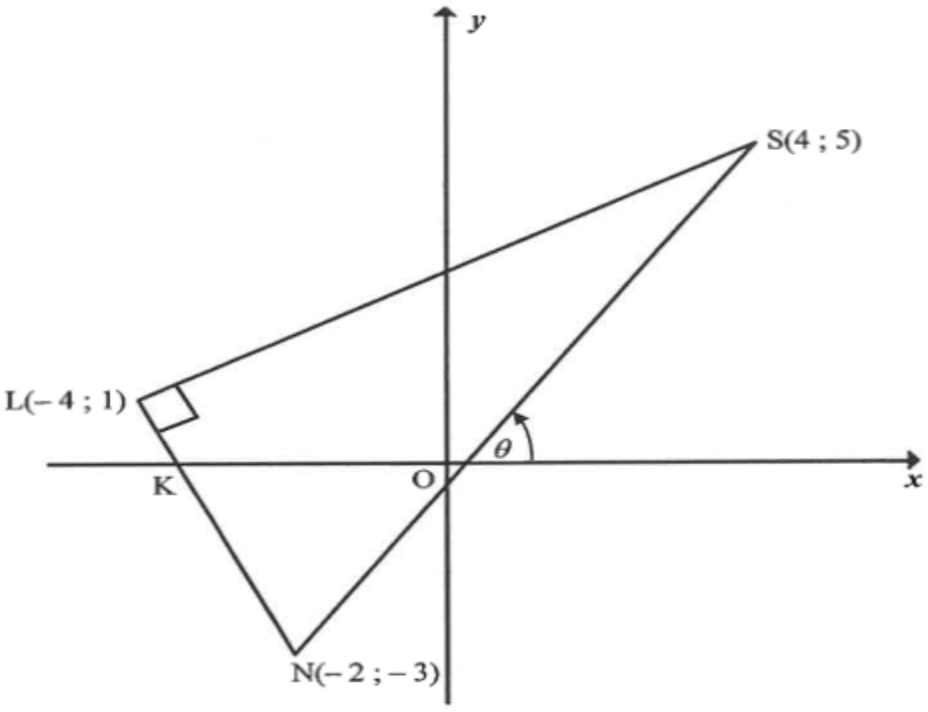
2.4 The staff committee observed that k teachers were absent on the day of the interviews. It was found that half of these k teachers drank from 0 to fewer than 2 (that is $0 \leq x < 2$) glasses of water per day, while the remainder of them drank from 4 to fewer than 6 (that is $4 \leq x < 6$) glasses of water per day. When these k teachers are included in the data, the estimated mean is 4 glasses of water per staff member per day.

How many teachers were absent on the day of the interviews? (4)
[8]

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QUESTION 3

In the figure, L(−4 ; 1), S(4 ; 5) and N(−2 ; −3) are the vertices of a triangle having $S\hat{L}N = 90°$. LN intersects the x-axis at K.



3.1 Calculate the length of SL. Leave your answer in surd form. (2)

3.2 Calculate the gradient of SN. (2)

3.3 Calculate the size of θ, the angle of inclination of SN. (2)

3.4 Calculate the size of $L\hat{N}S$. (3)

3.5 Determine the equation of the line which passes through L and is parallel to SN. Write your answer in the form $y = mx + c$. (3)

3.6 Calculate the area of ΔLSN. (3)

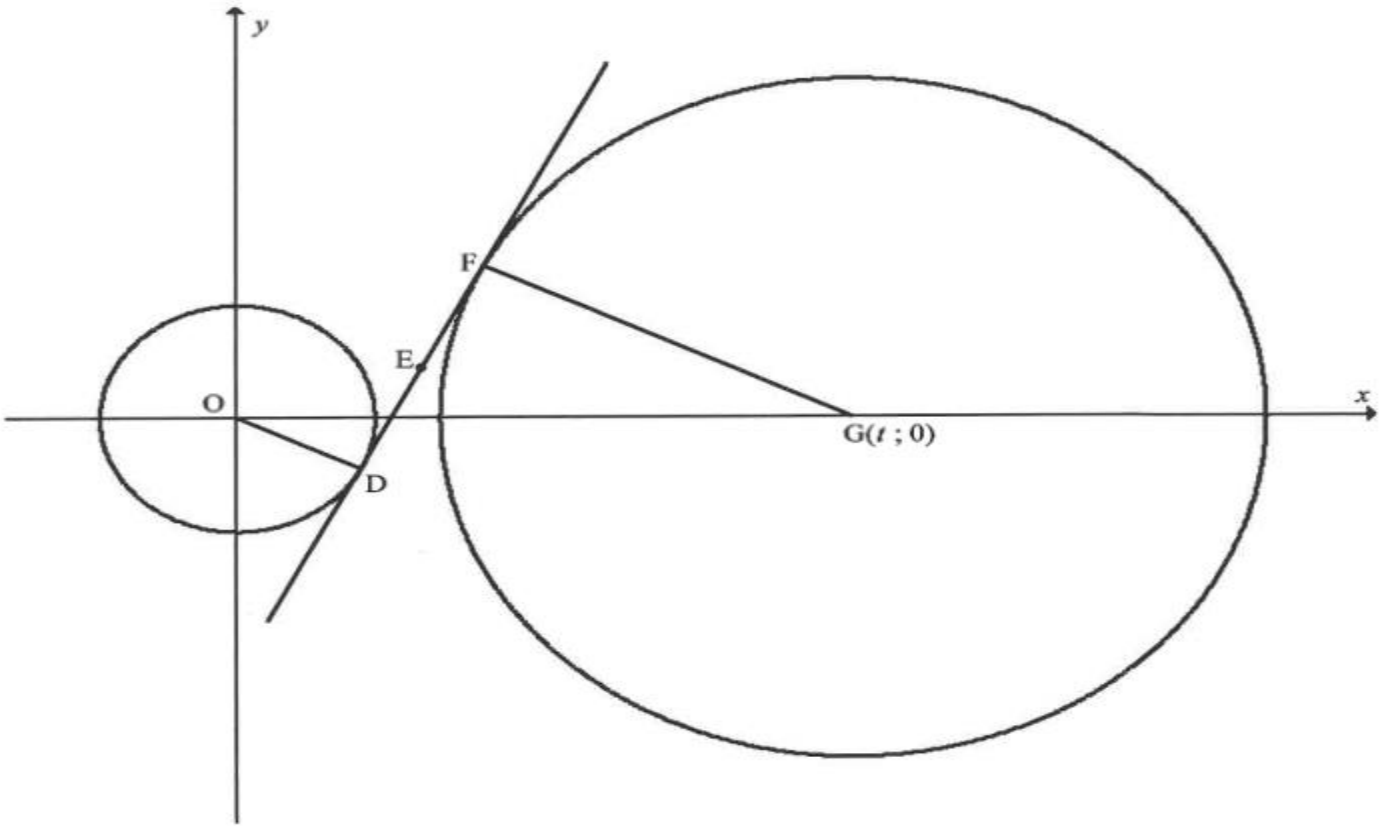
3.7 Calculate the coordinates of point P, which is equidistant from L, S and N. (3)

3.8 Calculate the size of $L\hat{P}S$. (2)
[20]

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QUESTION 4

In the diagram, the circle with centre O has the equation $x^2 + y^2 = 20$. G$(t\ ;\ 0)$ is the centre of the larger circle. A common tangent touches the circles at D and F respectively, such that D$(p\ ;\ -2)$ lies in the 4th quadrant.



4.1 Given that D$(p\ ;\ -2)$ lies on the smaller circle, show that $p = 4$. (2)

4.2 E$(6\ ;\ 2)$ is the midpoint of DF. Determine the coordinates of F. (3)

4.3 Determine the equation of the common tangent, DF, in the form $y = mx + c$. (4)

4.4 Calculate the value of t. Show ALL working. (3)

4.5 Determine the equation of the larger circle in the form $ax^2 + by^2 + cx + dy + e = 0$. (4)

4.6 The smaller circle must be translated by k units along the x-axis to touch the larger circle internally. Calculate the possible values of k. (4)
[20]

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QUESTION 5

5.1 Given: $\sin\beta = \frac{1}{3}$, where $\beta \in (90° ; 270°)$

Without using a calculator, determine each of the following:

5.1.1 $\cos\beta$ (3)

5.1.2 $\sin 2\beta$ (3)

5.1.3 $\cos(450° - \beta)$ (3)

5.2 Given: $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$

5.2.1 Prove that $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x} = 1 - \sin x$ (4)

5.2.2 For what value(s) of x in the interval $x \in [0° ; 360°]$ is $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ undefined? (2)

5.2.3 Write down the minimum value of the function defined by $y = \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ (2)

5.3 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

5.3.1 Use the above identity to deduce that $\sin(A - B) = \sin A\cos B - \cos A\sin B$ (3)

5.3.2 Hence, or otherwise, determine the general solution of the equation $\sin 48°\cos x - \cos 48°\sin x = \cos 2x$ (5)

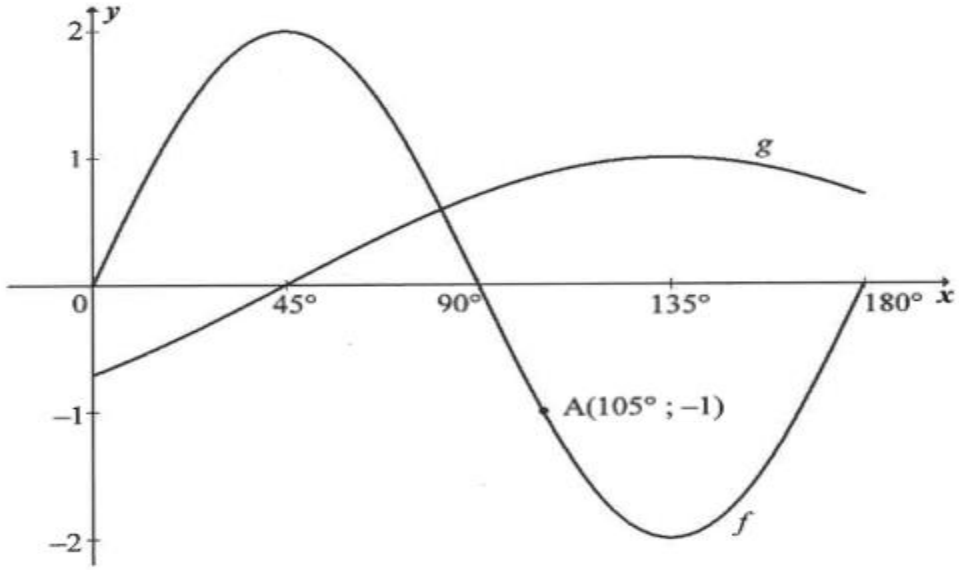
5.4 Simplify $\frac{\sin 3x + \sin x}{\cos 2x + 1}$ to a single trigonometric ratio. (6)

[31]

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QUESTION 6

In the diagram, the graphs of $f(x)=2\sin 2x$ and $g(x)=-\cos(x+45°)$ are drawn for the interval $x\in[0°;180°]$. A(105° ; –1) lies on f.



6.1 Write down the period of f. (1)

6.2 Determine the range of g in the interval $x \in [0° ; 180°]$. (2)

6.3 Determine the values of x, in the interval $x \in [0° ; 180°]$, for which:

6.3.1 $f(x).g(x)>0$ (2)

6.3.2 $f(x)+1\leq 0$ (2)

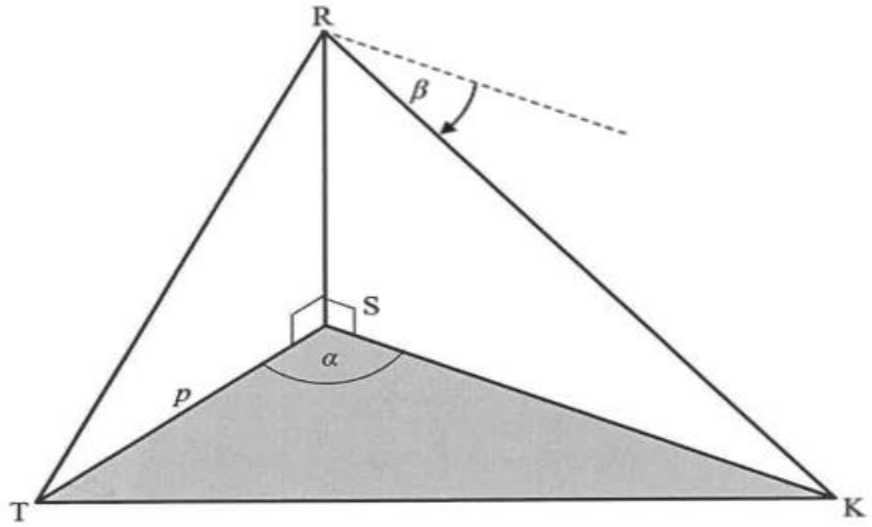
6.4 Another graph p is defined as $p(x)=-f(x)$. D(k ; –1) lies on p. Determine the value(s) of k in the interval $x \in [0° ; 180°]$. (3)

6.5 Graph h is obtained when g is translated 45° to the left. Determine the equation of h. Write your answer in its simplest form. (2)
[12]

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QUESTION 7

In the diagram, S, T and K lie in the same horizontal plane. RS is a vertical tower. The angle of depression from R to K is β. $\hat{TSK} = \alpha$, TS = p metres and the area of ΔSTK is q m^2.



7.1 Determine the length of SK in terms of p, q and α. (2)

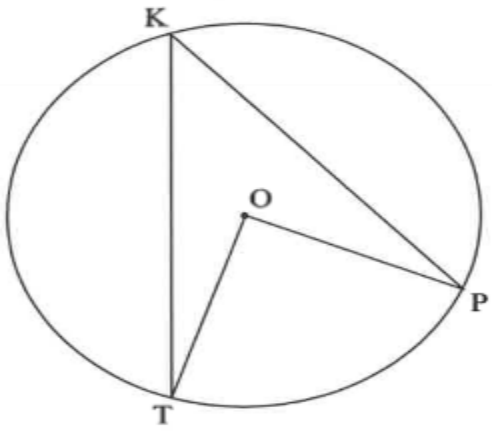
7.2 Show that $RS = \dfrac{2q \tan \beta}{p \sin \alpha}$ (2)

7.3 Calculate the size of α if $\alpha < 90°$ and RS = 70 m, p = 80 m, q = 2 500 m^2 and $\beta = 42°$. (3)
[7]

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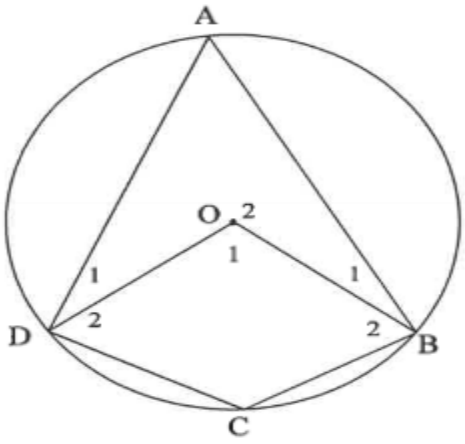
QUESTION 8

8.1 In the diagram, O is the centre of the circle.



Use the diagram above to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that $T\hat{O}P = 2T\hat{K}P$. (5)

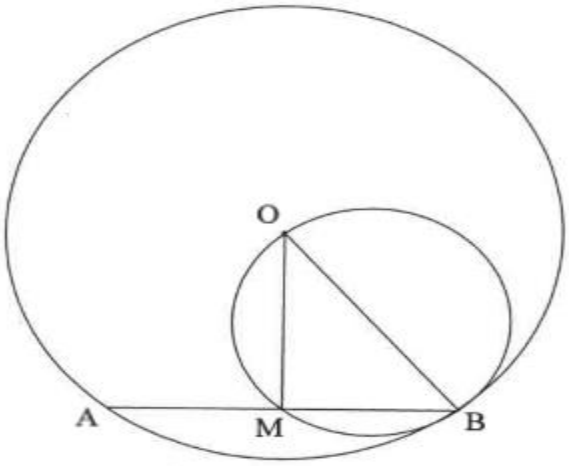
8.2 In the diagram, O is the centre of the circle and ABCD is a cyclic quadrilateral. OB and OD are drawn.



If $\hat{O}_1 = 4x + 100°$ and $\hat{C} = x + 34°$, calculate, giving reasons, the size of x. (5)

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8.3 In the diagram, O is the centre of the larger circle. OB is a diameter of the smaller circle. Chord AB of the larger circle intersects the smaller circle at M and B.



8.3.1 Write down the size of $O\hat{M}B$. Provide a reason. (2)

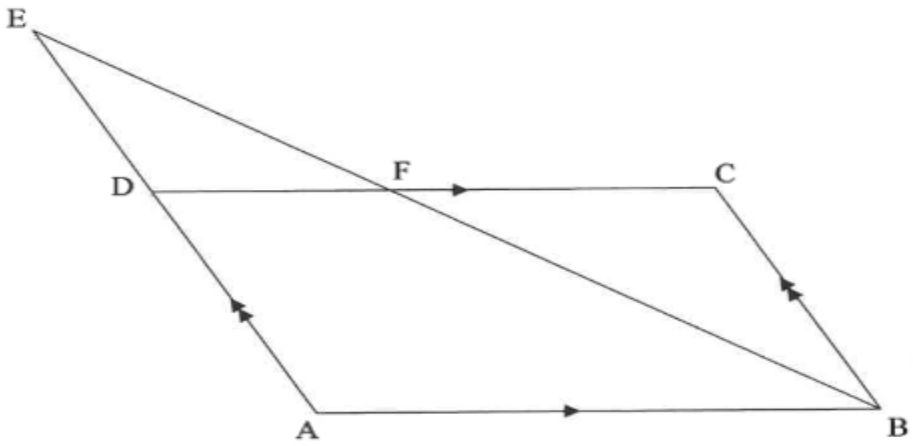
8.3.2 If $AB = \sqrt{300}$ units and $OM = 5$ units, calculate, giving reasons, the length of OB. (4)

[16]

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QUESTION 9

In the diagram, ABCD is a parallelogram with AB = 14 units. AD is produced to E such that AD : DE = 4 : 3. EB intersects DC in F. EB = 21 units.



9.1 Calculate, with reasons, the length of FB. (3)

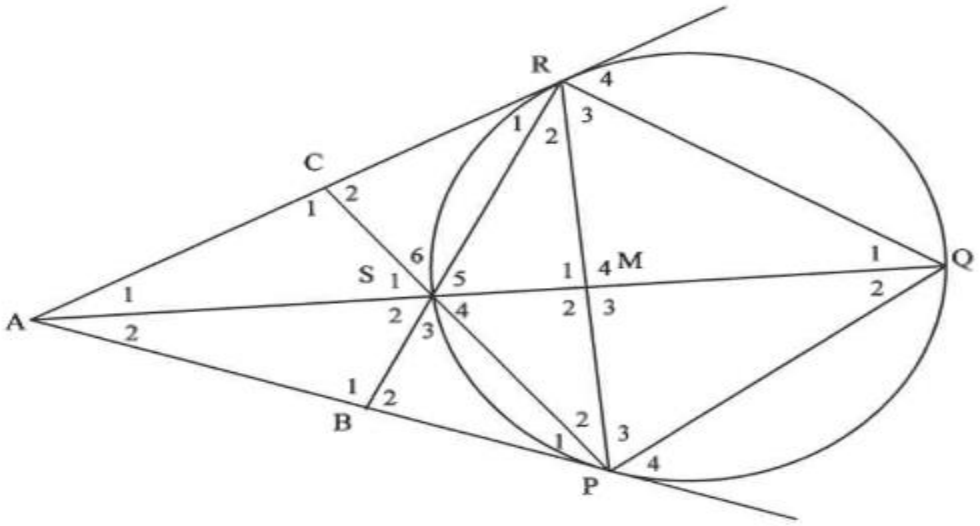
9.2 Prove, with reasons, that $\Delta EDF \;|||\; \Delta EAB$. (3)

9.3 Calculate, with reasons, the length of FC. (3)
[9]

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QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.1 $\hat{S}_3 = \hat{S}_4$ (5)

10.2 SMRC is a cyclic quadrilateral (4)

10.3 RP is a tangent to the circle passing through P, S and A at P (6)
[15]

TOTAL: 150

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 23 pages./
Hierdie nasienriglyne bestaan uit 23 bladsye.

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QUESTION/*VRAAG 1*

1.1	$a=-23,846\ldots$ $b=0,227\ldots$ $\hat{y}=-23,85+0,23x$	✓ $a=-23,846\ldots$ ✓ $b=0,227\ldots$ ✓ equation (3)
1.2	$\hat{y}=-23,85+0,23(550)$ $y=102,65$ **OR** $y=101,02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y=101,02$ (calculator) (2)
1.3	$r=0,98$	✓ $r=0,98$ (1)
1.4	Very strong positive correlation	✓ strong positive (1)

50	100	130	150	180	190	200	200

1.5.1	$\bar{x}=\frac{1200}{8}$ $\bar{x}=150$ **OR** $\bar{x}=150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x}=150$ (2)
1.5.2	$\sigma=50,50$	✓ $\sigma=50,50$ (1)
1.5.3	$\bar{x}-\sigma$ $=150-50,50$ $=99,50$ ∴ 1 stop	✓ calculation of $\bar{x}-\sigma$ ✓ answer (2)
		[12]

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QUESTION/*VRAAG 2*

2.1

Number of glasses of water per day	Number of staff members	Cumulative frequency
$0 \le x < 2$	5	5
$2 \le x < 4$	15	20
$4 \le x < 6$	13	33
$6 \le x < 8$	5	38
$8 \le x < 10$	2	40

✓ 5; 20

✓ 40 (2)

2.2 40 staff members

✓ answer (1)

2.3 33 staff members

✓ answer (1)

2.4

$$\bar{x} = \frac{\left(1\times\left(5+\frac{k}{2}\right)\right)+(3\times15)+\left(5\times\left(13+\frac{k}{2}\right)\right)+(7\times5)+(9\times2)}{40+k} = 4$$

✓ answer from Q2.2 + k

✓ $\left(1\times\left(5+\frac{k}{2}\right)\right)$

✓ $\left(5\times\left(13+\frac{k}{2}\right)\right)$

$$5+\frac{k}{2}+45+65+\frac{5k}{2}+35+18 = 160+4k$$
$$3k+168 = 160+4k$$
$$k = 8$$

✓ answer (4)

OR

$$\bar{x} = \frac{(1\times5)+(15\times3)+(13\times5)+(5\times7)+(2\times9)}{40}$$
$$= 4{,}2$$

✓ 4,2

$$\bar{x}_{old} - \bar{x}_{current} = 4{,}2-4$$
$$= 0{,}2$$

✓ $\bar{x}_{old} - 4$

✓ difference

$$\therefore 0{,}2\times40$$
$$= 8 \text{ teachers}$$

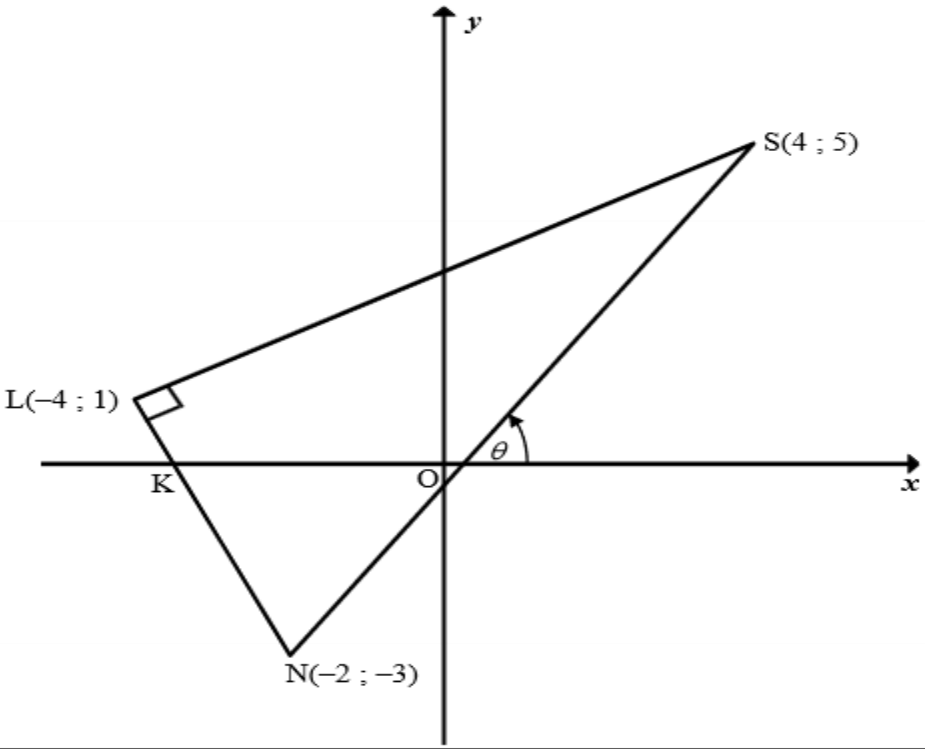
✓ answer (4)

[8]

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QUESTION/*VRAAG 3*



3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4-(-4))^2 + (5-1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8,94$ units	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN} = \frac{5-(-3)}{4-(-2)}$ $m_{SN} = \frac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m = \tan\theta = \frac{4}{3}$ $\theta = 53,13°$	✓ $\tan\theta = m_{SN}$ ✓ answer (2)
3.4	$m_{LN} = \frac{1-(-3)}{-4-(-2)}$ $m_{LN} = -2$ $L\hat{K}O = 116,565...°$ $L\hat{N}S = 116,565...° - 53,13°$ $L\hat{N}S = 63,44°$	✓ $m_{LN} = -2$ ✓ size of $L\hat{K}O$ ✓ answer (3)

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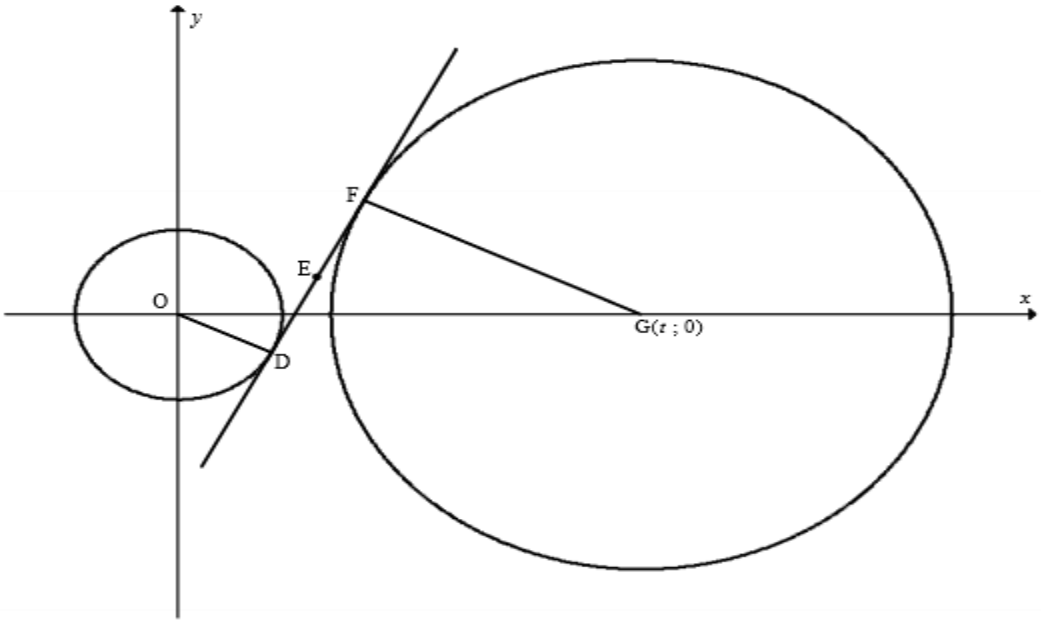
	OR SN = 10 units $\sin L\hat{N}S = \frac{4\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$ **OR** $LN = 2\sqrt{5}$ units $\tan L\hat{N}S = \frac{4\sqrt{5}}{2\sqrt{5}}$ $L\hat{N}S = 63{,}44°$ **OR** SN = 10 units $LN = 2\sqrt{5}$ units $\cos L\hat{N}S = \frac{2\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units ✓ correct trig ratio ✓ answer (3) ✓ $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3) ✓ SN = 10 units and $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
3.5	$m = \frac{4}{3}$ $1 = \frac{4}{3}(-4) + c$ **OR** $y - 1 = \frac{4}{3}(x - (-4))$ $c = \frac{19}{3}$ $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$	✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)
3.6	$SL = 4\sqrt{5}$ $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$ $= 20$ units^2 **OR**	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)

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	SN = 10 units $LN = \sqrt{(-4-(-2))^2+(1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(10)(2\sqrt{5})\sin 63,44°$ $= 20$ units^2	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
3.7	$\hat{L} = 90°$ SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse ∠ in semi-circle] Centre of circle = $P\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ $= P(1;1)$ **OR** Let the coordinates of P be $(a;b)$. Then, PL = PN: $(-4-a)^2+(1-b)^2=(-2-a)^2+(-3-b)^2$ $a-2b=-1$ .....equation 1 If PS = PN, then: $4a+2b=6$ ..... equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$ **OR** If PL = PN, then: $a-2b=-1$ .....equation 1 If PS = PL, then: $2a+b=3$ .....equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$	✓ SN is a diameter of circle S, L, N ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3)
3.8	$L\hat{P}N = \theta = 53,13°$ [alt ∠s; LP ∥ x –axis] $\therefore L\hat{P}S = 126,87°$ **OR** $L\hat{N}S = 63,44°$ $\therefore L\hat{P}S = 126,88°$ [∠ at centre = 2 × ∠ at circumference] **OR** $L\hat{S}N = 26,56°$ [sum of ∠s in Δ] $S\hat{L}P = 26,56°$ [∠s opp equal radii] $\therefore L\hat{P}S = 126,88°$ [sum of ∠s in Δ] **OR** $(4\sqrt{5})^2 = 5^2+5^2-2(5)(5)\cos L\hat{P}S$ $\cos L\hat{P}S = -\frac{3}{5}$ $\therefore L\hat{P}S = 126,87°$	✓ $L\hat{P}N$ ✓ answer (2) ✓ $L\hat{N}S$ ✓ answer (2) ✓ $L\hat{S}N$ ✓ answer (2) ✓ correct substitution into cosine formula ✓ answer (2)
		[20]

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QUESTION/*VRAAG 4*



4.1	$D(p\ ;\ -2)$ $x^2+y^2=20$ $p^2+(-2)^2=20$ $p^2=16$ $p=\pm 4$ $p=4$	✓ substitution of point $D(p\ ;-2)$ ✓ $p^2=16$ (2)
4.2	$\frac{4+x_F}{2}=6$ $\quad$ $\frac{-2+y_F}{2}=2$ $x_F=8$ $\quad$ $y_F=6$ $F(8;6)$ **OR** $x_E-x_D=6-4$ $\quad$ $y_E-y_D=2-(-2)$ $=2$ $\quad$ $=4$ $x_F=6+2=8$ $\quad$ $y_F=2+4=6$ $F(8;6)$	✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

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4.3	$m_{DE} = \frac{-2-2}{4-6}$ $m_{DE} = 2$	✓ correct substitution ✓ gradient of DE, DF or EF
	$-2 = 2(4)+c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y+2 = 2x-8$	✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6)
	$y = 2x-10$ $y = 2x-10$ **OR**	✓ answer (4)
	$m_{OD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2$ [tan ⊥ radius]	✓ correct gradient of OD ✓ gradient of DE
	$-2 = 2(4)+c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y+2 = 2x-8$ $y = 2x-10$ $y = 2x-10$	✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE} = 2$ $\therefore m_{GF} = -\frac{1}{2}$ [tan ⊥ radius]	✓ correct gradient of GF
	$\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$	✓ substitution of F ✓ answer (3)
	OR	
	$y = 2x-10$ $0 = 2x-10$ $x = 5$ A(5 ; 0)	✓ x-intercept of DF
	In ΔAFG: FA ⊥ FG $FA^2 = (6-0)^2+(8-5)^2 = 45$ $FG^2 = (t-8)^2+(0-6)^2$ $= t^2-16t+100$ $GA^2 = (t-5)^2$ $= t^2-10t+25$ $\therefore GA^2 = GF^2+FA^2$ $t^2-10t+25 = t^2-16t+100+45$ $6t = 120$ $t = 20$	✓ substitution into Pythagoras ✓ answer (3)

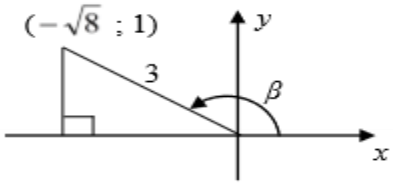
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4.5	F(8;6) G(20 ; 0) $(8-20)^2+(6-0)^2=r^2$ $r^2=180$ $(x-20)^2+y^2=180$ $x^2+y^2-40x+220=0$	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r=2\sqrt{5}$ Larger circle $r=6\sqrt{5}$ G(20 ; 0) $k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ $=11{,}06$ units $=28{,}94$ units **OR** Smaller circle $r=2\sqrt{5}$ $k=2(2\sqrt{5})+20-8\sqrt{5}$ or $k=2(6\sqrt{5})+20-8\sqrt{5}$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ $=11{,}06$ units $=28{,}94$ units **OR** $x^2+y^2-40x+220=0$ $y=0$ $\therefore x^2-40x+220=0$ $\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$ $\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$ $\therefore k=20+4\sqrt{5}$ $\therefore k=20-4\sqrt{5}$ $=11{,}06$ units $=28{,}94$ units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ x-intercepts ✓ method ✓ answer ✓ answer (4)
		[20]

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QUESTION/VRAAG 5

5.1.1	$\sin\beta=\frac{1}{3}$ $\quad\beta\in(90°;270°)$  $x=-\sqrt{8}=-2\sqrt{2}$ $\cos\beta$ $=\frac{-2\sqrt{2}}{3}$ **OR** $\sin\beta=\frac{1}{3}$ $\quad\beta\in(90°;270°)$ $\cos^2\beta=1-\sin^2\beta$ $\cos^2\beta=1-\left(\frac{1}{3}\right)^2$ $\cos^2\beta=\frac{8}{9}$ $\cos\beta=\frac{-\sqrt{8}}{3}$ $=\frac{-2\sqrt{2}}{3}$	✓ $x^2+y^2=r^2$ ✓ $x=-2\sqrt{2}$ ✓ answer (3) ✓ square identity ✓ $\cos^2\beta$ ✓ answer (3)
5.1.2	$\sin 2\beta$ $=2\sin\beta\cos\beta$ $=2\left(\frac{1}{3}\right)\left(\frac{-\sqrt{8}}{3}\right)$ $=\frac{-2\sqrt{8}}{9}$ **OR** $2\left(\frac{-2\sqrt{2}}{9}\right)$ $=\frac{-4\sqrt{2}}{9}$	✓ double angle ✓ substitution ✓ answer (3)
5.1.3	$\cos(450°-\beta)$ $=\cos(90°-\beta)$ $=\sin\beta$ $=\frac{1}{3}$ **OR**	✓ $\cos(90°-\beta)$ ✓ co-ratio ✓ answer (3)

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	$\cos(450° - \beta)$ $= \cos 450° \cos \beta + \sin 450° \sin \beta$ $= \cos 90° \cos \beta + \sin 90° \sin \beta$ $= \sin \beta$ $= \frac{1}{3}$	✓ expansion ✓ reduction ✓ answer (3)
5.2.1	$\text{LHS} = \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x(\cos^2 x + \sin^2 x)}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ $= \text{RHS}$	✓ factors ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\cos^2 x = 1 - \sin^2 x$ ✓ factors (4)
	OR	
	$\text{LHS} = \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + (1-\cos^2 x)\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x - \cos^4 x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ $= \text{RHS}$	✓ $\sin^2 x = 1 - \cos^2 x$ ✓ expansion ✓ $\cos^2 x = 1 - \sin^2 x$ ✓ factors (4)
	OR	
	$\text{RHS} = 1-\sin x$ $= (1-\sin x) \times \frac{1+\sin x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x(\sin^2 x + \cos^2 x)}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x.\sin^2 x}{1+\sin x}$ $= \text{LHS}$	✓ $\times \frac{1+\sin x}{1+\sin x}$ ✓ product ✓ $1 - \sin^2 x = \cos^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$ (4)

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5.2.2	$\sin x+1=0$ $\sin x=-1$ ref. $\angle=90°$ $x=270°$	✓ $\sin x+1=0$ ✓ $x=270°$ (2)
5.2.3	$y=\dfrac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ $y=1-\sin x$ $\therefore$ Minimum = 0	✓✓ Minimum = 0 (2)
5.3.1	$\sin(A-B)$ $=\cos[90°-(A-B)]$ $=\cos[(90°-A)-(-B)]$ $=\cos(90°-A)\cos(-B)+\sin(90°-A)\sin(-B)$ $=\sin A\cos B+\cos A(-\sin B)$ $=\sin A\cos B-\cos A\sin B$ **OR** $\sin(A-B)$ $=\cos[90°-(A-B)]$ $=\cos[(90°+B)-A]$ $=\cos(90°+B)\cos A+\sin(90°+B)\sin A$ $=-\sin B\cos A+\cos B\sin A$ $=\sin A\cos B-\cos A\sin B$	✓ co-ratio ✓ compound angle ✓ reduction (3) ✓ co-ratio ✓ compound angle ✓ reduction (3)
5.3.2	$\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin(48°-x)=\cos 2x$ $\sin(48°-x)=\sin(90°-2x)$ $48°-x=90°-2x+k.360°$ or $48°-x=180°-(90°-2x)+k.360°$ $x=42°+k.360°$ $\quad -3x=42°+k.360°$ $x=-14°-k.120°$; $k\in Z$ **OR** $\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin(48°-x)=\cos 2x$ $\cos(90°-48°+x)=\cos 2x$ $\cos(42°+x)=\cos 2x$ $42°+x=2x+k.360°$ or $42°+x=360°-2x+k.360°$ $-x=-42°+k.360°$ $\quad 3x=318°+k.360°$ $x=42°-k.360°$ $\quad x=106°+k.120°$; $k\in Z$	✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5) ✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5)

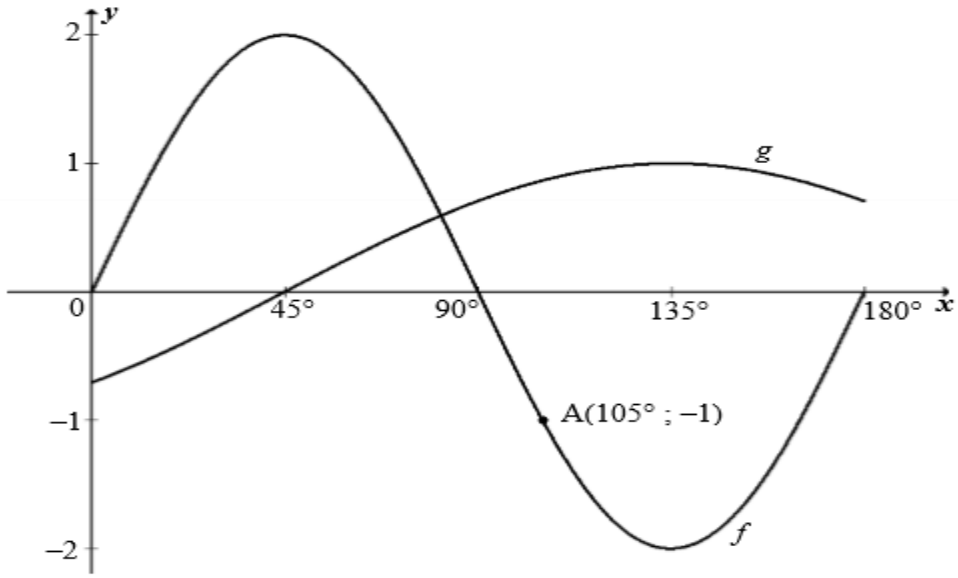
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5.4	$\frac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\frac{\sin(2x+x)+\sin(2x-x)}{\cos 2x+1}$	✓ $3x=(2x+x)$
	$=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin 2x\cos x-\cos 2x\sin x}{2\cos^2 x-1+1}$	✓ expansion ✓ double angle of cos2x
	$=\frac{2\sin 2x\cos x}{2\cos^2 x}$	✓ simplification
	$=\frac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\frac{4\sin x\cos^2 x}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
	OR	
	$\frac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\frac{\sin(2x+x)+\sin x}{2\cos^2 x-1+1}$	✓ $3x=(2x+x)$ ✓ double angle of cos2x
	$=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ expansion
	$=\frac{2\sin x\cos x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\frac{\sin x(2\cos^2 x+\cos 2x+1)}{2\cos^2 x}$	✓ common factor
	$=\frac{\sin x(2\cos^2 x+2\cos^2 x-1+1)}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
		[31]

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QUESTION/*VRAAG 6*

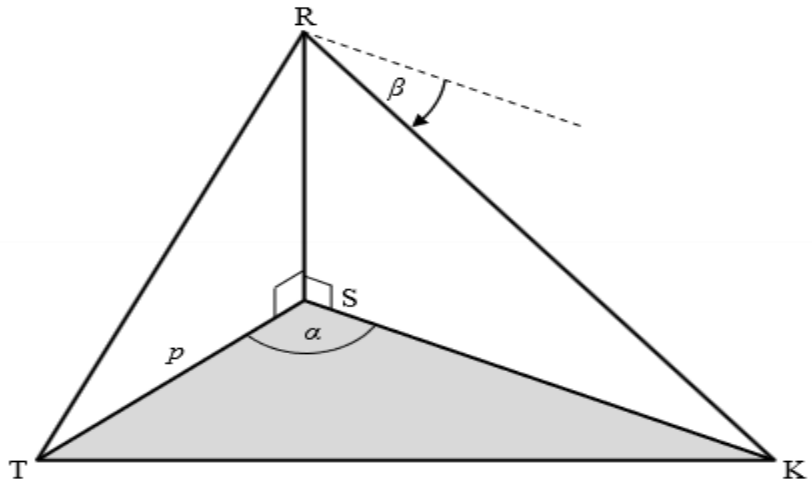


6.1	Period = 180°	✓ 180° (1)
6.2	$y \in \left[-\frac{\sqrt{2}}{2}\,;1\right]$ **OR** $y \in [-0,71;1]$ **OR** $-\frac{\sqrt{2}}{2} \le y \le 1$	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y \in \left[-\frac{\sqrt{2}}{2}\,;1\right]$ (2)
6.3.1	$x \in (45° ; 90°)$ **OR** $45° < x < 90°$	✓✓ $x \in (45° ; 90°)$ (2)
6.3.2	$f(x)+1 \le 0$ $f(x) \le -1$ $x \in [105° ; 165°]$ **OR** $105° \le x \le 165°$	✓✓ $x \in [105° ; 165°]$ (2)
6.4	$p(x) = -2\sin 2x$ $-2\sin 2x = -1$ **OR** $2\sin 2x = 1$ $k = 15°$ or $k = 75°$	✓ reading off $f(x) = 1$ or $-f(x) = -1$ ✓ 15° ✓ 75° (3)
6.5	$g(x) = -\cos(x+45°)$ $h(x) = -\cos(x+90°)$ $h(x) = \sin x$	✓ $-\cos(x+90°)$ ✓ answer (2)
		[12]

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QUESTION/*VRAAG 7*

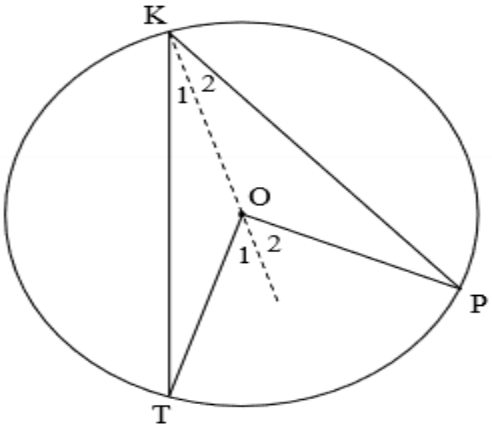


7.1	Area $\Delta STK = \frac{1}{2}p(SK)\sin\alpha$ $q = \frac{1}{2}p(SK)\sin\alpha$ $SK = \frac{q}{\frac{1}{2}p\sin\alpha}$ $= \frac{2q}{p\sin\alpha}$	✓ substitution into the correct formula ✓ answer (2)
7.2	$R\hat{K}S = \beta$ $\frac{RS}{SK} = \tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$ **OR** $\frac{RS}{\sin\beta} = \frac{SK}{\sin(90° - \beta)}$ $RS\cos\beta = SK\sin\beta$ $RS = SK\tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$	✓ $R\hat{K}S = \beta$ ✓ correct trig ratio (2) ✓ $R\hat{K}S = \beta$ ✓ $\tan\beta = \frac{\sin\beta}{\cos\beta}$ (2)
7.3	$70 = \frac{2(2500)\tan 42°}{80\sin\alpha}$ $\sin\alpha = \frac{25}{28}\tan 42°$ **OR** $\sin\alpha = 0,80...$ $\alpha = 53,51°$	✓ correct substitution of values into RS ✓ value of $\sin\alpha$ ✓ answer (3)
		[7]

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QUESTION/*VRAAG 8*

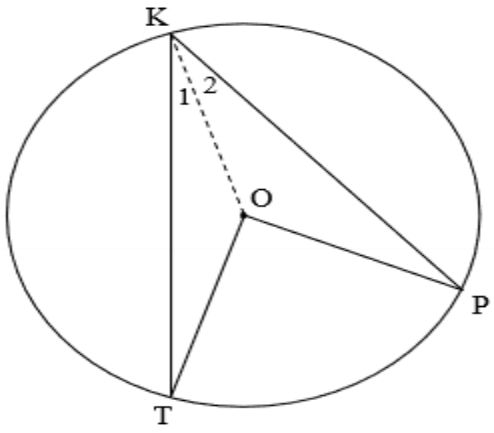
8.1



8.1	Construction: Draw KO produced	✓ construction
	$\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext ∠ of Δ]	
	But $\hat{K}_1 = \hat{T}$ [∠s opp equal sides]	✓ S / R
	$\therefore \hat{O}_1 = 2\hat{K}_1$	✓ S
	$\hat{O}_2 = \hat{K}_2 + P$ [ext ∠ of Δ]	
	But $\hat{K}_2 = P$ [∠s opp equal sides]	
	$\therefore \hat{O}_2 = 2\hat{K}_2$	✓ S
	$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$ $= 2(\hat{K}_1 + \hat{K}_2)$	✓ S
	$\therefore T\hat{O}P = 2T\hat{K}P$	(5)
	OR	

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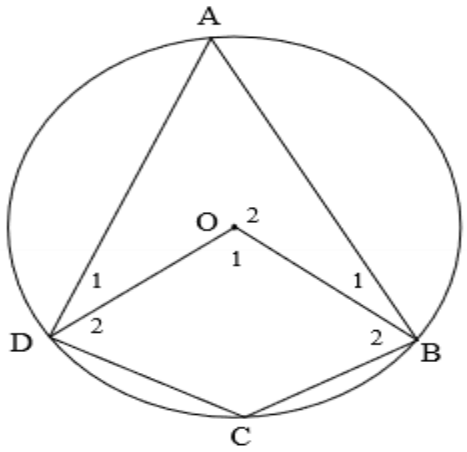
8.1



8.1	Construction: Draw KO		✓ construction
	$\hat{T} = \hat{K}_1$	[$\angle$s opp. equal sides]	✓ S / R
	$\therefore K\hat{O}T = 180° - 2\hat{K}_1$	[sum of $\angle$s of ΔKOT]	✓ S
	$\hat{P} = \hat{K}_2$	[$\angle$s opp. equal sides]	
	$\therefore K\hat{O}P = 180° - 2\hat{K}_2$	[sum of $\angle$s of ΔKOP]	✓ S
	$T\hat{O}P = 360° - (K\hat{O}T + K\hat{O}P)$	[$\angle$s around a point]	
	$= 360° - (180° - 2\hat{K}_1 + 180° - 2\hat{K}_2)$		✓ S
	$= 2\hat{K}_1 + 2\hat{K}_2$		
	$= 2(\hat{K}_1 + \hat{K}_2)$		
	$\therefore T\hat{O}P = 2T\hat{K}P$		(5)

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8.2

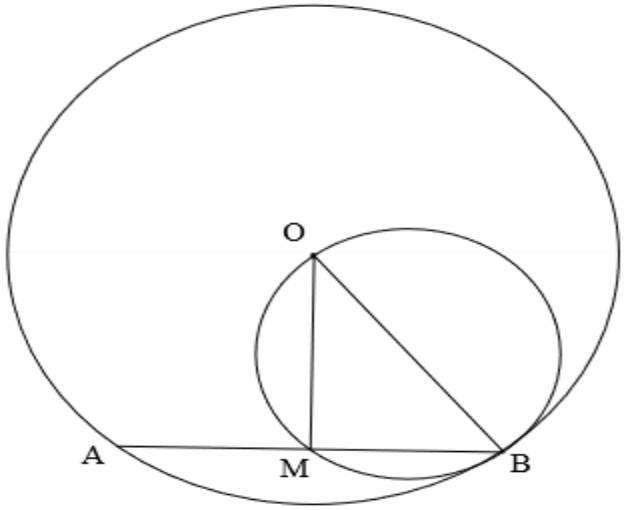


8.2	$\hat{O}_1 = 4x + 100°$	[given]	
	$\therefore \hat{A} = 2x + 50°$	[∠ at centre = 2 ×∠ at circumference]	✓ S ✓ R
	$x + 34° + 2x + 50° = 180°$	[opp ∠s of cyclic quad]	✓ S ✓ R
	$3x = 96°$		
	$x = 32°$		✓ answer (5)
	OR		
	$\hat{O}_2 = 2x + 68°$	[∠ at centre = 2 ×∠ at circumference]	✓ S ✓ R
	$4x + 100° + 2x + 68° = 360°$	[∠s round a pt]	✓ S ✓ R
	$6x = 192°$		
	$x = 32°$		✓ answer (5)
	OR		
	$\hat{O}_2 = -4x + 260°$	[∠s round a pt]	✓ S ✓ R
	$2\hat{C} = -4x + 260°$	[∠ at centre = 2 ×∠ at circumference]	✓ S ✓ R
	$\hat{C} = -2x + 130°$		
	$x + 34° = -2x + 130°$		
	$3x = 96°$		
	$x = 32°$		✓ answer (5)

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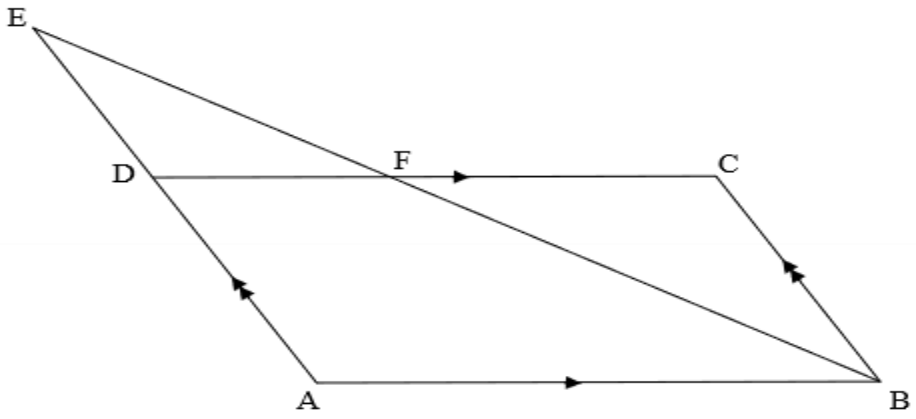
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8.3



8.3.1	$O\hat{M}B = 90°$ [$\angle$ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre $\perp$ to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$ OB = 10 units	✓ S ✓ R ✓ S ✓ answer (4)
		[16]

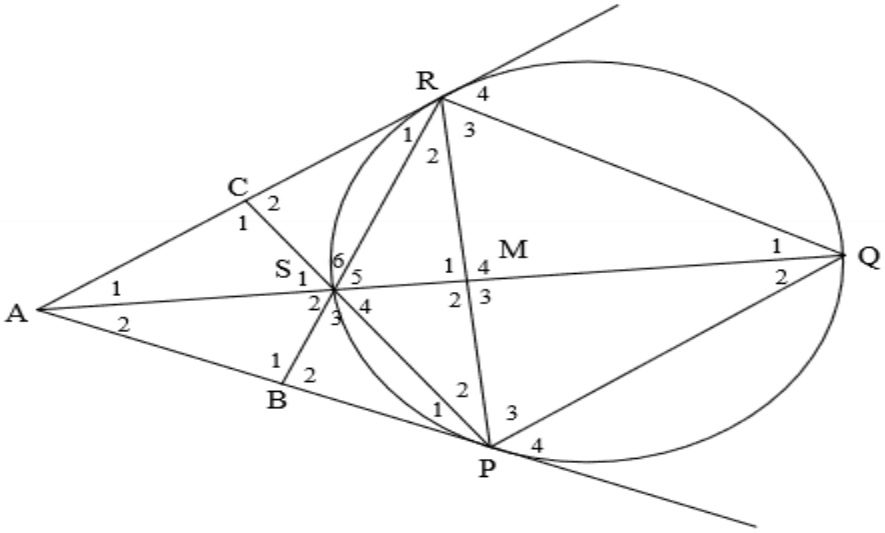
QUESTION/*VRAAG 9*



9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ∥ AB] **OR** [line ∥ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB: $\hat{E}$ is common $E\hat{D}F=\hat{A}$ [corresp ∠s; EA ∥ CB] $E\hat{F}D=E\hat{B}A$ [corresp ∠s; DC ∥ AB] ΔEDF ||| ΔEAB [∠;∠;∠]	✓ S ✓ S/R ✓ S **OR** R (3)
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [||| Δs] $DF=\frac{3p\times 14}{7p}$ DF = 6 units FC = 8 units [DC = AB = 14 units; opp sides of ||^m] **OR** ΔEDF ||| ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [||| Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of ||^m] $3FC=56-4FC$ $FC=8$	✓ S ✓ DF = 6 ✓ FC = 14 − DF (3) ✓ ΔEDF ||| ΔBCF ✓ $\frac{3}{4}=\frac{14-FC}{FC}$ ✓ answer (3)
		[9]

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QUESTION/*VRAAG 10*



10.1	$\hat{S}_3 = P\hat{Q}R$	[ext ∠ of cyclic quad]	✓ S ✓ R
	$\hat{R}_3 = P\hat{Q}R$	[∠s opp equal sides]	✓ S / R
	$\therefore \hat{S}_3 = \hat{R}_3$		
	But $\hat{S}_4 = \hat{R}_3$	[∠s in the same seg]	✓ S ✓ R
	$\therefore \hat{S}_3 = \hat{S}_4$		(5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$	[tan chord theorem]	✓ S ✓ R
	$\hat{S}_4 = P\hat{Q}R$	[proved in 10.1]	
	$\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$		✓ S
	SMRC is a cyclic quad	[converse ext ∠ of cyclic quad]	✓ R
			(4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$	[ext ∠ of Δ]	✓ S ✓ R
	$\hat{S}_4 = \hat{P}_1 + \hat{A}_2$	[ext ∠ of Δ]	✓ S
	$\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$		
	But $\hat{P}_1 = \hat{R}_2$	[tan chord theorem]	✓ S ✓ R
	$\therefore \hat{P}_2 = \hat{A}_2$		
	RP is a tangent to the circle	[converse tan chord theorem]	✓ R
		OR	
		[∠ between line and chord]	
		OR	
		[converse alt seg theorem]	(6)
	OR		

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	In ΔMSP and ΔMPA		
	$\hat{M}_2$ is common		✓ S
	$AR = AP$	[tans from same point]	✓ S / R
	$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$	[$\angle$s opp equal sides]	✓ S
	$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$	[proved in 10.2]	
	$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$		✓ S
	$\therefore \hat{P}_2 = \hat{A}_2$	[sum of $\angle$s in Δ]	✓ S
	RP is a tangent to the circle	[converse tan chord theorem]	✓ R
			(6)
			[15]

TOTAL/TOTAAL: 150

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